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Assignment 2 2.3.1 b
http://people.tamu.edu/~jewlia/2_3_1b.pdf

Consider the following partial differential equation:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} - v_0\frac{\partial u}{\partial x}.$$

What ordinary differential equations are implied by the method of separation of variables?

From the PDE, we attempt to determine solutions in the product form

$$u(x,t) = \varphi(x)G(t)$$

We then calculate the differentials used in our PDE:

$$\frac{\partial u}{\partial t} = \varphi(x)\frac{dG}{dt}$$

$$\frac{\partial u}{\partial x} = \frac{d\varphi}{dx}G(t)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{d^2\varphi}{dx^2}G(t)$$

Plugging these in to our original equation, we get

$$\varphi(x)\frac{dG}{dt} = k\frac{d^2\varphi}{dx^2}G(t) - v_0\frac{d\varphi}{dx}G(t)$$

Through some algebraic manipulation to separate the variables:

$$\varphi(x)\frac{dG}{dt} = \left[k\frac{d^2\varphi}{dx^2} - v_0\frac{d\varphi}{dx}\right]G(t)$$

$$\frac{1}{G}\frac{dG}{dt} = \frac{1}{\varphi}\left(k\frac{d^2\varphi}{dx^2} - v_0\frac{d\varphi}{dx}\right)$$

While it makes very little sense to say that a function dependent only on x is equal to a function dependent only on t, we can claim that they are both equal to the an arbitrary constant $-\lambda$

$$\frac{1}{G}\frac{dG}{dt} = \frac{1}{\varphi}\left(k\frac{d^2\varphi}{dx^2} - v_0\frac{d\varphi}{dx}\right) = -\lambda$$

This yields two ordinary differential equations:

$$\frac{dG}{dt} = -\lambda G$$

$$k\frac{d^2\varphi}{dx^2} - v_0\frac{d\varphi}{dx} = -\lambda\varphi$$