12.2.2

Question:

Solve

$$\frac{\partial w}{\partial t} - 3\frac{\partial w}{\partial x} = 0 \, with \, w(x,0) = \cos x.$$

Solution:

$$\frac{dx}{dt} = c = -3$$

which indicates that:

$$\frac{dw}{dt} = 0$$

$$\int dw = 0$$

This indicates that w is constant along the characteristic

Then $\frac{dx}{dt} = -3$ is integrated:

$$\int dx = -3 \int dt$$

$$x = -3t + x_o$$

$$x_o = x + 3t$$

Then because $w(x,t) = P(x - ct)$the solution is:

$$w(x,t) = cos(x + 3t)$$

12.2.3

Question:

Solve

$$\frac{\partial w}{\partial t} + 4\frac{\partial w}{\partial x} = 0 \, with \, w(0,t) = \sin 3t.$$

Solution:

$$\frac{dx}{dt} = c = 4$$

which indicates that:

$$\frac{dw}{dt} = 0$$

$$\int dw = 0$$

This indicates that w is constant along the characteristic

Then $\frac{dx}{dt} = 4$ is integrated:

$$\int dx = 4 \int dt$$

$$x = 4t + x_o$$

$$x_o = x - 4t$$

Next using $w(x,t) = P(x - ct)$we find:

$$w(x,0) = P(x - 4t)$$

$$P(-4t) = \sin 3t$$

$$P(y) = \sin(-\frac{3}{4}y) = -\sin\frac{3}{4}y$$

Finally, we put this back into $w(x,t) = P(x - ct)$, and the solution is:

$$w(x,t) = -sin[\frac{3}{4}(x - 4t)] = -sin[\frac{3}{4}x - 3t]$$