

Construct a Green function to solve the ODE problem

$$\frac{d^2 y}{dx^2} + 4y = f(x), \quad y(0) = 0, \quad y(L) = 0.$$

(That is, find G so that $y(x) = \int_0^L G(x, z)f(z) dz$.) For what values of L is this problem impossible? Explain why those values are special.