Grant Helmreich NetID: granthelmreich Math 412 Homework 12 11/26/07

Available online at http://people.tamu.edu/~granthelmreich/exercise7.10.10

$$\nabla^2 u = 0 \tag{1}$$
$$u(a,\theta,\phi) = F(\theta,\phi) \tag{2}$$
$$\rho > a \tag{3}$$

Assume that a solution may be found through separation of variables of the form,

$$u(\rho,\theta,\phi) = R(\rho)\Theta(\theta)\Phi(\phi) \tag{4}$$

The radial portion of this equation may be expressed as 7.10.28 in Haberman,

$$\frac{d}{d\rho}\left(\rho^2 \frac{dR}{d\rho}\right) - n(n+1)R = 0 \tag{5}$$

The two solutions to Equation 5 are,

$$R = \rho^n \tag{6}$$
$$R = \rho^{-n-1} \tag{7}$$

Requiring the solution to be bounded as ρ approaches infinity eliminates Equation 6 as a solution, leaving only Equation 7.

The solutions of the angular depended portions are well known, and thus the final solution may be written as,

$$u(\rho,\theta,\phi) = \sum_{m=0}^{\infty}\sum_{n=m}^{\infty} \rho^{-n-1}\left(A_{mn}\cos(m\theta) + B_{mn}\sin(m\theta)\right)P_n^m \cos(\phi) \tag{8}$$

The constants within the solution may be determined from the boundary condition as,

$$a^{-n-1}A_{mn} = \frac{\iint F(\theta,\phi)\cos(m\theta)P_n^m \cos(\phi)\sin\phi d\phi d\theta}{\iint \cos^2(m\theta)\left[P_n^m \cos(\phi)\right]^2 \sin\phi d\phi d\theta} \tag{9}$$

$$a^{-n-1}B_{mn} = \frac{\iint F(\theta,\phi)\sin(m\theta)P_n^m \cos(\phi)\sin\phi d\phi d\theta}{\iint \sin^2(m\theta)\left[P_n^m \cos(\phi)\right]^2 \sin\phi d\phi d\theta} \tag{10}$$