Gary Chen superchen
Assignment:6 10.2.2 and 10.3.6
People.tamu.edu/~superchen/HW6

10.2.2

Question

If c(-w) = $\bar{c}$(w) (see Exercise 10.2.1), show that u(x,t) given by (10.2.11) is real.

Solution:

Form 10.2.1, we get that c(w) = { $\frac{1}{2}$ [A(-w) - iB(-w)], w<0

{ $\frac{1}{2}$ [A(w) + iB(w)], w>0

c(-w) = $\frac{1}{2}$ [A(w) – iB(w)]

from (10.2.11) and (10.2.12) we get

$u(x,t) = \int_{-\infty}^{\infty} c(w) e^{-iwx} e^{-kw^2 t} dw$ and

for c(-w), we get u(x,t) = $\int_{-\infty}^{\infty} c(w) e^{iwx} e^{-kw^2 t} dw$

to find the conjugate of u(x,t), we get

$\bar{u}(x,t) = \int_{-\infty}^{\infty} \bar{c}(w) e^{-iwx} e^{-kw^2 t} dw$

since c(-w) = $\bar{c}$(w), then we get u(x,t) = $\bar{u}(x,t)$

so the function u(x,t) is real, since if z = x + iy

and $\bar{z}$ = x – iy

if z = $\bar{z}$, then y must be equals to zero which makes z is real

Gary Chen superchen
Assignment:6 10.2.2 and 10.3.6
People.tamu.edu/~superchen/HW6

10.3.6

Question

If

$$f(x) = \{ 0 \quad |x|>a$$
$$= \{1 \quad |x|<a,$$

Determine the Fourier transform of f(x). [The answer is given in the table of Fourier transforms in Section 10.4.4.]

Solution:

Form equation (10.3.6), we can obtain the F(w), the Fourier transform of f(x) as

$$F(w) \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\bar{x})e^{iw\bar{x}}d\bar{x},$$

So $F(w) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\bar{x})e^{iw\bar{x}}d\bar{x}$

$$= \frac{1}{2\pi}\int_{-a}^{a} e^{iwx}dx = \frac{e^{iwx}}{2\pi iw}\Big|_{-a}^{a}$$

$$= \frac{1}{2\pi iw}(e^{iwa} - e^{-iwa})$$

Since the complex transformation equation, $\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$

So $\sin(wa) = \frac{1}{2i}(e^{iwa} - e^{-iwa})$

So $F(w) = \frac{1}{\pi w}\sin(wa)$