

**Proposition 3.3 (Second Result).** Given  $A * B * C$  and  $A * C * D$ . Then  $A * B * D$ .

*Proof.* We have  $A, B, C,$  and  $D$  as four distinct, collinear points. By Proposition 2.3, there exists some point  $E$  which does not lie on the line through  $A, B, C,$  and  $D$ . Consider the line  $\overleftrightarrow{EB}$ . Since  $\overleftrightarrow{EB}$  intersects  $AC$  at point  $B$  and  $A * B * C$ , then  $A$  and  $C$  are on opposite sides of  $\overleftrightarrow{EB}$ .

We wish to show that  $C$  and  $D$  are on the same side of  $\overleftrightarrow{EB}$ . Suppose by RAA hypothesis that  $C$  and  $D$  were on opposite sides of  $\overleftrightarrow{EB}$ . Then there must be a point on segment  $CD$  at which  $\overleftrightarrow{EB}$  intersects  $CD$ . By Proposition 2.1, this point must be  $B$ , so  $C * B * D$ . But by the first result of Proposition 3.3 (as proved by Greenberg),  $B * C * D$ , a contradiction (as per B-3). Thus,  $C$  and  $D$  must be on the same side of  $\overleftrightarrow{EB}$ . Since plane separation is an equivalence relation and  $A * B * C$  by hypothesis,  $A * B * D$ . □

