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Assignment (week) 7 Problem: 4.5.12
URL: http://people.tamu.edu/~x0z0680/hw7a(revised).pdf Revised Version

4.5.12 Find the matrix of L with respect t to the primed basis for $\Re^3$ given in Exercise 4.4.18 if its matrix with respect to the unprimed basis in that exercise is

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$$

Solution:

Since computer used is expected for this problem, many steps involving heavy calculations are omitted (i.e. use maple to find the inverse matrix).

To solve this problem, I need to solve 4.4.18 first:

Exercise 4.4.18:

The set $\{\vec{b_1}, \vec{b_2}, \vec{b_3}\}$ is a basis, and so is the set $\{\vec{b_1'}, \vec{b_2'}, \vec{b_3'}\}$. Find the formulas of an arbitrary vector with respect to the primed basis into coordinates with respect to the unprimed basis.

$$\vec{b_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \vec{b_2} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \vec{b_3} = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}; \vec{b_1'} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \vec{b_2'} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}, \vec{b_3'} = \begin{pmatrix} -6 \\ 1 \\ 1 \end{pmatrix}$$

Solution to 4.4.18:

Put $\{\vec{b_1}, \vec{b_2}, \vec{b_3}\}$ into one big matrix B, where $B = \begin{pmatrix} 1 & 2 & -1 \\ 1 & -1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$

And then put $\{\vec{b_1'}, \vec{b_2'}, \vec{b_3'}\}$ into one big matrix B', where $B' = \begin{pmatrix} 1 & 1 & -6 \\ -1 & 2 & 1 \\ 1 & 5 & 1 \end{pmatrix}$

Because

$$B\begin{pmatrix} A \\ B \\ C \end{pmatrix} = B'\begin{pmatrix} A' \\ B' \\ C' \end{pmatrix} \quad \Rightarrow \quad C = B^{-1}B'$$

$$C = \begin{pmatrix} 1 & 2 & -1 \\ 1 & -1 & 3 \\ 1 & 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 1 & -6 \\ -1 & 2 & 1 \\ 1 & 5 & 1 \end{pmatrix} = \begin{pmatrix} 2 & \frac{3}{2} & \frac{-5}{2} \\ -1 & -1 & 2 \\ -1 & -\frac{1}{2} & \frac{3}{2} \end{pmatrix} \begin{pmatrix} 1 & 1 & -6 \\ -1 & 2 & 1 \\ 1 & 5 & 1 \end{pmatrix} = \begin{pmatrix} -2 & -7.5 & -13 \\ 2 & 7 & 7 \\ 1 & 5.5 & 7 \end{pmatrix}$$

Now, back to problem 4.5.12

If B is the coordinate transformation from b to natural; and B' is the coordinate transformation from b' to natural (both of which are transparent); and A is the original matrix, then the new matrix is $B'^{(-1)}BAB^{-1}B'$.

Similarly:

$L = C^{-1}AC$ The order of the factors is the most essential part of this problem.

$\Rightarrow$

$$L = \begin{pmatrix} -2 & -7.5 & -13 \\ 2 & 7 & 7 \\ 1 & 5.5 & 7 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} -2 & -7.5 & -13 \\ 2 & 7 & 7 \\ 1 & 5.5 & 7 \end{pmatrix} = \begin{pmatrix} -10.15 & -47.80 & -35.02 \\ 6.10 & 28.54 & 22.68 \\ -2.34 & -10.88 & -9.39 \end{pmatrix}$$

$$L = \begin{pmatrix} -10.15 & -47.80 & -35.02 \\ 6.10 & 28.54 & 22.68 \\ -2.34 & -10.88 & -9.39 \end{pmatrix}$$