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Assignment 7 Problem 5.1.8
http://calclab.math.tamu.edu/~b-young/5_1_8.pdf

Problem Statement: Show that the solution space of of $x + 2y^2 = 0$ is *not* a subspace of $\mathcal{R}^2$

Solution: In order to show that the solution space is not a subspace, it must be shown that that the solutions are not closed under addition and scalar multiplication. This is easier if $\vec{v}$ is defined parametrically. That is:

$$\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix}_{y=t} = (\, -2t^2, t \,)$$

Now two arbitrary vectors in the subset, $\vec{v}_1$ and $\vec{v}_2$ can be added together:

$$\vec{v}_1 + \vec{v}_2 = \begin{pmatrix} -2{t_1}^2 \\ t_1 \end{pmatrix} + \begin{pmatrix} -2{t_2}^2 \\ t_2 \end{pmatrix} = \begin{pmatrix} -2({t_1}^2 + {t_2}^2) \\ t_1 + t_2 \end{pmatrix}$$

Then the values are placed back into the original equation $x + 2y^2 = 0$:

$$-2({t_1}^2 + {t_2}^2) + 2t_1 + {t_2}^2 = 0$$

$$-2{t_1}^2 - 2{t_2}^2 + 2{t_1}^2 + 4t_1t_2 + 2{t_2}^2 = 0$$

$$4t_1t_2 = 0$$

This equation is true only where t_1 or t_2 is equal to zero, and thus not *any* arbitrary vector. This means that any two vectors in the subset where both $t_1 \neq 0$ and $t_2 \neq 0$ will not sum to another vector, thus making the solution space open for addition, and showing it not be a subspace.