

Solve (by Fourier series) the wave equation describing vibrations in a ring:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial \theta^2} \quad \text{for } 0 \leq \theta < 2\pi, \quad -\infty < t < \infty,$$

$$u(\theta, 0) = f(\theta), \quad \frac{\partial u}{\partial t}(\theta, 0) = g(\theta),$$

and periodic boundary conditions in θ . Also, solve the same problem by d'Alembert's method.