Proposition 3.7. Given an angle $\sphericalangle CAB$ and a point D lying on line $\overleftrightarrow{BC}$. Then D is in the interior of $\sphericalangle CAB$ if and only if $B * D * C$.

Proof. Let $\sphericalangle CAB$ be an angle with a point D lying on $\overleftrightarrow{BC}$. Begin by assuming that D is in the interior of $\sphericalangle CAB$. Then C and D are on the same side of $\overleftrightarrow{AB}$, meaning segment CD does not intersect $\overleftrightarrow{AB}$. Since B, C, and D are all collinear (and thus, $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$ only at point B), this means that $B * D * C$, $B * C * D$. (Note that since plane separation implies the existence of a segment CD, we may rule out the case in which $C = D$.) But by a similar argument, B and D must be on the same side of $\overleftrightarrow{AC}$, so $B * C * D$ is a contradiction, as it implies that segment BD intersects $\overleftrightarrow{AC}$ at C. Thus, $B * D * C$.

For the converse direction, let $B * D * C$. Then $\overleftrightarrow{AC}$ cannot intersect segment BD, and $\overleftrightarrow{AB}$ cannot intersect segment CD. Thus, B and D are on the same side of $\overleftrightarrow{AC}$, and C and D are on the same side of $\overleftrightarrow{AC}$, so D is in the interior of $\sphericalangle CAB$ by definition. □