Brian Young b-young@calclab.math.tamu.edu
Assignment 8 Problem 5.3.7
http://calclab.math.tamu.edu/~b-young/5_3_7.pdf

Problem Statement: Find the general solution of $\frac{d^4y}{dx^4} = \sin(3x)$

Using the second superposition principle, the general solution of the equation can be found by summing a solution to the equation and the general solution to the corresponding homogenous equation.

First find a solution to the equation:

$$\frac{d^4y}{dx^4} = \sin(3x) \longrightarrow d^4y = \sin(3x)dx^4$$

$$y = \int\int\int\int \sin(3x)dx^4$$

$$y = \frac{1}{81}\sin(3x)$$

This solution can be checked easily by taking the fourth derivative.

The solution to the corresponding homogenous equation, $\frac{d^4y}{dx^4} = 0$ is a simple problem. A third order polynomial is the general solution for this equation, that is:

$$y = C_1x^3 + C_2x^2 + C_3x + C_4$$

The general solution to this problem is therefore simply the sum of these two equations. That is:

$$y = \frac{1}{81}\sin(3x) + C_1x^3 + C_2x^2 + C_3x + C_4$$

This solution can also easily be confirmed by taking the derivatives. Any other solution to the original equation will result in the same final equation, since the constant terms created through integration will create only polynomials of degree less than or equal to three.