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Assignment 2 2.3.6
http://people.tamu.edu/~jewlia/2_3_6.pdf

Evaluate the expression

$$\int_0^L \cos\frac{n\pi x}{L}\cos\frac{m\pi x}{L}dx$$

For n and m both being non-negative integers using the trigonometry identity

$$\cos a\cos b = \frac{1}{2}\left[\cos(a+b)+\cos(a-b)\right]$$

In attacking this problem, we need to be careful of three cases:
Neither a-b or a+b equal zero (m≠n)
a+b=0 (m=n=0 is the only way this can happen with non-negative integers)
a-b=0 (m=n, we will focus on m=n≠0 to differentiate from previous case)

For any case, we use the suggested trig identity to simplify the original integral

$$\int_0^L \cos\frac{n\pi x}{L}\cos\frac{m\pi x}{L}dx = \int_0^L \frac{1}{2}\left[\cos\frac{(n+m)\pi x}{L}+\cos\frac{(n-m)\pi x}{L}\right]dx$$

For the first case (m≠n):

$$\frac{1}{2}\int_0^L\left[\cos\frac{(n+m)\pi x}{L}+\cos\frac{(n-m)\pi x}{L}\right]dx$$

$$\frac{1}{2}\int_0^L\cos\frac{(n+m)\pi x}{L}dx+\frac{1}{2}\int_0^L\cos\frac{(n-m)\pi x}{L}dx$$

$$=\frac{1}{2}\sin\frac{(m+n)\pi x}{L}\bigg|_0^L+\frac{1}{2}\sin\frac{(m-n)\pi x}{L}\bigg|_0^L$$

Since sin(2πn) for any integer n (or sum/difference of integers) is equal to 0:

$$\frac{1}{2}\int_0^L\left[\cos\frac{(n+m)\pi x}{L}+\cos\frac{(n-m)\pi x}{L}\right]dx=0$$

For the second case (m=n=0):

$$\frac{1}{2}\int_0^L\left[\cos\frac{(n+m)\pi x}{L}+\cos\frac{(n-m)\pi x}{L}\right]dx$$

$$=\frac{1}{2}\int_0^L(\cos 0+\cos 0)dx$$

$$=\frac{1}{2}\int_0^L 2dx$$

$$=\frac{1}{2}2x\bigg|_0^L$$

$$\frac{1}{2}\int_0^L\left[\cos\frac{(n+m)\pi x}{L}+\cos\frac{(n-m)\pi x}{L}\right]dx=L$$

For the third case (m=n≠0):

$$\frac{1}{2}\int_0^L\left[\cos\frac{(n+m)\pi x}{L}+\cos\frac{(n-m)\pi x}{L}\right]dx$$

$$=\frac{1}{2}\int_0^L\left[\cos\frac{2\pi nx}{L}+\cos 0\right]dx$$

$$=\frac{1}{2}\int_0^L\cos\frac{2\pi nx}{L}dx+\frac{1}{2}\int_0^L 1dx$$

$$=\frac{1}{2}\sin\frac{2\pi nx}{L}\bigg|_0^L+\frac{1}{2}x\bigg|_0^L$$

$$=\frac{1}{2}\left[\sin(2\pi n)-\sin(0)\right]+\frac{1}{2}L$$

Since sin(0) and sin(2πn) for any integer n are still equal to 0:

$$\int_0^L\cos\frac{n\pi x}{L}\cos\frac{m\pi x}{L}dx=\frac{L}{2}$$