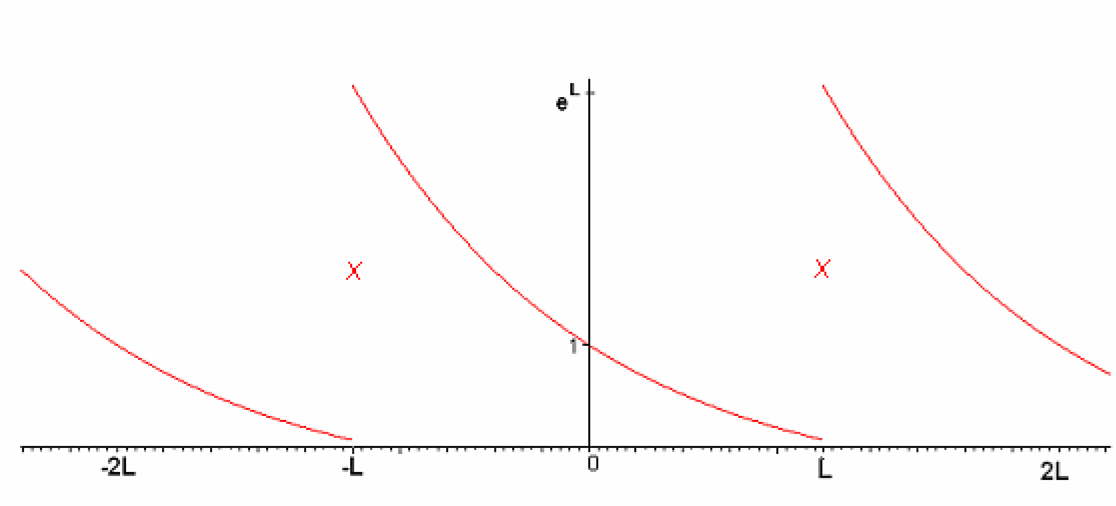
Jason Hill
Assignment 3
http://yojox.com/math412/3-2-2.pdf

In this problem, we are asked to sketch the Fourier series of $f(x)$ (on the interval $-L \le x \le L$)and determine the Fourier coefficients.

(b) $f(x) = e^{-x}$

The sketch of the Fourier series is the periodic extension of $f(x)$, with period 2L.



The Fourier coefficients are found by:

$$a_0 = \frac{1}{2L}\int_{-L}^{L} e^{-x} dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} e^{-x} \cos\left(\frac{n\pi x}{L}\right) dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} e^{-x} \sin\left(\frac{n\pi x}{L}\right) dx$$

a_0:

$$a_0 = \frac{1}{2L}\int_{-L}^{L} e^{-x} dx = \frac{1}{2L}[-e^{-x}]_{-L}^{L}$$

$$a_0 = \frac{1}{2L}\left[e^{L} - e^{-L}\right]$$

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a_n and b_n are found using the integration by parts technique. The solutions generated by Maple 10 software, then simplified, are as follows:

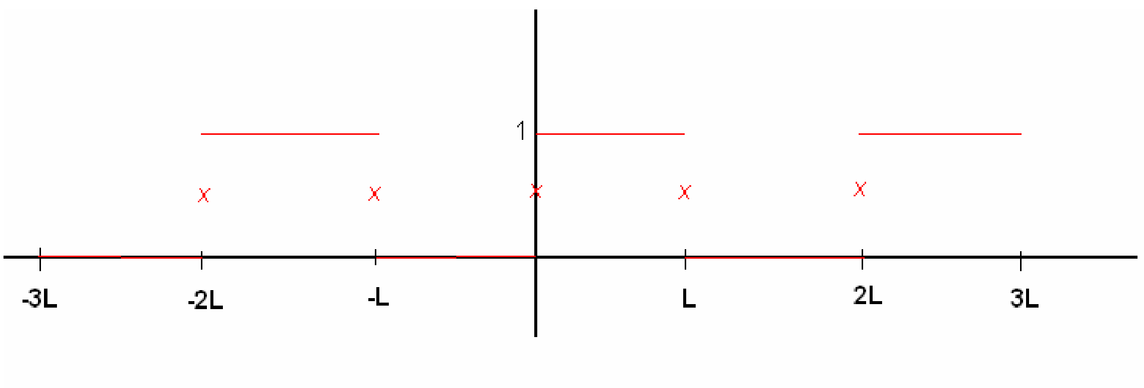
$$a_n = \frac{2\cos(n\pi)\sinh(L)}{L + \frac{n^2\pi^2}{L}}$$

$$b_n = \frac{2n\pi\cos(n\pi)\sinh(L)}{L^2 + n^2\pi^2}$$

(f)

$$f(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases}$$

The Fourier series sketch of this is again the 2L periodic extension:



The Fourier coefficients are found by:

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx = \frac{1}{2L}\int_{0}^{L} 1dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx = \frac{1}{L}\int_{0}^{L} 1\cos\left(\frac{n\pi x}{L}\right)dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx = \frac{1}{L}\int_{0}^{L} 1\sin\left(\frac{n\pi x}{L}\right)dx$$

Solving each of these integrals is much simpler than in part (b). The solutions are:

$$a_0 = \frac{1}{2}$$

$$a_n = \frac{1}{n\pi}\sin\left(\frac{n\pi x}{L}\right) = 0$$

$$b_n = \frac{1-\cos(n\pi)}{n\pi} = \begin{cases} \dfrac{2}{n\pi} & , n \text{ is odd} \\ 0 & , n \text{ is even} \end{cases}$$