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Assignment (week) 12 Problem: 7.6.14
URL: http://people.tamu.edu/~x0z0680/hw12a.pdf

**7.6.14** Do Example 3 of Sec. 7.4 by the parametric method.

Example 3: Evaluate $\iint_S \vec{B}\cdot d\vec{S}$ when $\vec{B}(\vec{r}) = y\hat{i}$ and S is the triangle with corners at (0,0,1), (1,0,0), and (1,1,0).

**Solution:**

$$\iint_S \vec{B}\cdot d\vec{S} = \iint_S [B_x dydz + B_y dzdx + B_z dxdy] = \iint_S (ydydz)$$

We can use x and y as coordinates on the surface (x and z can be used equally well). The plane (in which the triangle lies) sits at a $45°$ angle, and we can regard the plane as a graph of $z = 1 - x$.

With $dz = -dx$ and

According to the formula:

$$I = \iint_{proj\_S} [-A_x \frac{\partial \psi}{\partial x} - A_y \frac{\partial \psi}{\partial y} + A_z]dxdy$$

$$\iint_x (-y\cdot(-1)+0)dxdy = \int_0^x dy \int_0^1 dx(y) = \int_0^1 \int_0^x (y)dydx = \frac{1}{6}$$

$$\Rightarrow$$

$$\iint_S \vec{B}\cdot d\vec{S} = \frac{1}{6} \text{ when } \vec{B}(\vec{r}) = y\hat{i}$$

Check: The solution we obtained above agrees with the solution from example 3 in 7.4