

Proposition 4.2 (Hypotenuse-Leg Criterion). Two right triangles are congruent if the hypotenuse and a leg of one are congruent, respectively, to the hypotenuse and a leg of the other.

Proof. Let $\triangle ABC$ be a right triangle such that $\sphericalangle BAC$ is a right angle. Let D be a point on the ray opposite to \overrightarrow{AC} such that $BD \cong BC$. (We know that such a D exists because the hypotenuse of any right triangle must be longer than either of its legs, ensuring that a segment congruent to BC would actually meet the ray.) $\triangle BDC$ is then an isosceles triangle, so we may apply the Base Angles Theorem to obtain that $\sphericalangle BDA \cong \sphericalangle BCA$. Since any right angle is congruent to its supplement, $\sphericalangle BAC \cong \sphericalangle BAD$, so $\triangle ABC \cong \triangle ABD$ by the SAA criterion. This proves the Hypotenuse-Leg Criterion, since the only assumptions we made about $\triangle ABD$ were that its hypotenuse and shared leg were congruent to that of $\triangle ABC$, and that $\sphericalangle BAD$ is right (since it is supplementary to a right angle). \square

