Corollary to Proposition 3.3. Given $A*B*C$ and $B*C*D$. Then $A*B*D$ and $A*C*D$.

Proof. (This proof is nearly identical to the proof of the second part of Proposition 3.3)

We have A, B, C, and D as four distinct, collinear points. By Proposition 2.3, there exists some point E which does not lie on the line through A, B, C, and D. Consider the line $\overleftrightarrow{EB}$. Since $\overleftrightarrow{EB}$ intersects AC at point B and $A*B*C$, then A and C are on opposite sides of $\overleftrightarrow{EB}$.

We wish to show that C and D are on the same size of $\overleftrightarrow{EB}$. Suppose by RAA hypothesis that C and D were on opposite sides of $\overleftrightarrow{EB}$. Then there must be a point on segment CD at which $\overleftrightarrow{EB}$ intersects CD. By Proposition 2.1, this point must be B, so $C*B*D$. But $B*C*D$, a contradiction (as per B-3). Thus, C and D must be on the same side of $\overleftrightarrow{EB}$. Since plane separation is an equivalence relation and $A*B*C$ by hypothesis, $A*B*D$.

Proof that $A*C*D$ follows symmetrically from the above paragraphs, using line $\overleftrightarrow{EC}$ rather than $\overleftrightarrow{EB}$. □