Grant Helmreich NetID: granthelmreich Math 412 Homework 3 11/19/07
Available online at: people.tamu.edu/~granthelmreich/exercise3.3.14

3.3.14 (a) Given that f(x) is even about $x = \frac{L}{2}$, the definitions of odd and even functions may be used to determine the nature of a_n using the equation,

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx \tag{1}$$

It is clear upon inspection that $\cos\left(\frac{n\pi x}{L}\right)$ will be odd about $\frac{L}{2}$ for any odd integer n.

For a function to be odd about $\frac{L}{2}$,

$$f(x) = -f(L-x) \tag{2}$$

For the function of interest,

$$\cos\left(\frac{n\pi x}{L}\right) = -\cos\left(\frac{n\pi(L-x)}{L}\right) \tag{3}$$

By distributing,

$$\cos\left(\frac{n\pi x}{L}\right) = -\cos\left(n\pi - \frac{n\pi x}{L}\right) \tag{4}$$

By the definition of cosines for any odd integer n and any u,

$$\cos(n\pi - u) = \cos(-u) \tag{5}$$

Thus,

$$\cos\left(\frac{n\pi x}{L}\right) = -\cos\left(n\pi - \frac{n\pi x}{L}\right) = -\cos\left(-\frac{n\pi x}{L}\right) \tag{6}$$

Once again by the definition of cosines,

$$\cos(u) = -\cos(-u) \tag{7}$$

Thus,

$$\cos\left(\frac{n\pi x}{L}\right) = -\cos\left(-\frac{n\pi x}{L}\right) \tag{8}$$

This equation is true for all odd integers n and proves that $\cos\left(\frac{n\pi x}{L}\right)$ is odd around $\frac{L}{2}$.

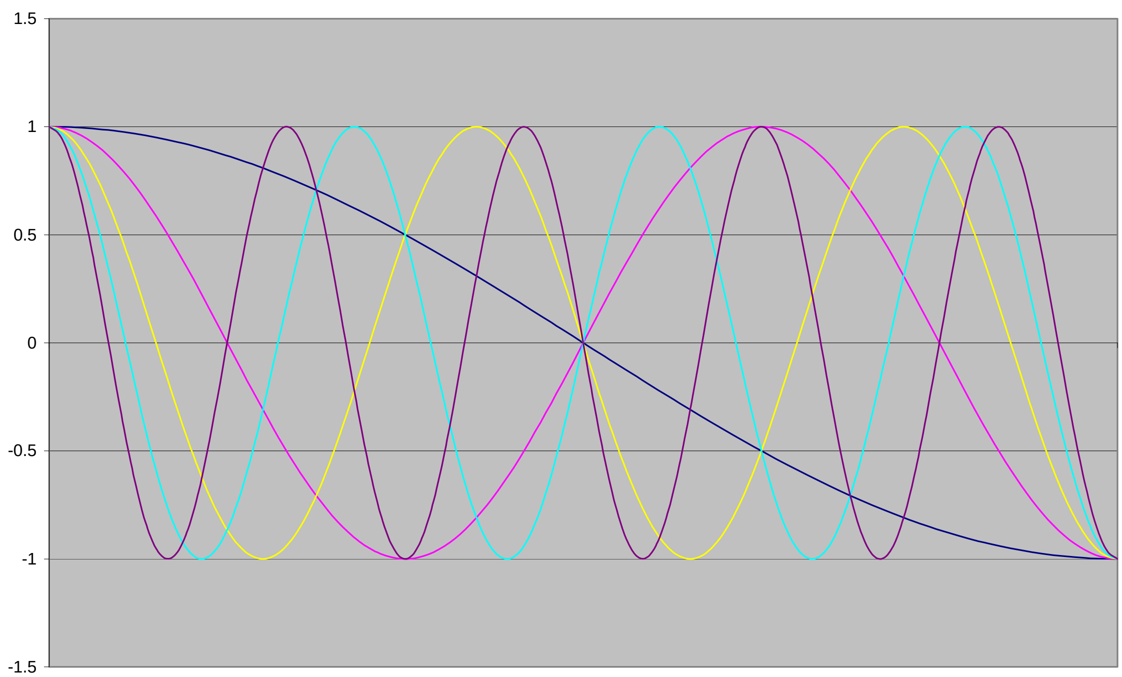
The definitions of even and odd functions define that any function formed by the multiplication of an odd function about a point by an even function about the same point will be an odd function about that original point. Thus, since the integrand of Equation

(1) is composed of an even function multiplied by an odd function, the integrand of Equation (1) will be odd around $\frac{L}{2}$ for all odd integers n.

Once more by the definition of odd functions, since the integral of Equation (1) takes place around $\frac{L}{2}$ and the integrand is odd around that same point, the result of that integral must equal zero for any odd integer n. Thus, since Equation (1) defines all coefficients of the Fourier cosine series of f(x), all of the odd coefficients in this case must equal zero.

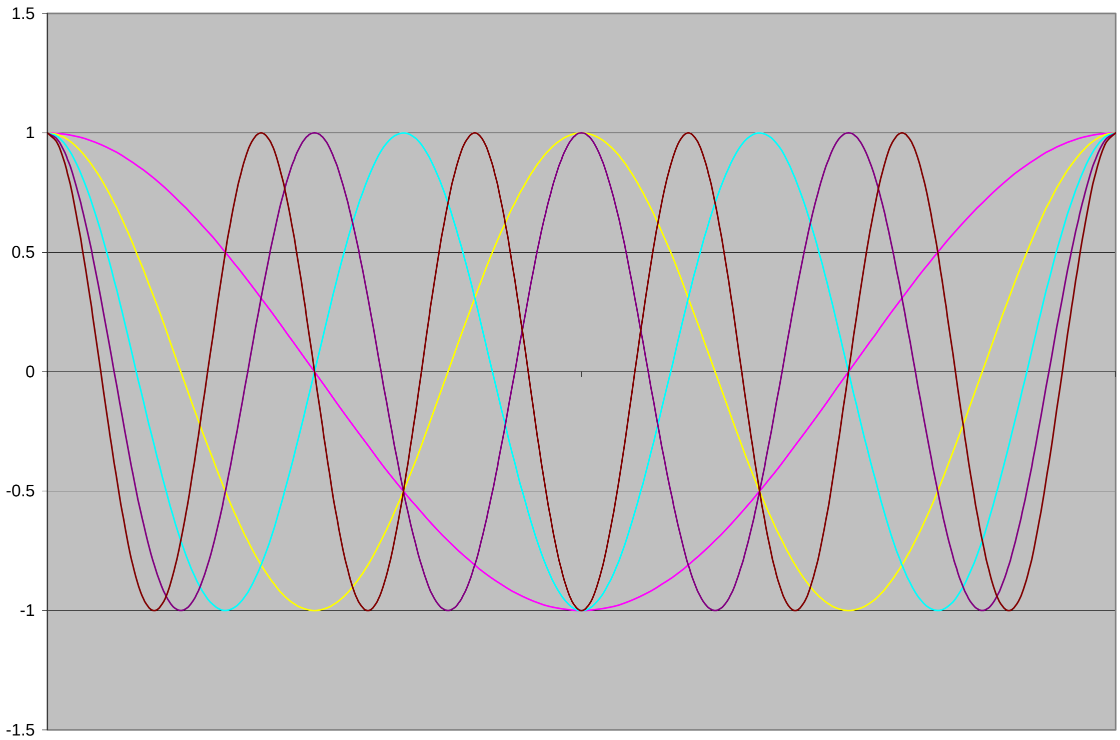
3.3.14 (b)

The result of part (a) may be simply visualized and explained by the aid of graphs of $\cos\left(\frac{n\pi x}{L}\right)$ for odd n. The following graph shows n = 1, 3, 5, 7, and 9 over a period of L.



Inspection of this graph reveals a common node for all odd n at $\frac{L}{2}$. In addition, comparison of the left and right sides of the graph show that the right side in each case is simply the odd extension of the left side. This confirms the more rigorous analysis of part (a) in showing that the function is indeed odd about $\frac{L}{2}$ for odd values of n.

In contrast, the next graph shows $\cos\left(\frac{n\pi x}{L}\right)$ for n = 2, 4, 6, 8, and 10 over a period of L.



Similar inspection of the left and right sides of this graph show that the function will be even around $\frac{L}{2}$ for even values of n. This in turn results in an even integrand in Equation (1) and a non-zero result for the Fourier coefficients.

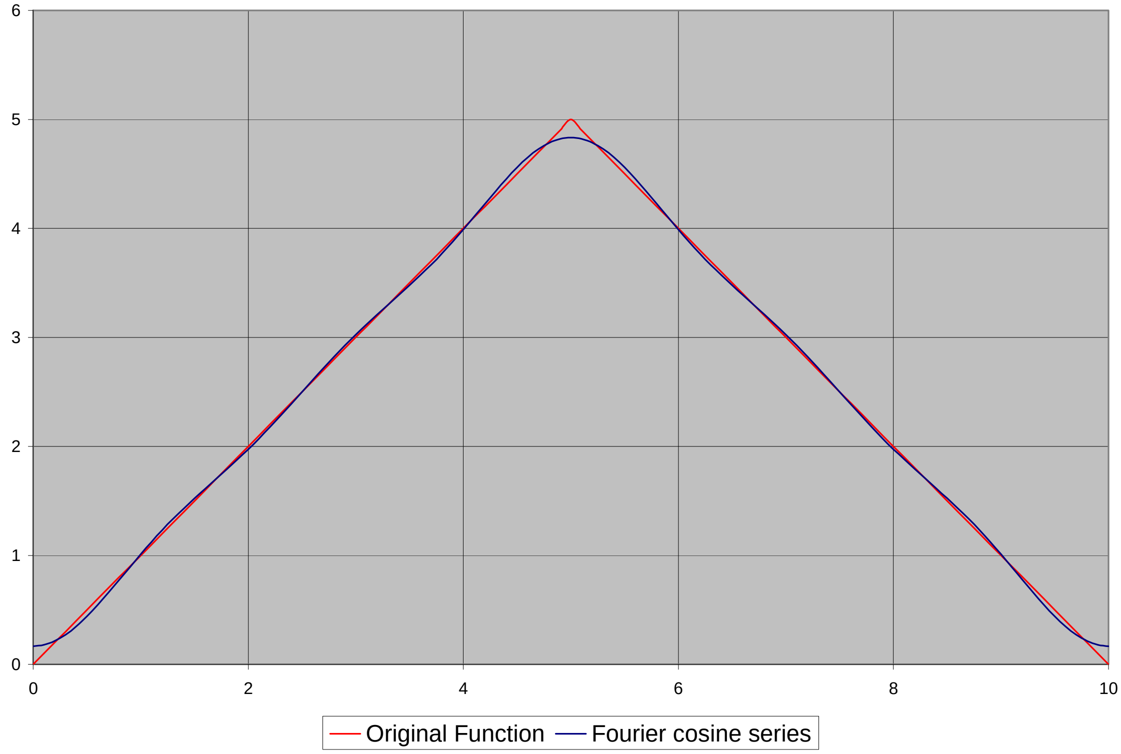
A simple test of this may be performed by creating the Fourier cosine series for the function,

$$f(x) = \left\{\frac{x \ldots 0 < x < 5}{10 - x \ldots 5 < x < 10}\right\}$$

The first 11 coefficients of the Fourier cosine series were found as follows:

a_0= 2.5, a_1= 0, a_2= -2.02642, a_3= 0, a_4= 0, a5= 0, a_6= -0.22516, a_7= 0, a_8= 0, a_9=0, a_{10}= -0.08106

All odd coefficients were zero as expected, in addition to a few of the even coefficients due to the simplicity of the function. The resulting graph showing the original function and Fourier transform follows.


6
5
4
3
2
1
0
0
2
4
6
8
10
Original Function
Fourier cosine series