Matthew Gidden mgidden http://yojox.com/mgidden/math412/12.5.3.pdf
Assignment 2 12.5.3 (a,c)

The general wave equation is held on the spatial domain from 0 to 10 and subject to initial conditions (1) and boundary conditions (2); for simplicity, we will assume L = 10.

$$\frac{\partial u}{\partial t}(x,0) = g(x) = 0, \quad u(x,0) = f(x) = \begin{cases} 1 & 4 < x < 5 \\ 0 & otherwise \end{cases} \tag{1}$$

$$u(0,t) = 0, \quad \frac{\partial u}{\partial x}(L,t) = 0 \tag{2}$$

This problem can initially be interpreted using the method of characteristics to show:

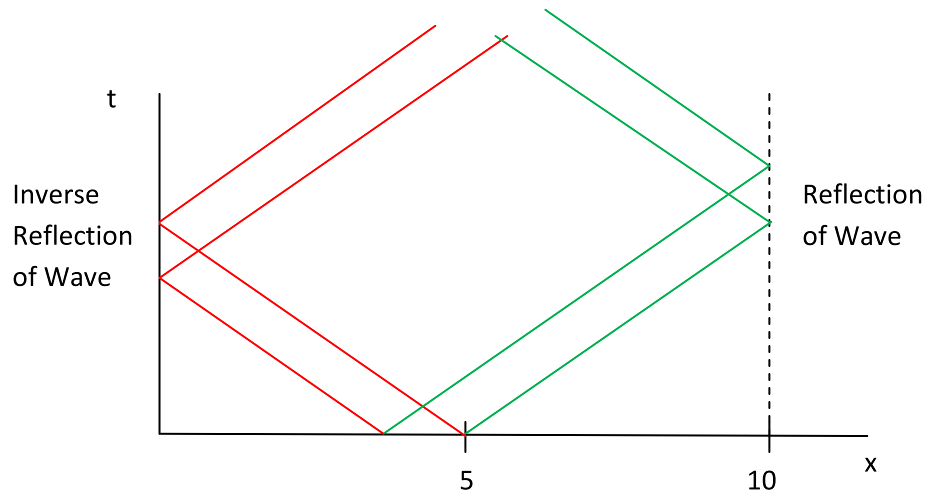

Figure 1: u(x,t) using the method of characteristics

Because of the Dirichlet and Neumann boundary conditions, we must create an odd extension of f around x=0 and an even extension of f around x=L which then imply the following conditions on f(x):

$$f(-x) = -f(x) \tag{3}$$

$$f(2L - x) = f(x) \tag{4}$$

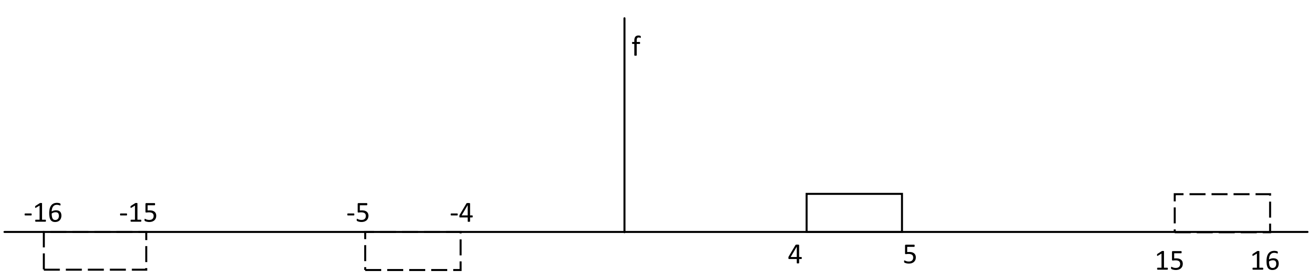

Figure 2: f(x) extended as an even and odd function around x=0 and x=L, respectively

Described graphically above, f(x), when extended, has a period of 4L, or 40 for this specific case. When introduced with the time dimension in the D'Alembert solution, f moves both to the right and left as B and C respectively.

$$u(x,t) = B(x - ct) + C(x + ct) \qquad (5)$$

Where

$$B(x - ct) = \frac{1}{2} f(x - ct) \qquad (6)$$

And

$$C(x + ct) = \frac{1}{2} f(x + ct) \qquad (7)$$

-5 -4

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