Matching within Limited Distance*

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March, 2024

Abstract

This study explores housing markets constrained by limited distances, a common scenario in real-world contexts, where agents' preferences over potential matches are single-peaked. We introduce a novel category of mechanisms, the ***r*-neighborhood mechanisms**, characterized by their group strategy-proofness, individual rationality, and Pareto efficiency. Within this framework, agents sequentially identify their most preferred house within a specified distance r, consistent with the linear organization of their single-peaked preferences. Our analysis reveals that the r-neighborhood mechanisms encapsulate Gale's Top Trading Cycles mechanism and Bade's Crawler mechanism as special cases at the extremes. Importantly, we demonstrate that of all r-neighborhood mechanisms, the 1-neighborhood mechanism (equivalent to the Crawler and its dual) stands out as the sole mechanism implementable under the principle of obvious strategy-proofness.

*We are grateful to an advisory editor and two anonymous referees whose insightful comments and suggestions have significantly enhanced the quality of this paper. Our appreciation also extends to Qianfeng Tang, Peng Liu, Jiaxin Wang, Yu Gu, Yongchao Zhang, Huaxia Zeng, Xiang Han, Gaoji Hu, Kang Rong, Bingyong Zheng, and Cuihong Fan for their valuable feedback.

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JEL classification: C78; D47; D61; D78

Keywords: Single-peaked preferences, Top Trading Cycles, Pareto efficiency, group strategy-proof, obvious dominance.

1 Introduction

We explore housing markets where each agent is endowed with a house and aims to exchange it for another, subject to distance constraints. In unrestricted Shapley-Scarf markets (Shapley and Scarf , 1974), Gale's Top Trading Cycles (TTC) mechanism is renowned for uniquely identifying matchings in the core, distinguished as the sole mechanism ensuring individual rationality, Pareto efficiency, and strategy-proofness (Roth and Postlewaite , 1977; Ma , 1994). Introducing distance constraints presents new challenges to this foundational framework.

Bade (2019) pioneers the examination of housing markets with single-peaked preferences, introducing a scenario where all houses are arrayed along a street from one end to the other. Agents have strict, single-peaked preferences, each with an ideal house on this linear spectrum. A house h is deemed less preferable than another house h' by an agent if both are situated on the same side of her ideal point, with h being farther away. Bade (2019) introduces the Crawler mechanism, where each agent selects their ideal house. The mechanism prioritizes agents pointing to occupied houses or those to the right, assigning the rightmost agent their ideal house. If this house is not the one that the agent occupied at the beginning of this step, then each occupant of a house between these two "crawls" to the left to the next house. This iterative process continues until all agents are matched. The Crawler, akin to the TTC mechanism, is shown to be individually rational, efficient, and strategy-proof within the context of single-peaked preferences.

This observation leads to a pivotal question: Ma's characterization is not directly applicable in domains constrained by distance. How, then, do the TTC mechanism and the Crawler relate? We observe that the Crawler's process closely resembles agents trading progressively with their immediate neighbors.[1] Viewing through this lens, the TTC mechanism and the Crawler

[1] A similar observation is provided by Schummer and Serizawa (2019), who offer an alternative algorithm for the

emerge as polar scenarios: while the former allows trading across the entire set of houses, the latter confines trades to direct neighbors. This contrast raises an intriguing inquiry: Are there other mechanisms that also satisfy the criteria of individual rationality, Pareto efficiency, and strategy-proofness within these constraints?

The TTC mechanism and the Crawler are predicated on two fundamentally different assumptions. The TTC mechanism permits trading between any two agents, irrespective of their geographical distance. In contrast, the Crawler mechanism confines trades to "adjacent" agents only. However, real-world allocation scenarios often feature constraints that limit the direct exchange scope among market participants, evident across various contexts. For instance, in kidney exchange programs, medical considerations such as the impact of anti-HLA donor-specific antibodies significantly influence transplant outcomes. Logistical challenges, including waiting list durations and the costs of identifying suitable donors, as highlighted in Roth et.al. (2004), further restrict the available kidney pool to specific donor-patient pairs. Also, in the liver exchange problem proposed by Ergin et.al. (2020), the set of feasible two-way exchanges for a specific donor-patient pair is directly restricted by medical indices, such as the volume of the liver lobe and the antigens in the blood. Similarly, in the housing market, the financial burden of engaging in direct exchanges with geographically distant agents can be prohibitive, leading to exchanges typically being confined within specific geographical areas, be it a district, city, or country, due to factors like workplace proximity or the availability of housing resources through different intermediaries. In school choice situations, preferences often lean towards schools closer to students' residences, influenced by Nearby Enrollment policies, despite varying admission criteria.

To address the realistic constraint of limited exchange scope, we introduce a comprehensive class of mechanisms tailored for housing markets with single-peaked preferences, designated as ***r*-neighborhood mechanisms**. These are individually rational, Pareto efficient, and strategy-proof, encapsulating both the TTC mechanism and the Crawler as their extremities. Conceptually, with houses arrayed along a street and the distance between any two adjacent houses defined as one,

Crawler. This version updates an allocation by allowing two "adjacent" agents to swap their assignments if they mutually agree to do so.

we propose scenarios where agents are limited to considering houses within a specific r-distance from their current locations, known as an r-neighborhood. Thus, agents aim to exchange for their most preferred house within this constrained range. Within this framework, the TTC mechanism aligns with an $\bar{n} - 1$-neighborhood mechanism, where $\bar{n}$ denotes the total agent count, while the Crawler is akin to a 1-neighborhood mechanism.

In Theorem 1, we show that, similar to the TTC mechanism and the Crawler, r-neighborhood mechanisms satisfy the properties of individual rationality, Pareto efficiency, and group strategy-proofness. We then broaden the scope of r-neighborhood mechanisms to introduce a more expansive class known as **neighborhood mechanisms**. These mechanisms are distinguished by neighborhood sizes that adjust based on the identities of agents and the properties they currently hold. In Theorem 3, we establish that neighborhood mechanisms also adhere to individual rationality, Pareto efficiency, and group strategy-proofness. This discovery stands in stark contrast to Ma (1994)'s characterization, which posits the TTC mechanism as the exclusive framework that achieves individual rationality, Pareto efficiency, and strategy-proofness within Shapley-Scarf housing markets with the domain of strict preference.

Obvious dominance, a concept introduced by Li (2017), is regarded as a beneficial attribute where strategies are considered optimal even by agents with limited cognitive abilities, who may not comprehend how strategy outcomes depend on unobserved contingencies. Bade (2019) shows that the Crawler can be implemented through obviously dominant strategies. In Theorem 2, we establish that among all r-neighborhood mechanisms, the 1-neighborhood mechanism, outcome equivalent to the Crawler, is the sole mechanism implementable via obviously dominant strategies.

The foundational assumption in neighborhood mechanisms, restricting agents to point to a limited range of objects, is well-supported both theoretically and empirically. Theoretical models often highlight a trade-off between the benefits of proximity and the costs associated with distant objects, leading agents to prefer objects within a manageable distance. These costs may include transportation (as discussed in Hotelling, 1929 and Pal, 1998) or search costs (see, for example, Stigler , 1961 and Ioannides , 1975). Empirical research also underscores distance as a critical factor influencing agents' decisions (Rosenthal and Strange , 2008; Robert and Goh , 2011; Sivey , 2012; Andersson et al. , 2014; He et al. , 2019), and ties these preferences to regulatory frameworks like the Nearby Enrollment Policy (Black , 1999; Bayer et al. , 2007). Notably, scant literature addresses this specific restriction within the context of allocation problems featuring single-peaked preferences.

Beyond the seminal work of Shapley and Scarf (1974), our paper significantly intersects with Ma (1994)'s characterization of the TTC mechanism, as well as with Bade (2019)'s analysis of the Crawler. Additionally, our research draws upon the findings of Tamura and Hosseini (2022) and Liu (2022). Tamura and Hosseini (2022) elucidate the equivalence between the Crawler and the Dual Crawler, alongside demonstrating the Crawler's consistency with the random priority rule from varied initial allocations. They further assert the Crawler's robustness to orderings over objects that maintain single-peaked preferences. Liu (2022) contributes to this dialogue by introducing a distinct class of individually rational, efficient, and strategy-proof mechanisms that consider a sequential availability of houses for exchange, contingent upon preceding transactions. The interplay between these two mechanism classes is thoroughly examined in Section 4.

This paper also aligns with broader inquiries into mechanism design under single-peaked preferences. Damamme et.al. (2015) propose an algorithm ensuring Pareto efficiency within this domain. Schummer and Serizawa (2019) differentiate the application of axiomatic principles in abstract versus specific market design analyses, offering the Iterative Swaps algorithm as an alternative to the Crawler. Mandal and Roy (2021) explore the constraints of designing strategy-proof, Pareto efficient, top-envy-proof, non-bossy, and pairwise reallocation-proof assignment rules within a narrowly defined single-peaked preference domain, proposing hierarchical exchange rules as a solution. Further, Tamura (2022) and Tamura (2023) investigate additional properties of the Crawler, with the latter identifying a set of rules termed "Crawler-jumper rules" and delineating their efficiency, adherence to endowment lower bounds, and obvious strategy-proofness criteria. Our discussion also touches upon the broader discourse surrounding Gale's

TTC mechanism, referencing contributions from Pápai (2000) and Sethuraman (2016), and delves into the concept of obvious strategy-proofness in trading mechanisms, engaging with the works of Li (2017), Troyan (2019), and Pycia and Troyan (2022).

The remainder of this paper is structured as follows: Section 2 lays out the basic framework, detailing Gale's Top Trading Cycles mechanism and the Crawler, before introducing r-neighborhood mechanisms and summarizing pertinent results in Section 3. We expand upon these mechanisms and explore their connections to other relevant mechanisms in Section 4. Section 5 provides concluding remarks. For completeness, all proofs are located in Appendix A.

2 Model

2.1 Notations

We consider the allocation problem within a Shapley-Scarf housing market (Shapley and Scarf (1974)), where each agent is endowed with an indivisible object, referred to as a house, and seeks to trade with others. Let $I = \{1, 2, \cdots, \bar{n}\}$ represent the set of agents, and $H = \{h_1, h_2, \cdots, h_{\bar{n}}\}$ the set of houses. Each agent $i \in I$ initially owns house h_i and possesses a strict preference P_i over H, alongside a symmetric extension R_i. For a given preference P_i, if agent i strictly (or weakly) prefers house h_j over h_k, we denote this as $h_j P_i h_k$ (or $h_j R_i h_k$, respectively). For any agent i, P_{-i} signifies the preference profile excluding agent i. Similarly, for a coalition of agents $C \subseteq I$, P_C and P_{-C} respectively denote the preference profiles within coalition C and outside of C.

In line with Bade (2019), we assume agents exhibit single-peaked preferences concerning the houses. Envision houses aligned along a street extending from left to right, with h_1 at the left extremity and $h_{\bar{n}}$ at the right. If $i < j$, then h_i is situated to the left of h_j. A preference P_i is **single-peaked** if there exists a pivotal house h_{i^*} making $h_j P_i h_k$ valid for $k < j \leq i^*$ or $i^* \geq j > k$. For clarity, $h_i < h_j$ indicates h_i is positioned to the left of h_j. Let $\mathbb{P}$ represent the set of all single-peaked preferences.

A **matching** (or allocation) μ is defined as a bijective function $\mu : I \to H$. If $\mu(i) = h_j$, then agent i is matched with house h_j. A matching μ is **individually rational** at preference profile P if $\mu(i) R_i h_i$ for every agent i. A matching ν **(Pareto) dominates** another matching μ at P if $\nu(i) R_i \mu(i)$ for all agents i, with $\nu(j) P_j \mu(j)$ for at least one agent j. A matching μ is **Pareto efficient** if no matching ν exists that Pareto dominates μ. For ease of reference, we sometimes denote a matching as $(i_1 h_1, i_2 h_2, \cdots, i_{\bar{n}} h_{\bar{n}})$, where $i_1, i_2, \cdots, i_{\bar{n}} \in I$ and $\mu(i_k) = h_k$ for each $k = 1, 2, \cdots, \bar{n}$. For example, $\{4h_1, 2h_2, 1h_3, 3h_4\}$ illustrates a matching when $|I| = 4$.

A **direct revelation mechanism**, or simply a **mechanism**, f, assigns a matching $f(P)$ for each preference profile P. Let $f_i(P)$ denote the house assigned to agent i in $f(P)$. A mechanism f is **(Pareto) efficient** (**individually rational**, respectively) if, for any preference profile P, the matching it selects is Pareto efficient (individually rational, respectively). A mechanism f is **strategy-proof** if for all $P_i, P_i^{'}$, and P_{-i}, the relation $f_i(P_i, P_{-i}) R_i f_i(P_i^{'}, P_{-i})$ is satisfied, indicating that no agent can benefit by unilaterally changing their reported preference.

The mechanism f is **group strategy-proof** if, for any preference profile P, there does not exist a coalition $C \subseteq I$ and an alternative preference profile $P_C^{'}$ such that for all $i \in C$, $f_i(P_C^{'}, P_{-C}) R_i f_i(P_C, P_{-C})$ is true, with at least one $j \in C$ for whom $f_j(P_C^{'}, P_{-C}) P_j f_j(P_C, P_{-C})$ holds. Additionally, f is **non-bossy** if for all P, i, and $P_i^{'}$, should $f_i(P) = f_i(P_i^{'}, P_{-i})$, then it must follow that $f(P) = f(P_i^{'}, P_{-i})$, indicating that no single agent's preference change can alter the allocation for others without altering their own. Pápai (2000) shows that a mechanism f is group strategy-proof if and only if it is both strategy-proof and non-bossy within the domain of strict preferences. Extending this, Alva (2017), Mandal and Roy (2021), and Tamura (2022) establish that a mechanism f satisfies group strategy-proofness if and only if it is strategy-proof and non-bossy within the single-peaked preference domain.

In the algorithm of our mechanisms, ownership can change. Let $\omega(i)$ represent the initial endowment for each agent i. For subsets $I_0 \subseteq I$ and $H_0 \subseteq H$ where $|I_0| = |H_0|$, an endowment profile ω is defined as a bijection $\omega : I_0 \to H_0$. The initial endowment, when $I_0 = I$ and $H_0 = H$, is specified by $\omega(i) = h_i$.

2.2 Gale's Top Trading Cycles and the Crawler

Shapley and Scarf (1974) introduced Gale's Top Trading Cycles (TTC) mechanism, proving its efficiency and individual rationality. Roth (1982) further established that the TTC mechanism is strategy-proof. The operation of the TTC mechanism is outlined as follows:

Step 1. Each agent indicates her most preferred house by pointing to its current owner. Given the finite number of agents, there will be at least one cycle, including possible self-cycles, without any intersecting cycles. Agents within these cycles trade houses and are then removed from the process.

Step $k, k \geq 2$**.** The process from Step 1 is repeated for the remaining agents until all agents have been allocated a house.

Ma (1994) further showed that the TTC mechanism is the only mechanism that is individually rational, efficient, and strategy-proof across the domain of strict preferences.

Bade (2019) introduced **the Crawler**, a novel mechanism, proving it to be individually rational, Pareto efficient, and strategy-proof within the context of single-peaked preferences. The Crawler operates as follows:

Step 1. Each agent selects her most preferred house from those remaining. Among agents who point to their current house or to houses on their right, the rightmost agent pointing to a house is matched with that house and removed along with the house. Occupants of houses between the vacated and removed houses move to the next house on the left, treating it as their new endowment.

Step $k, k \geq 2$**.** This step is repeated for the remaining agents until all have been successfully matched.

The Dual Crawler operates inversely, identifying the leftmost agent who prefers to move leftward or who prefers her current house the most. This agent is then allocated her most preferred house. Subsequently, occupants between the two relevant houses “crawl” towards the next house on the right. Bade (2019) establishes that both the Crawler and its dual variant are individually rational, Pareto efficient, and strategy-proof mechanisms.

3 The r-neighborhood Mechanism

3.1 Definition

The Crawler mechanism introduced by Bade (2019) limits agents to trading for houses that are adjacent to their own. Building upon this concept, we propose a series of generalized mechanisms. These mechanisms progressively permit agents to engage in trades exclusively within an r-neighborhood of their current house, broadening the scope of potential exchanges beyond immediate adjacency.

To begin with, we define the concept of an r-neighborhood for a house. The **r-neighborhood** of a house h comprises the nearest r houses on each side of h. Formally, for a given house h, a set of houses $\bar{H}$, and an integer r, we define the set $N(h, \bar{H}; r)$ as the r-neighborhood of h within $\bar{H}$. A house h' is included in $N(h, \bar{H}; r)$ if either $|\{\hat{h} \in \bar{H} : h < \hat{h} < h'\}| < r$ or $|\{\hat{h} \in \bar{H} : h' < \hat{h} < h\}| < r$. The house $\tau(P_i, \bar{H})$ represents the most preferred house of agent i within the set $\bar{H}$, according to preference P_i, such that $\tau(P_i, \bar{H}) R_i h$ for all $h \in \bar{H}$.

For each integer $r \geq 1$, the **r-neighborhood mechanism** operates as follows:

Step 0. Initialize with $H_1 = H$, and $\omega_1(i) = h_i$ for all $i = 1, 2, ..., \bar{n}$.

Step 1. **Pointing:** Each agent i points to an agent $a_1(i)$ whose initial endowment is agent i's most preferred house within the r-neighborhood according to P_i, i.e., $a_1(i) = \omega_1^{-1}(\tau(P_i, N(h_i, H; r)))$.

Trading: There exists at least one cycle (including self-cycles), and cycles do not intersect. Allow agents in cycles to trade. Define the new endowment for each agent i as follows:

1. If i is in a cycle in step 1, then agent i is endowed with the house she points to, i.e., $\omega_2(i) = \omega_1(a_1(i))$.
2. If i is not in any cycle in step 1, then agent i retains her initial endowment, i.e., $\omega_2(i) = h_i$.

Matching: Remove each agent i along with the house $\omega_2(i)$ if $\omega_2(i) = \tau(P_i, H_1)$. Terminate if all agents are matched. Otherwise, update H_2 to include only the remaining houses and proceed to step 2.

Step $k, k \geq 2$. **Pointing:** Direct each agent i to point to an agent $a_k(i)$ as follows:

1. If $a_{k-1}(i)$ was in a cycle in step $k-1$, then let agent i point to the agent whose endowment is her most preferred house in the r-neighborhood according to P_i, i.e., $a_k(i) = \omega_k^{-1}(\tau(P_i, N(h_i, H_k; r)))$.
2. If $a_{k-1}(i)$ was not in any cycle in step $k-1$, then let agent i continue to point to the same agent as in step $k-1$, i.e., $a_k(i) = a_{k-1}(i)$.

Trading: There exists at least one cycle (including self-cycles), and cycles do not intersect. Let agents in cycles trade. For each agent i, define the new endowment as follows:

1. If i is in a cycle in step k, then let agent i be endowed with the house she points to, i.e., $\omega_{k+1}(i) = \omega_k(a_k(i))$.
2. If i is not in any cycle in step k, then let agent i retain her current endowment, i.e., $\omega_{k+1}(i) = \omega_k(i)$.

Matching: For each agent i, if $\omega_{k+1}(i) = \tau(P_i, H_k)$ is satisfied, remove agent i along with the house $\omega_{k+1}(i)$. If all agents have been matched, terminate the process. Otherwise, update H_{k+1} to include only the remaining houses and proceed to step $k+1$.

That is, at the outset of the r-neighborhood mechanism in step 1, all houses are unmatched, allowing for the definition of r-neighborhoods across the entire set H. Each agent points to her most preferred house within the r-neighborhood and engages in trading if she forms part of a cycle. An agent matched with her most preferred house is then removed along with this house. Remaining agents consider their current house as the new endowment and advance to the next step. In subsequent step k, the r-neighborhood is defined over the set of unmatched houses H_k. An agent's target is contingent upon the involvement of her previous target in a trade during step $k-1$: If the previous target was not part of a trade, the agent continues to point to the same agent as before. This constraint is critical for ensuring the strategy-proofness of the mechanism. If the previous target was involved in a trade, the agent may then point to her most preferred house within the r-neighborhood. Following trades within cycles, those occupying their most preferred houses are removed. The rest regard their new house as the endowment and proceed to the subsequent step. Given the definition of single-peaked preferences, preferences within any subset $H_k \subseteq H$ remain single-peaked, meaning each agent has a distinct peak at every step. The algorithm concludes once all agents have been matched.

As houses are arranged along a street, it is conceivable that each agent can only perceive the nearest r houses from her current location, thus limiting her choices to her most preferred house within the r-neighborhood. The r-neighborhood mechanisms facilitate agents progressively trading for their most preferred house within their respective r-neighborhoods.

The following example illustrates how the r-neighborhood mechanisms encompass the TTC mechanism, the Crawler, and the Dual Crawler. Additionally, it reveals the existence of r-neighborhood mechanisms that are distinct from these established mechanisms.

Example 1. Consider a housing market with a set of agents $I = \{1, 2, 3, 4\}$. Their preference profiles are as follows:

P_1	P_2	P_3	P_4
h_4	h_4	h_4	h_1
h_3	h_3	h_3	h_2
h_2	h_2	h_2	h_3
h_1	h_1	h_1	h_4

In the TTC mechanism and the 3-neighborhood mechanism, all agents select their most preferred houses from the set $\{h_1, h_2, h_3, h_4\}$. Thus, agents 1, 2, and 3 point to h_4, while agent 4 points to h_1. Consequently, agent 1 and 4 trade in a cycle, receiving h_4 and h_1 respectively. Subsequently, agents 3 and 2 match with h_3 and h_2 in successive steps. The final matching is $(4h_1, 2h_2, 3h_3, 1h_4)$.

In the Crawler mechanism, no agent initially occupies their most preferred house. Agents 1, 2, and 3 aim to move rightwards, with agent 3 being the rightmost. Hence, agent 3 is selected first, matched with h_4, causing agent 4 to "crawl" to h_3. In the next step, agent 2 is matched with h_3, and agent 4 "crawls" to h_2. Finally, agent 1 is matched with h_2, and agent 4 secures h_1. The Crawler's final matching is $(4h_1, 1h_2, 2h_3, 3h_4)$, which is equivalent to a sequential trading process ending with agent 4 receiving h_1.

The Crawler mechanism can be replicated using the 1-neighborhood mechanism. Here, agent 1 chooses between $\{h_1, h_2\}$, pointing to h_2. Agent 2 selects from $\{h_1, h_2, h_3\}$, pointing to h_3. Agent 3, with options $\{h_2, h_3, h_4\}$, points to h_4. Agent 4, choosing from $\{h_3, h_4\}$, points to h_3 in the first step. This leads to a trade between agents 3 and 4, with agent 3 being matched with h_4. In the subsequent steps, agents follow a similar process, eventually leading to the matching $(4h_1, 1h_2, 2h_3, 3h_4)$, identical to the outcome of the Crawler.

For the Dual Crawler, agent 4 uniquely desires to move leftwards, with none of the agents initially occupying their most preferred house. Consequently, agent 4 acquires house h_1 in the first step, prompting agents 1, 2, and 3 to "crawl" to their next right house. Subsequently, agent 3 (now with h_4) is the sole agent occupying her most preferred unmatched house, with no agents looking to move leftwards. Thus, agent 3 secures h_4. Similar steps ensue for agents 2 and 1 in the subsequent steps, resulting in the final matching of $(4h_1, 1h_2, 2h_3, 3h_4)$, identical to the outcomes from both the Crawler and the 1-neighborhood mechanism.

The TTC mechanism and the Crawler can be emulated within the r-neighborhood framework by setting $r \geq 3$ and $r = 1$, respectively. However, with $r = 2$, the 2-neighborhood mechanism yields a matching distinct from those produced by the TTC mechanism and the Crawler. Initially, agent 1 selects from $\{h_1, h_2, h_3\}$, pointing to h_3, and agent 4 chooses from $\{h_2, h_3, h_4\}$, pointing to h_2. The remaining agents, having access to the full set $\{h_1, h_2, h_3, h_4\}$, both point to h_4. This results in agents 2 and 4 exchanging houses first, with agent 2 being matched and removed with h_4. In the next step, agent 1 continues to point to h_3 as agent 3 was not part of any cycle previously. Agent 4 (now with h_2) and agent 3, selecting from $\{h_1, h_2, h_3\}$, point to h_1 and h_3, respectively. This leads to agent 3 being matched and removed with h_3. In the final step, agents 1 (with h_1) and 4 (with h_2) point to each other and trade in a cycle, finalizing the matching $(4h_1, 1h_2, 3h_3, 2h_4)$, which diverges from the TTC and Crawler outcomes.

As demonstrated in Example 1, r-neighborhood mechanisms can replicate the matchings of the TTC mechanism and the Crawler by setting $r = 3$ and $r = 1$, respectively. Additionally, the Dual Crawler is also effectively reproduced with $r = 1$ in our framework, showcasing the versatility of r-neighborhood mechanisms in accommodating various trading dynamics.

3.2 Results

Proposition 1 presents the equivalence results between the r-neighborhood mechanisms, the TTC mechanism, the Crawler, and the Dual Crawler.

Proposition 1. *Under single-peaked preferences, the following equivalences are observed:*

- *The TTC mechanism and the $\bar{n} - 1$-neighborhood mechanism yield identical outcomes.*
- *The Crawler, the Dual Crawler, and the 1-neighborhood mechanism produce equivalent outcomes.*

The equivalence of the TTC mechanism and the $\bar{n} - 1$-neighborhood mechanism, as stated in Proposition 1, stems directly from their definitions. The parallel between the Crawler and the 1-neighborhood mechanism arises from the observation that each step of the Crawler can be replicated by allowing agents to sequentially trade with their immediate neighbors. This rationale equally applies to the Dual Crawler and the 1-neighborhood mechanism, underscoring the second statement's implication of equivalence between the Crawler and the Dual Crawler, a significant finding of Tamura and Hosseini (2022).

Schummer and Serizawa (2019) introduce an Iterative Swaps algorithm as an alternative depiction of the Crawler, updating allocations by enabling adjacent agent pairs to swap assignments upon mutual consent. This algorithm is outcome equivalent to the 1-neighborhood mechanism, where each agent similarly engages in sequential trades with adjacent agents. However, the underlying motivations of the two mechanisms diverge. The Iterative Swaps algorithm selects "adjacent" agent pairs from all potential trading cycles for Pareto improvements, whereas the 1-neighborhood mechanism independently directs each agent within the 1-neighborhood, facilitating cycle trades. Consequently, while it is straightforward to extend the 1-neighborhood mechanism to r-neighborhood mechanisms for any $r \geq 2$, a similar extension of the Iterative Swaps algorithm proves more challenging.

The r-neighborhood mechanisms constitute a broad class of mechanisms distinguished by their individual rationality, Pareto efficiency, and group strategy-proofness.

Theorem 1. *An r-neighborhood mechanism f inherently guarantees individual rationality, Pareto efficiency, and group strategy-proofness.*

Theorem 1 confirms that r-neighborhood mechanisms not only retain the desirable properties of the TTC mechanism and the Crawler but also significantly expand the spectrum of mechanisms accommodating single-peaked preferences.

3.3 Obviously Strategy-Proofness

Certain implementations of strategy-proof mechanisms are more readily apparent. Li (2017) introduces the concept of obvious dominance to identify strategies that agents would choose even without understanding how the outcome of each strategy depends on unobserved contingencies. A strategy is considered **obviously dominant** if, at any point where two strategies diverge, the best possible outcome from deviating is no better than the worst outcome under the dominant strategy. A mechanism is **obviously strategy-proof (OSP)** if it admits an equilibrium in obviously dominant strategies. Bade (2019) proves that the Crawler is OSP. Proposition 1 reveals that the Crawler and the 1-neighborhood mechanism are outcome equivalent. Below, we establish that the 1-neighborhood mechanism is the sole OSP-implementable mechanism within the

r-neighborhood mechanisms class.

An **extensive-form game** G models a series of **histories** in a rooted tree structure. If θ extends θ', denoted as $\theta' \succ \theta$, then θ' is a **subhistory** of θ. Histories without continuations are **terminal**, each associated with a specific matching. At any history θ, an agent has a set of **actions** $A(h)$. A strategy S_i selects an action for agent i at every information set. An **extensive-form mechanism** combines an extensive-form game G with a strategy profile $(S_i)_{i\in I}$. Following Li (2017), a strategy S_i is **obviously dominant** over another strategy S_i' if, at the initial divergence point, the worst payoff from S_i is at least as good as the best payoff from S_i'. A strategy profile $(S_i(\cdot))_{i\in I}$ is obviously dominant if, for any agent i and preference P_i, the strategy $S_i(P_i)$ obviously dominates all others for P_i. An extensive-form mechanism is **obviously strategy-proof** if it admits a profile of obviously dominant strategies $(S_i(\cdot))_{i\in I}$. Let $G(S(P))$ denote the matching selected through game G, strategy profile $(S_i(\cdot))_{i\in I}$, and preference profile P. A (direct revelation) mechanism f is OSP-implementable if an extensive-form mechanism G and an obviously dominant strategy profile $(S_i(\cdot))_{i\in I}$ exist such that $G(S(P)) = f(P)$ for all P. In this context, G **OSP-implements** f, and f is **obviously implemented** in obviously dominant strategies $(S_i(P_i))_{i\in I}$.

Not all strategy-proof mechanisms are OSP-implementable. Li (2017) finds that Gale's Top Trading Cycles mechanism with at least three agents cannot be OSP-implemented within the domain of all linear preferences. With single-peaked preferences, Bade (2019) shows that the TTC mechanism with four or more agents is not OSP-implementable, whereas the Crawler can be.

While r-neighborhood mechanisms significantly expand the set of individually rational, Pareto efficient, and group strategy-proof mechanisms, most are not OSP-implementable. The following theorem specifies that only the 1-neighborhood mechanism achieves OSP within the r-neighborhood framework.

Theorem 2. *For $|I| \geq 4$ and any r-neighborhood mechanism f, f is implementable in obviously dominant strategies if and only if it is a 1-neighborhood mechanism.*

Theorem 2 implies that the 1-neighborhood mechanism, outcome equivalent to the Crawler, stands as the only OSP-implementable mechanism among the r-neighborhood mechanisms for settings with at least four agents. For $|I| \leq 3$, an r-neighborhood mechanism either aligns with the Crawler or the TTC mechanism, each being OSP-implementable as shown by Bade (2019).

4 Extension

4.1 Generalization

The r-neighborhood mechanism, by design, keeps the size of r-neighborhoods constant across all agents. We propose a further generalization by allowing the sizes of neighborhoods to vary depending on the agents, the houses, and the directions. For agent i and house h_j, let the positive integers l_i^j and r_i^j denote the left and right sizes of a neighborhood, respectively. A size function is defined as $Size : I \times H \to \mathbb{Z}_+ \times \mathbb{Z}_+$, assigning (l_i^j, r_i^j) to each pair of agent i and house h_j.

Given a pair of positive integers (l, r) and a set of houses $\bar{H}$, let $N(h, \bar{H}, (l, r))$ denote the **neighborhood** of size (l, r) for house h. A house h' is included in $N(h, \bar{H}, (l, r))$ if it satisfies either $|\{\hat{h} \in \bar{H} : h' < \hat{h} < h\}| < l$ or $|\{\hat{h} \in \bar{H} : h < \hat{h} < h'\}| < r$.

Using a size function $S : I \times H \to \mathbb{Z}_+ \times \mathbb{Z}_+$, the generalized r-neighborhood mechanisms, henceforth referred to simply as the **neighborhood mechanisms**, operate as follows:

Step 0. Initialize with $H_1 = H$, and $\omega_1(i) = h_i$ for all $i = 1, 2, ..., \bar{n}$.

Step $k, k \geq 1$. **Pointing:** For each agent i, determine a set of houses $N_k^i = N(\omega_k(i), H_k, S(i, \omega_k(i)))$, where N_k^i represents the neighborhood with size $S(i, \omega_k(i))$ for the current endowment $\omega_k(i)$. Agents point to another agent $a_k(i)$ based on the following:

1. If $k = 1$ or $a_{k-1}(i)$ was part of a cycle in step $k-1$, agent i points to the owner of her most preferred house within N_k^i, i.e., $a_k(i) = \omega_k^{-1}(\tau(P_i, N_k^i))$.
2. If $k \geq 2$ and $a_{k-1}(i)$ was not in a cycle in step $k-1$, agent i continues to point to the same agent as in the previous step, i.e., $a_k(i) = a_{k-1}(i)$.

Trading: There exist at least one cycle (including self-cycles), with cycles not intersecting. Allow agents in cycles to trade. Update each agent i's new endowment accordingly:

1. If i is in a cycle in step k, then i is endowed with the house she points to, i.e., $\omega_{k+1}(i) = \omega_k(a_k(i))$.
2. If i is not in any cycle in step k, then i retains her current endowment, i.e., $\omega_{k+1}(i) = \omega_k(i)$.

Matching: Remove agent i along with house $\omega_{k+1}(i)$ if $\omega_{k+1}(i) = \tau(P_i, H_k)$ is satisfied. Terminate if all agents have been matched; otherwise, update H_{k+1} to include only the remaining houses and proceed to the next step.

In neighborhood mechanisms, agents may target different neighborhoods at various houses, allowing for distinct perspectives on the same house among different agents. At each step k, neighborhoods are defined based on the set of unmatched houses H_k. An agent's target is influenced by her previous target's actions in step $k-1$: if the previous target $a_{k-1}(i)$ engaged in a trade during step $k-1$, the agent then points to her most preferred house within the current house's neighborhood $\omega_k(i)$; otherwise, the agent continues to target the same house as in the previous step. This rule is crucial for maintaining the strategy-proofness of the neighborhood mechanisms.

Additionally, we can envision scenarios in which an agent's visibility is limited to houses within her neighborhood. Unlike the r-neighborhood mechanisms, the scope of visible houses in neighborhood mechanisms is determined by the agent's identity and the houses they occupy. Agents naturally aim for their most preferred house within these neighborhoods, engaging in progressive trades to secure these optimal houses.

Hence, r-neighborhood mechanisms are effectively special cases of the broader category of neighborhood mechanisms. Theorem 3 confirms that neighborhood mechanisms retain the desirable attributes of the r-neighborhood mechanisms, such as individual rationality, Pareto efficiency, and group strategy-proofness.

Theorem 3. *A neighborhood mechanism f is individually rational, Pareto efficient, and group strategy-proof.*

Given that all r-neighborhood mechanisms fall under the umbrella of neighborhood mechanisms, Theorem 3 substantiates Theorem 1. This allows for an expansion of the class of mechanisms that are individually rational, efficient, and group strategy-proof to include neighborhood mechanisms. While Theorem 2 identifies the 1-neighborhood mechanism as the only OSP-implementable mechanism within the r-neighborhood framework, this direct generalization is not universally applicable. Specifically, the 1-neighborhood mechanism is not the sole OSP-implementable neighborhood mechanism for scenarios when $|I| \geq 4$.[2].

4.2 Generalized Neighborhood Mechanism

Neighborhood mechanisms define an agent i's neighborhood in relation to a specific house h, with each agent targeting her most preferred house within this neighborhood. Liu (2022) introduces a class of mechanisms where, at each step, only a subset of houses is available for exchange among the agents occupying those houses. This subset, termed "neighborhood" by Liu (2022), is defined by proximity.

The NTTC mechanisms detailed in Liu (2022) allow the subset of available houses for exchange to vary based on previous exchanges, indicating that not all NTTC mechanisms qualify as neighborhood mechanisms due to the fixed neighborhood size criterion. Motivated by Liu (2022), we propose an even broader generalization of neighborhood mechanisms by permitting the neighborhood range to adapt based on previous cycles and incorporating any non-negative neighborhood sizes, encompassing NTTC mechanisms as a subset.

In this generalized framework, neighborhood sizes evolve based on prior sub-allocations and exchanges. However, not every size configuration supports strategy-proof outcomes. To address this, we introduce a sufficient condition known as the **available set expanding condition**, which

[2]For an illustration of an OSP-implementable neighborhood mechanism distinct from the 1-neighborhood mechanism, refer to Section A.4

ensures strategy-proofness within these generalized neighborhood mechanisms. The elaboration on generalized neighborhood mechanisms and the available set expanding condition is deferred to Section A.5.

The following theorem underscores the attributes of generalized neighborhood mechanisms:

Theorem 4. *A generalized neighborhood mechanism f guarantees individual rationality and Pareto efficiency. Moreover, if f adheres to the available set expanding condition, it also ensures strategy-proofness.*

We further examine the interplay between our proposed mechanisms and the NTTC mechanisms from Liu (2022), with the latter's definition detailed in Section A.6.

While neighborhood mechanisms are defined by agent-specific neighborhood sizes, NTTC mechanisms may exhibit path-dependent neighborhood structures. Thus, an NTTC mechanism might not fit within the neighborhood mechanisms framework. Conversely, neighborhood mechanisms can encapsulate strategies not available in NTTC mechanisms due to their overlapping vision ranges among agents within the same step. Nonetheless, every NTTC mechanism aligns with a generalized neighborhood mechanism that meets the available set expanding condition, as stated below:

Proposition 2. *Under single-peaked preferences, the following are true:*

- *The set of NTTC mechanisms and the set of r-neighborhood mechanisms are incomparable.*
- *The set of NTTC mechanisms and the set of neighborhood mechanisms are incomparable.*
- *For every NTTC mechanism f, there exists an equivalent generalized neighborhood mechanism g that satisfies the available set expanding condition.*

An illustrative example in Section A.7 demonstrates a 2-neighborhood mechanism that does not align with any NTTC mechanism, further showing that some NTTC mechanisms fall outside the neighborhood mechanisms class. The proof for the third claim is presented in Section A.8.

Our mechanisms, compared to NTTC mechanisms, offer several advantages. Primarily, the generalized neighborhood mechanism class encompasses NTTC mechanisms as special cases, offering a broader range of strategic options. Additionally, while NTTC mechanisms restrict each step's trading to agents with a unified vision range, neighborhood mechanisms enable variable vision ranges among agents in the same step. Unlike Liu (2022), which necessitates separate neighborhood trees to replicate the Crawler and Dual Crawler, our approach integrates both within the 1-neighborhood mechanism.Furthermore, although NTTC mechanisms are strategy-proof, we show that neighborhood mechanisms extend to being group strategy-proof[3].

5 Conclusion

This paper enhances the literature on market mechanisms by extending matching within limited distances for individuals with single-peaked preferences. We introduce a comprehensive class of mechanisms, known as r-neighborhood mechanisms, which encapsulate the TTC mechanism and the Crawler as special cases, and further broaden this concept to define neighborhood mechanisms. These mechanisms are demonstrated to be group strategy-proof, individually rational, and Pareto efficient. Our analysis reveals that the Crawler, Dual Crawler, and 1-neighborhood mechanism yield equivalent outcomes. Within the spectrum of r-neighborhood mechanisms, the 1-neighborhood mechanism stands out as the sole mechanism implementable in obviously dominant strategies. Moreover, we highlight that certain neighborhood mechanisms, distinct from the Crawler, are also OSP-implementable.

The paper also opens a pathway for future inquiry into the full characterization of strategy-proof, efficient, and individually rational mechanisms beyond the scope of neighborhood mechanisms. This unresolved characterization problem presents a fertile ground for further research, suggesting that the landscape of such mechanisms is rich and potentially harbors yet undiscovered structures

[3]A counterexample showcasing a NTTC mechanism failing group strategy-proofness is provided in Section A.9

A Appendix

A.1 Proof of Proposition 1

Proof. The equivalence of the $\bar{n}-1$-neighborhood mechanism and the TTC mechanism is straightforward, given their definitions. In both mechanisms, every agent can point to and trade with the owner of her most preferred house among all houses at every step, leading to identical outcomes.

For the second statement, consider a preference profile $P = (P_1, P_2, \ldots, P_{\bar{n}})$. Let $C_i(P)$ denote the house received by agent i in the matching selected by the Crawler, and $f_i(P)$ denote the house received in the matching selected by the 1-neighborhood mechanism. The goal is to show $C_i(P) = f_i(P)$ for all agents i. Denote by i_k the agent matched in the k^{th} step of the Crawler, and by m_k the number of agents "crawling" in that step. Assume agents $i_k^1, i_k^2, \ldots, i_k^{m_k}$ move from houses $h_k^1, h_k^2, \ldots, h_k^{m_k}$ to $h_k^0, h_k^1, h_k^2, \ldots, h_k^{m-1}$ respectively, ordered such that $i_k^1 < i_k^2 < \cdots < i_k^{m_k}$. Then, agent i_k starts at house h_k^0 and ultimately receives $h_k^{m_k} = C_{i_k}(P)$.

Firstly, we establish $C_{i_1}(P) = f_{i_1}(P)$ and show that agent i_1 trades with agent i_1^m for house h_1^m for $m = 1, 2, \ldots, m_1$ in the 1-neighborhood mechanism. If agent i_1 already occupies her most preferred house, implying no "crawling" occurs in the first step of the Crawler, then $C_{i_1}(P)$ and $f_{i_1}(P)$ both equal h_{i_1}. If agent i_1 prefers to move rightward, it implies agents $i_1^1, i_1^2, \ldots, i_1^{m_1}$ aim leftward according to the Crawler's definition. In the 1-neighborhood mechanism's first step, agent $i_1^{m_1}$ and agent i_1 being direct neighbors point to each other, facilitating a trade. This iterative process leads to agent i_1 trading up to house $h_1^{m_1}$, hence $C_{i_1}(P) = f_{i_1}(P) = h_1^{m_1}$.

By induction, assuming for all $j < k$, agent i_j completes trades in the 1-neighborhood mechanism as in the Crawler, i.e., $C_{i_j}(P) = f_{i_j}(P)$, we aim to prove $f_{i_k}(P) = C_{i_k}(P)$. If $i_k > \min\{i_1, i_2, \ldots, i_{k-1}\}$, then agent i_k prefers the currently occupied house to the house of the right direct neighbor at any step of the Crawler. Thus, no agent crawls in step k, and $h_k^0 = C_{i_k}(P)$. By the induction hypothesis, for all $j < k$, if i_k crawls from a house h to another house h' in step j of the Crawler, then agent i_k trades with i_j from h to h' in the 1-neighborhood mechanism. Consequently, agent i_k occupies $C_{i_k}(P)$ at some step of the 1-neighborhood mechanism. Since

$f_{i_j}(P) = C_{i_j}(P)$ holds for all $j < k$, agent i_k does not trade for any house strictly better than $C_{i_j}(P)$ in the 1-neighborhood mechanism. Hence, no agent crawls in step k, and $C_{i_k}(P) = f_{i_k}(P)$.

If $i_k < \min\{i_1, i_2, \ldots, i_{k-1}\}$, then agent i_k does not crawl in any step of the Crawler, and h_k^0 is the initial endowment h_{i_k}. For any $n = 0, 1, \ldots, m_k$, we must show that agent i_k trades with i_k^n from the house h_k^{n-1} to h_k^n in the 1-neighborhood mechanism. By the induction hypothesis, for any $j < k$ and $n < m_k$, if an agent i_k^n crawls from a house h to another house h' in step j of the Crawler, then agent i_k^n trades with i_j in the 1-neighborhood mechanism. Let $i_k^0 = i_k$. Then, for any $n = 0, 1, 2, \ldots, m_k$, there exists an integer t_n such that agent i_k^n occupies h_k^n at the beginning of step t_n of the 1-neighborhood mechanism. Now, we show that agent i_k trades with i_k^1 from h_k^0 to h_k^1 in step $\max\{t_0, t_1\}$ of the 1-neighborhood mechanism. If $t_0 = t_1$, then agent i_k and i_k^1 point to each other in step t_0 and trade in a cycle. If $t_0 > t_1$, we must demonstrate that agent i_k^1 does not trade from h_k^1 to h_k^0 before step t_0 of the 1-neighborhood mechanism. Assuming the contrary, and agent i_k^1 trades with an agent i_p, chosen in step p of the Crawler, from h_k^1 to h_k^0 before step t_0. Since agent i_k^1 does not crawl from h_k^1 to h_k^0 in the first $k-1$ steps of the Crawler, we have $p \notin \{1, 2, \ldots, k-1\}$ by induction, implying $p > k$. However, agent i_p occupying the house h_k^0 before step t_0 implies $i_p > i_k$. The assumption that i_p trades with i_k^1 suggests i_p prefers h_k^1 over h_k^0, thus i_p should be screened out before i_k, leading to a contradiction as $p < k$.

Conversely, if $t_0 < t_1$, we must prove that agent i_k does not trade from h_k^0 to h_k^1 before step t_1. Assume otherwise, and agent i_k trades with an agent i_p from h_k^0 to h_k^1 before step t_1. Given that each agent trades with their direct neighbor in the 1-neighborhood mechanism, the house h_{i_p} is situated between h_{i_k} (which is h_k^0) and $h_{i_k^1}$. Let h^* denote the house owned by agent i_p in step k of the Crawler. The assumption $i_k < \min\{i_1, i_2, \ldots, i_{k-1}\}$ implies that h^* is located on the right side of h_k^0. If $p > k$ holds, then agent i_k^1 cannot move to the left side of agent i_p before step k of the Crawler. Thus, the house h^* is positioned on the left side of h_k^1. However, no remaining house exists between h_k^0 and h_k^1 in step k of the Crawler. Therefore, $p > k$ cannot hold. If $p < k$, then agent i_p is removed before step k of the Crawler. Consequently, $f_{i_p}(P) = C_{i_p}(P) = h^*$, and $f_{i_p}(P) R_{i_p} h_k^0$ holds. Yet, the assumption that agent i_p trades with agent i_k for house h_k^0 suggests that $h_k^0 R_{i_p} f_{i_p}(P)$, contradicting the single-peaked preference. Thus, agent i_k trades with i_k^1 from h_k^0 to h_k^1 in step $\max\{t_0, t_1\}$ of the 1-neighborhood mechanism. Subsequently, agent i_k occupies house h_k^1 in step $\max\{t_0, t_1\} + 1$, and agent i_k^2 occupies h_k^2 in step t_2. For the same reason, agent i_k trades with i_k^2 from h_k^1 to h_k^2 in step $\max\{t_0 + 1, t_1 + 1, t_2\}$. By induction, we establish that agent i_k sequentially trades with $i_k^1, i_k^2, \ldots, i_k^{m_k}$ for houses $h_k^1, h_k^2, \ldots, h_k^{m_k}$ in the 1-neighborhood mechanism, where $h_k^{m_k} = C_{i_k}(P)$. Since $f_{i_j}(P) = C_{i_j}(P)$ for all $j < k$, agent i_k cannot trade for any house superior to $C_{i_k}(P)$, ensuring $C_{i_k}(P) = f_{i_k}(P)$.

By induction, we confirm that $C_i(P) = f_i(P)$ for every agent i, thus validating the second statement of the proof.

□

A.2 Proof of Theorem 1, Theorem 3 and Theorem 4

We complete the proof with the following three parts.

Lemma 1. *If a mechanism f is a generalized neighborhood mechanism, then f is individually rational and Pareto efficient.*

Proof. **Individual rationality:** The generalized neighborhood mechanisms ensure individual rationality, as the initial endowment always lies within the agent's neighborhood until the first trade. Thus, an agent only trades for houses that are better than or equal to her initial endowment, resulting in a final allocation that is weakly preferred to the initial one.

Pareto efficiency: The Pareto efficiency of the generalized neighborhood mechanisms can be demonstrated through induction. All agents removed in the first step are matched with their most preferred houses. By removing houses matched in the first step and proceeding to the second step, agents removed in this step receive their most preferred houses among the remaining ones. Thus, these agents cannot achieve a strictly better outcome without making at least one of the agents removed in the first step worse off. By applying this reasoning inductively, it is shown that agents matched in step k receive their most preferred houses among those available at the beginning of step k. Hence, the generalized neighborhood mechanisms are Pareto efficient. $\square$

Lemma 2. *If a mechanism f is a generalized neighborhood mechanism satisfying the available set expanding condition, then f is strategy-proof.*

Proof. To show strategy-proofness, we must prove that $f_i(P_i, P_{-i})R_i f_i(P_i^{'}, P_{-i})$ holds for any preferences P_i, $P_i^{'}$, P_{-i}, and for any agent i. We define a house h as the (right) **boundary** of a neighborhood N_i^k if $h \in N_i^k$ and for any $h^{'} > h$, $h^{'} \notin N_i^k$. A house h is in the **interior** of N_i^k if it is not a boundary of N_i^k.

Given agent i, a preference profile $P = (P_1, P_2, \cdots, P_{\bar{n}})$, and another preference $P_i^{'}$, let $h_i^k(P_i, P_{-i})$ denote the house occupied by agent i at the end of step k with preference profile (P_i, P_{-i}), and let $H^k(P_i, P_{-i})$ denote the remaining houses at the beginning of step k with (P_i, P_{-i}). Without loss of generality, assume the peak of P_i is to the right side of h_i. Due to single-peakedness, the left neighbor of agent i is worse than the occupied one at any step, thus agent i will never point to the left side of the occupied house in any step. If misreporting leads to the same house, the condition $f_i(P_i, P_{-i})R_i f_i(P_i^{'}, P_{-i})$ is satisfied. Otherwise, assume the first deviation occurs at step k_0, where $k_0 = \min\{k \mid h_i^k(P_i, P_{-i}) \neq h_i^k(P_i^{'}, P_{-i})\}$. This implies that agent i is in a cycle at step $k_0 - 1$ with either (P_i, P_{-i}) or $(P_i^{'}, P_{-i})$ if $k_0 \geq 2$, so agent i points to the owner of the most preferred house in the neighborhood at step k_0 with either P_i or $P_i^{'}$. There are three possible cases at step k_0: (1) agent i with P_i moves to the interior of the neighborhood, (2) agent i with P_i moves to the boundary of the neighborhood, or (3) agent i with P_i does not participate in a cycle at step k_0.

We show that if agent i with P_i moves to the interior of the neighborhood at step k_0, then $f_i(P_i, P_{-i})R_i f_i(P_i^{'}, P_{-i})$ holds. If truth-telling results in a house $h_i^k(P_i, P_{-i})$ in the interior, then agent i occupies the most preferred house in the neighborhood after the trade, which is also the most preferred house among all remaining houses at the beginning of step k_0 due to single-peakedness. Hence, misreporting to $P_i^{'}$ cannot result in a strictly better house.

In the case where agent i, reporting preference P_i, moves to the boundary of the neighborhood in step k_0, agent i reporting preference $P_i^{'}$ points to the interior of the neighborhood at step k_0. Consequently, the peak of $P_i^{'}$ among all remaining houses, $H^k(P_i, P_{-i})$, denoted as $\tau(P_i^{'}, H^k(P_i, P_{-i}))$, is situated within the interior of the neighborhood. Hence, $\tau(P_i^{'}, H^k(P_i, P_{-i}))$ is positioned to the left of $h_i^k(P_i, P_{-i})$. Assume the boundary house is occupied by agent $i^{'}$ at the onset of step k_0. A chain forms from agent $i^{'}$ to agent i during step k_0, persisting until agent i engages in trading. In accordance with the available set expanding condition, $h_i^k(P_i, P_{-i})$ consistently remains within agent i's neighborhood prior to the subsequent trade. Given that a cycle emerges if agent i directs from the current house to either agent $i^{'}$ or herself, agent i, reporting $P_i^{'}$, ultimately secures a house weakly better than $h_i^k(P_i, P_{-i})$, implying $f_i(P_i^{'}, P_{-i})P_i^{'}h_i^k(P_i, P_{-i})$ holds. The premise that $\tau(P_i^{'}, H^k(P_i, P_{-i}))$ is located to the left of $h_i^k(P_i, P_{-i})$ suggests that for any house $h > h_i^k(P_i, P_{-i})$, the relationship $h_i^k(P_i, P_{-i})P_i^{'}h$ is maintained. Thus, $f_i(P_i^{'}, P_{-i})$ is either positioned to the left of $h_i^k(P_i, P_{-i})$ or is identically $h_i^k(P_i, P_{-i})$. Recalling that $\tau(P_i, H^k(P_i, P_{-i}))$ resides on the right of $h_i^k(P_i, P_{-i})$ or is exactly $h_i^k(P_i, P_{-i})$, it follows that $f_i(P_i, P_{-i})R_i h_i^k(P_i, P_{-i})R_i f_i(P_i^{'}, P_{-i})$, rendering the deviation unprofitable.

In the final case, wherein the agent with P_i does not engage in trading during step k_0, but the agent with $P^{'}$ conducts a trade with agent $i^{''}$ for the house $h^{''}$ within a cycle, a sequence extends from agent $i^{''}$ to agent i until agent i trades, with the house $h^{''}$ remaining available until a trade materializes for P_i. If agent i, with P_i, targets the left side of $h^{''}$ in step k_0, she is eliminated after the ensuing trade, establishing $f_i(P_i, P_{-i})R_i h^{''} R_i f_i(P_i^{'}, P_{-i})$. Conversely, if agent i, with P_i, aims at the right side of $h^{''}$ in step k_0, we deduce $f_i(P_i^{'}, P_{-i}) = h^{''}$. The continuous availability of $h^{''}$ until agent i's next trade with P_i implies $f_i(P_i, P_{-i})R_i f_i(P_i^{'}, P_{-i})$.

Hence, the mechanism f is verified as strategy-proof. □

Lemma 3. *If a mechanism f is a neighborhood mechanism, then f is group strategy-proof.*

Proof. **Non-bossiness:** We show that f is non-bossy. Assume a preference profile P and a preference $P_i^{'}$ such that $f_i(P_i, P_{-i}) = f_i(P_i^{'}, P_{-i})$. Our goal is to prove that $f(P_i, P_{-i}) = f(P_i^{'}, P_{-i})$. If agent i points to the same agent in every step, the same matching is selected. Otherwise, consider the first step k where she points to different agents. Assume agent i, with preferences P_i and $P_i^{'}$, moves to houses h and $h^{'}$ in the next *trade* of step k, respectively. Without loss of generality, if agent

i moves rightwards and h is closer to the currently occupied house, then either $h < h^{'}$ or $h = h^{'}$ holds. There is always a chain from the owner of h to agent i before the next trade of $P^{'}$. Therefore, the premise that the house h is closer to the currently occupied house implies that agent i with P_i does not point to the boundary of the neighborhood in step k and will be removed after the next trade. Thus, $f_i(P_i, P_{-i}) = h$ holds. As agents in neighborhood mechanisms always move in the same direction, $f_i(P_i, P_{-i}) = f_i(P_i^{'}, P_{-i}) = h$ implies $h^{'} = h$. Consequently, agent i, whether with P_i or $P_i^{'}$, trades in the same cycle in the next trade and is removed after this trade in each scenario. Since the preference profile of other agents remains unchanged, the deviation does not affect the cycles in the algorithm. Hence, $f(P_i, P_{-i}) = f(P_i^{'}, P_{-i})$, establishing that f is non-bossy.

Mandal and Roy (2021) and Tamura (2022) show that a mechanism f is group strategy-proof if and only if it is strategy-proof and non-bossy. Since f is both strategy-proof and non-bossy, it follows that f is group strategy-proof. □

A.3 Proof of Theorem 2

Proof. We aim to show that an r-neighborhood mechanism f with $|I| \geq 4$ is OSP-implementable if and only if $r = 1$. When $r = 1$, the 1-neighborhood mechanism f is outcome equivalent to the Crawler, as shown in Proposition 1. According to Bade (2019), the Crawler is OSP-implementable, making the 1-neighborhood mechanism OSP-implementable as well.

We then prove that if $|I| = 4$ and $r \geq 2$, the r-neighborhood mechanism cannot be implemented in obviously dominant strategies. Assume an extensive-form game G with a profile of obviously dominant strategies $(S_i(\cdot))_{i \in I}$ implements f, where $I = \{1, 2, 3, 4\}$ and $H = \{a, b, c, d\}$. Each agent i is endowed with a distinct house from the set $\{a, b, c, d\}$. If $i < j$, the endowment of agent i is located to the left of agent j's endowment.

Consider a subset of the preference domain where each agent i has two possible preferences P_i and $P_i^{'}$ as follows:

$$
\begin{aligned}
&P_1 : bR_1cR_1dR_1a \quad && P_1' : dR_1'cR_1'bR_1'a \\
&P_2 : cR_2dR_2bR_2a \quad && P_2' : dR_2'cR_2'bR_2'a \\
&P_3 : bR_3aR_3cR_3d \quad && P_3' : aR_3'bR_3'cR_3'd \\
&P_4 : cR_4bR_4aR_4d \quad && P_4' : aR_4'bR_4'cR_4'd
\end{aligned}
$$

Suppose there is a history θ in game G where agent 1 must choose between actions corresponding to P_1 and P_1'. We claim that such a history θ cannot precede all equivalent histories for agents 2, 3, and 4. This means, given such a history θ for agent 1, there exists a subhistory $\theta' \succ \theta$ and $j \in \{2,3,4\}$ where agent j must choose between actions corresponding to P_j and P_j'.

By contradiction, assume agent 1 with P_1 chooses the action corresponding to P_1, facing agents with preferences P_2', P_3, and P_4, then agent 1 receives house a. If agent 1 chooses the other action and faces P_2', P_3', and P_4, agent 1 receives house d, and dP_1a holds, indicating it is not an obviously dominant strategy to choose the action corresponding to P_1. Therefore, agent 1 cannot be the first to have such a non-singleton action set.

Now consider a history θ at which agent 2 moves, and two actions correspond to P_2 and P_2'. We need to show that agent 2 also cannot be the first to face such a decision. Suppose that with P_2', agent 2 chooses the action corresponding to P_2' and encounters preferences P_1', P_3', and P_4. In this scenario, agent 2 would receive house b. However, if agent 2, opting for the action corresponding to P_2, faces preferences P_1, P_3, and P_4', she would receive house c, and thus $cP_2'b$ holds, indicating a preference for c over b under P_2'. Therefore, agent 2 also cannot be the first to have such a non-singleton action set.

By symmetry, applying similar logic to agents 3 and 4, we find that no agent can be the first to have a non-singleton action set. This leads to a contradiction, as it implies no such game G and profile of obviously dominant strategies can implement f. Hence, an r-neighborhood mechanism with $r \geq 2$ cannot be OSP-implementable when there are 4 agents.

If $r \geq 3$, then an r-neighborhood mechanism f is outcome equivalent to the TTC mechanism when $|I| = 4$. The TTC mechanism cannot be implemented in obviously dominant strategies when there are at least 4 agents (Bade , 2019). Therefore, r-neighborhood mechanisms with $r \geq 3$ are not OSP-implementable when $|I| = 4$.

Next, we show that an r-neighborhood mechanism f cannot be implemented in obviously dominant strategies with $r \geq 2$ and $|I| > 4$. Assume the contrary, and let an extensive-form game G along with a profile of obviously dominant strategies $(S_i(\cdot))_{i \in I}$ implement f. Fix the preferences of agent i as $\bar{P}_i$ for any $i > 4$. Then, the game G and $(S_i(\cdot))_{i \in I}$ induce an extensive-form game $\bar{G}$, where only agents $1, 2, 3$, and 4 make choices at the nodes. Consequently, $\bar{G}$ and the profile of obviously dominant strategies $(S_i(\cdot))_{i \in \{1,2,3,4\}}$ implement f with 4 agents, leading to a contradiction.

In conclusion, the 1-neighborhood mechanism is the only OSP-implementable mechanism with $|I| \geq 4$ in the class of r-neighborhood mechanisms.

□

A.4 An Example of an OSP-Implementable Neighborhood Mechanism Other Than the 1-Neighborhood Mechanism

Consider $I = \{1, 2, 3, 4\}$ and $H = \{h_1, h_2, h_3, h_4\}$. The neighborhood mechanism f corresponds to the size function $S : I \times H \to \mathbb{Z}_+ \times \mathbb{Z}_+$ defined as:

$$S(i, h) = \begin{cases} (3, 3), & \text{if } i = 1 \text{ or } 4, \\ (1, 1), & \text{if } i = 2 \text{ or } 3. \end{cases}$$

This implies that agents 1 and 4 can choose from all remaining houses, while agents 2 and 3 can only choose from their direct neighbors. To demonstrate that the neighborhood mechanism f is not outcome equivalent to the 1-neighborhood mechanism, consider the following preference profile:

P_1	P_2	P_3	P_4
h_4	h_4	h_4	h_1
h_3	h_3	h_3	h_2
h_2	h_2	h_2	h_3
h_1	h_1	h_1	h_4

In the 1-neighborhood mechanism, the matching $(4h_1, 1h_2, 2h_3, 3h_4)$ is selected. In the neighborhood mechanism f, agents 1 and 4 trade in step 1, resulting in the matching $(4h_1, 2h_2, 3h_3, 1h_4)$. Hence, f is not outcome equivalent to the 1-neighborhood mechanism.

To demonstrate that f is OSP-implementable, consider the following algorithm[4]:

Step 1. Sequentially ask agents 1, 2, 3, and 4 whether their initial endowment h_i is their top choice, i.e., whether $\tau(i, H) = h_i$ holds for $i = 1, 2, 3, 4$. Once an agent i replies yes, assign h_i to her. According to Bade (2019), both TTC and the Crawler can be implemented in obviously dominant strategies when $|I| = 3$. If all agents say no, proceed to step 2.

Step 2. Sequentially ask agents 2 and 3 whether their top choice is on the left or right side of their initial endowment. If both agent 2 and 3 choose left, go to step 2.1; If both agent 2 and 3 choose right, go to step 2.2; If agent 2 choose left and 3 choose right, go to step 2.3; If agent 2 choose right and 3 choose left, go to step 2.4.

Step 2.1 If both agent 2 and 3 choose left, let agent 1 choose her most-preferred house in $\{h_2, h_3, h_4\}$. If agent 1 chooses h_2, select the matching $(2h_1, 1h_2, 3h_3, 4h_4)$. If agent 1 chooses h_3, select the matching $(2h_1, 3h_2, 1h_3, 4h_4)$. If agent 1 chooses h_4, go to step 2.1.a.

Step 2.1.a If agent 1 chooses h_4, let agent 4 choose her most-preferred house in $\{h_1, h_2, h_3\}$. If agent 4 chooses h_1, select the matching $(4h_1, 2h_2, 3h_3, 1h_4)$. If agent 4 chooses h_2,

[4] This algorithm's construction is inspired by Example 1 in Troyan (2019), proposing a mechanism that OSP-implements the TTC mechanism when $|I| = 3$.

select the matching $(2h_1, 4h_2, 3h_3, 1h_4)$. If agent 4 chooses h_4, select the matching $(2h_1, 3h_2, 4h_3, 1h_4)$.

Step 2.2 If both agent 2 and 3 choose right, let agent 4 choose her most-preferred house in $\{h_1, h_2, h_3\}$. If agent 4 chooses h_2, select the matching $(1h_1, 4h_2, 2h_3, 3h_4)$. If agent 4 chooses h_3, select the matching $(1h_1, 2h_2, 4h_3, 3h_4)$. If agent 4 chooses h_1, go to step 2.2.a.

Step 2.2.a If agent 4 chooses h_1, let agent 1 choose her most-preferred house in $\{h_2, h_3, h_4\}$. If agent 1 chooses h_2, select the matching $(4h_1, 1h_2, 2h_3, 3h_4)$. If agent 1 chooses h_3, select the matching $(4h_1, 2h_2, 1h_3, 3h_4)$. If agent 1 chooses h_4, select the matching $(4h_1, 2h_2, 3h_3, 1h_4)$.

Step 2.3 If agent 2 chooses left and 3 choose right, ask agent 1 whether $h_2 R_1 h_3$ holds. If yes, select the matching $(2h_1, 1h_2, 4h_3, 3h_4)$. If no, go to step 2.3.a.

Step 2.3.a Ask agent 4 whether $h_3 R_4 h_2$ holds. If yes, select the matching $(2h_1, 1h_2, 4h_3, 3h_4)$. If no, go to step 2.3.b.

Step 2.3.b Ask agent 1 whether $h_3 R_1 h_4$ holds. If yes, go to step 2.3.c. If no, go to step 2.3.d.

Step 2.3.c Ask agent 4 whether $h_1 R_4 h_2$ holds. If yes, select the matching $(4h_1, 2h_2, 1h_3, 3h_4)$. If no, select the matching $(2h_1, 4h_2, 1h_3, 3h_4)$.

Step 2.3.d Ask agent 4 whether $h_1 R_4 h_2$ holds. If yes, select the matching $(4h_1, 2h_2, 3h_3, 1h_4)$. If no, select the matching $(2h_1, 4h_2, 3h_3, 1h_4)$.

Step 2.4 If agent 2 chooses right and 3 choose left, sequentially ask agent 2(agent 3, respectively) whether $h_3 R_2 h_4$ ($h_2 R_3 h_1$, respectively) holds. If agent 2(3, respectively) responds yes, assign h_3(h_2, respectively) to her. According to Bade (2019), either TTC and the Crawler can be implemented in obviously dominant strategies when $|I| = 3$. If both agents responds no, go to step 2.4.a.

Step 2.4.a ask agent 1 whether $h_2 R_1 h_3$ holds. If yes, select the matching $(3h_1, 1h_2, 4h_3, 2h_4)$. If no, go to step 2.4.b.

Step 2.4.b Ask agent 4 whether $h_3 R_4 h_2$ holds. If yes, select the matching $(3h_1, 1h_2, 4h_3, 2h_4)$. If no, go to step 2.4.c.

Step 2.4.c Ask agent 1 whether $h_3 R_1 h_4$ holds. If yes, go to step 2.4.d. If no, go to step 2.4.e.

Step 2.4.d Ask agent 4 whether $h_1 R_4 h_2$ holds. If yes, select the matching $(4h_1, 3h_2, 1h_3, 2h_4)$. If no, select the matching $(3h_1, 4h_2, 1h_3, 2h_4)$.

Step 2.4.e Ask agent 4 whether $h_1 R_4 h_2$ holds. If yes, select the matching $(4h_1, 3h_2, 2h_3, 1h_4)$. If no, select the matching $(3h_1, 4h_2, 2h_3, 1h_4)$.

Now, we show that truth-telling is an obviously dominant strategy in each step. In Step 1, if $\tau(i, H) = h_i$ holds for an agent i and she responds "yes," she receives her top choice h_i. Otherwise, according to individual rationality, she will receive a house weakly better than h_i. In each case, truth-telling is the obviously dominant strategy for each agent. Together with the argument in Bade (2019), deviations in this step are obviously dominated by truth-telling.

In Step 2, according to single-peakedness and individual rationality, reporting a wrong direction leads to a house weakly worse than the initial endowment. In Step 2.1, no matter which house agent 1 chooses, she ultimately receives the chosen house. Similarly, in Step 2.1.a, no matter which house agent 4 chooses, she ultimately receives the chosen house. Thus, truthful reports are obviously dominant strategies in these steps. By symmetry, the same argument applies to Steps 2.2 and 2.2.a.

In Step 2.3, if agent 1 responds $h_2 R_1 h_3$, she receives h_2. If agent 1 responds "no," she receives either house h_2 or the most preferred house if reporting truthfully. In Steps 2.3.a, 2.3.c, and 2.3.d, agent 4 ultimately receives the most-preferred house if reporting truthfully, regardless of how other agents choose in the following steps. The same argument applies to Step 2.2.b for agent 1. Thus, truthful reports are obviously dominant strategies in these steps.

In Step 2.3, agents 2 and 3 exchange their houses. If we regard h_2 and h_3 as the initial endowments of agents 3 and 2, respectively, the argument about Step 1 applies to Step 2.4, and the argument about Steps 2.3, 2.3.a, 2.3.b, 2.3.c, 2.3.d applies to Steps 2.4.a to 2.4.e, respectively.

Therefore, the deviation in each step is obviously dominated. Thus, f is OSP-implementable.

A.5 The Generalized Neighborhood Mechanism and Available Set Expanding Condition

Let e be the initial allocation. Given a subset of agents $\hat{I}$ and a subset of houses $\hat{H}$ such that $|\hat{I}| = |\hat{H}|$, a sub-allocation m is a one-to-one mapping from $\hat{I}$ to $\hat{H}$, and let I_m and H_m denote the agents and houses involved in m, respectively. A sub-allocation m is said to be nested in another sub-allocation m' if $H_m \subseteq H_{m'}$ and $I_m \subseteq I_{m'}$. The set of all sub-allocations is denoted by M. A sequence of sub-allocations $m_1 m_2 \cdots$ is called a sub-allocation history if $m_1 = e$ and m_{k+1} is nested in m_k for $k = 1, 2, \cdots$. If a sub-allocation history γ is created by appending some sub-allocations to history γ', we denote $\gamma' \succ \gamma$ and say that γ' is a sub-history of γ. For example, $m_1 m_2$ is a sub-history of both itself and $m_1 m_2 m_3$. Let Γ^m denote the set of all sub-allocation histories. Let $|\gamma|$ denote the number of allocations in the history γ. Let $\bar{\Gamma} = \{\gamma \in \Gamma : |\gamma| = \infty\}$ be the set of all infinite histories and $\Gamma_k = \{\gamma \in \Gamma : |\gamma| = k\}$. Similarly, a sequence of sets of agents $\phi = I_1 I_2 \cdots$ is called a self-cycle history. Let Φ denote the set of all self-cycle histories, and let $|\phi|$ denote the number of sets of agents in ϕ. Let $\bar{\Phi} = \{\phi \in \Phi : |\phi| = \infty\}$ be the set of all infinite histories and $\Phi_k = \{\phi \in \Phi : |\phi| = k\}$.

A **generalized size function** is a function $S : I \times \bigcup_{k \in \mathbb{Z}_+} (\Gamma_k \times \Phi_k) \to \mathbb{N} \times \mathbb{N}$ such that for any $i \in I$ and $\bar{\gamma} \in \bar{\Gamma}$, there exists a history $\gamma' \succ \bar{\gamma}$ such that $S(i, \gamma, \phi) = (\bar{n}, \bar{n})$ for any $\gamma' \succ \gamma$ and $\phi \in \Phi_{|\gamma|}$.

In a generalized neighborhood mechanism, the sizes of neighborhoods depend on the previous sub-allocation and previous exchanges. Given the history of sub-allocation $e m_1 m_2 \cdots$, the cycles consisting of at least 2 agents in step k can be derived from m_k and m_{k-1}. When considering the set of self-cycles, the sizes are allowed to depend on the exchanges in the previous steps. Meanwhile, it is more flexible to consider the history of sub-allocation and self-cycles than only the previous exchanges, as the size may change according to the set of removed agents.

Given a generalized size function S, the **generalized neighborhood mechanism** operates as follows:

Step 0. Initialize with $H_1 = H$, $m_0 = e$, $\tilde{I}_0 = \emptyset$, and $\omega_1(i) = h_i$ for all $i = 1, 2, ..., \bar{n}$.

Step k, for $k \geq 1$. Pointing: For each agent i, let N_k^i denote the neighborhood with the pair of sizes $(l, r) = S(i, em_1m_2 \cdots m_{k-1}, \tilde{I}_0\tilde{I}_1 \cdots \tilde{I}_{k-1})$ for the current endowment $\omega_k(i)$, i.e., $N_k^i = N(\omega_k(i), H_k, (l, r))$. Each agent i points to an agent $a_k(i)$ as follows:

1. If $k = 1$ or $a_{k-1}(i)$ is in a cycle in step $k-1$, then let agent i point to the agent whose endowment is her most preferred house in the neighborhood according to P_i, i.e., $a_k(i) = \omega_k^{-1}(\tau(P_i, N_k^i))$.
2. If $k \geq 2$ and $a_{k-1}(i)$ is not in any cycle in step $k-1$, then let agent i point to the same agent as in step $k-1$, i.e., $a_k(i) = a_{k-1}(i)$.

Trading: If there is at least one cycle (including self-cycles) and cycles do not intersect, let agents in cycles trade. For each agent i, define:

1. If i is in a cycle in step k, then let agent i be endowed with the house she points to, i.e., $\omega_{k+1}(i) = \omega_k(a_k(i))$.
2. If i is not in any cycle in step k, then let agent i retain her current endowment, i.e., $\omega_{k+1}(i) = \omega_k(i)$.

Matching: For each agent i, if $\omega_{k+1}(i) = \tau(P_i, H_k)$ holds, remove agent i with the house $\omega_{k+1}(i)$. If all agents are removed, terminate the process. Otherwise, let H_{k+1} be the set of remaining houses. Define m_k as the sub-allocation such that I_{m_k} is the set of unmatched agents at the end of step k, and $m_k(i) = \omega_{k+1}(i)$ for all $i \in I_{m_k}$. Let $\tilde{I}_k$ be the set of agents who are in self-cycles in step k. Proceed to step $k+1$.

By definition, the size of neighborhoods in a neighborhood mechanism depends only on the identity of agents and houses. Therefore, a neighborhood mechanism with $S_1 : I \times H \to \mathbb{Z}_+ \times \mathbb{Z}_+$ is equivalent to the generalized neighborhood mechanism with $S_2 : I \times \bigcup_{k \in \mathbb{Z}_+}(\Gamma_k \times \Phi_k) \to \mathbb{N} \times \mathbb{N}$ such that $S_2(i, \gamma, \phi) = S_1(i, h)$ for all $h = H_m(\gamma)$ and for any $\phi \in \Phi_{|\gamma|}$, where m is the last sub-allocation of γ.

In a generalized neighborhood mechanism, the sizes of neighborhoods depend on previous cycles. However, not all size functions correspond to a strategy-proof mechanism. The following example illustrates a generalized neighborhood mechanism that is not strategy-proof.

Example 2. Suppose $I = \{1, 2, 3, 4\}$ and $H = \{h_1, h_2, h_3, h_4\}$. Consider the generalized neighborhood mechanism f corresponding to the following size function S:

$$S(i, \gamma, \phi) = \begin{cases} (0, 1), & \text{if } i = 1, \phi = \tilde{I}_0 \tilde{I}_1 \text{ such that } \tilde{I}_1 = \{3\}, \forall \gamma, \\ (3, 3), & \text{otherwise.} \end{cases}$$

In words, if agent 3 is the only agent in a self-cycle in step 1, then the neighborhood of agent 1 in step 2 contains only the adjacent house. Otherwise, all agents can point to any house. Consider the following preference profile P and an alternative preference P_1':

P_1	P_2	P_3	P_4	P_1'
h_3	h_4	h_3	h_1	h_4
h_4	h_3	h_4	h_2	h_3
h_2	h_2	h_2	h_3	h_2
h_1	h_1	h_1	h_4	h_1

If each agent i reports P_i, then in the first step of f, agent 1 (2, 3, 4, respectively) points to agent 3 (4, 3, 2, respectively). The only cycle in step 1 is agent 3 pointing to herself, and $\tilde{I}_1 = \{3\}$ holds. By definition, agent 2 (and 4, respectively) still points to agent 4 (and 1, respectively). As $S(1, \gamma, \tilde{I}_0 \tilde{I}_1) = (0, 1)$ holds, agent 1 points to agent 2 in step 2. Thus, $1 \to 2 \to 4 \to 1$ is the only cycle in step 2. Consequently, we have $f_1(P) = h_2$.

If agent 1 reports P_1', she trades with agent 4 (with h_4) and is removed in step 1. Therefore, we have $f_1(P_1', P_{-1}) = h_4$. Since $h_4 R_1 h_2$ holds, the generalized neighborhood mechanism f is not strategy-proof.

In Example 2, agent 1 is able to point to agent 4 (with house h_4) in step 1, but she cannot point to agent 4 in step 2. This situation allows agent 1 the potential to manipulate her preferences to be assigned to h_4 in step 1. To prevent such scenarios, we introduce a constraint on the size function to ensure that the range of an agent's view in a specific house cannot decrease, except for those in a self-cycle. Given a generalized neighborhood mechanism f, a preference profile P, an agent i, and step k, let $\tilde{H}_k$ denote the set of houses involved in self-cycles in step k, i.e., $\tilde{H}_k = \{h : \omega_k^{-1}(h) \in \tilde{I}_k\}$.

Condition 1. (**Available Set Expanding Condition**) Suppose S is a generalized size function and f is the corresponding generalized neighborhood mechanism. The mechanism f satisfies the Available Set Expanding Condition if, for any preference profile P, agent i, and step k, $N_k^i \setminus \tilde{H}_k \subseteq N_{k+1}^i$ when $\omega_k(i) = \omega_{k+1}(i)$.

The Available Set Expanding Condition mandates that if we consider the neighborhood at step k as the available set for an agent, then in the subsequent step, agents can view at least as many houses as they could in the current step, excluding those whose owners are in self-cycles, as long as they are endowed with the same house.

In a neighborhood mechanism, for an agent i and step k, if $\omega_k(i) = \omega_{k+1}(i)$, then the size of the neighborhoods remains constant. Thus, the neighborhood in step $k+1$ (i.e., N_{k+1}^i) includes the neighborhood in step k (i.e., N_k^i), excluding those removed in step k. Therefore, all neighborhood mechanisms satisfy the Available Set Expanding Condition. Theorem 4 demonstrates that generalized neighborhood mechanisms are individually rational and efficient. Moreover, if a generalized neighborhood mechanism satisfies the Available Set Expanding Condition, then it is strategy-proof.

Although the Available Set Expanding Condition is a sufficient condition for the incentive compatibility of a generalized neighborhood mechanism, it is not a necessary condition. The following example illustrates this point.

Example 3. Consider a set of agents $I = \{1, 2, 3\}$ and a set of houses $H = \{h_1, h_2, h_3\}$. We analyze a generalized neighborhood mechanism f defined by the following size function S:

$$S(i, \gamma, \phi) = \begin{cases} (0,0), & \text{if } i = 1 \text{ and } \phi = \tilde{I}_0 \tilde{I}_1 \text{ such that } \tilde{I}_1 = \{2\}, \forall \gamma, \\ (2,2), & \text{otherwise.} \end{cases}$$

To show that mechanism f violates the Available Set Expanding Condition, consider the preference profile:

P_1	P_2	P_3
h_2	h_2	h_1
h_3	h_3	h_2
h_1	h_1	h_3

In step 1, agent 1 can point to any agent, but the only cycle that forms is $2 \to 2$. Consequently, agent 1 can only point to herself in step 2. Thus, h_3 is in the set $N_1^1 \setminus \tilde{H}_1 = \{h_1, h_3\}$ but not in $N_2^1 = \{h_1\}$, indicating that the Available Set Expanding Condition is not satisfied.

To establish that f is strategy-proof, consider a preference profile $P = (P_1, P_2, P_3)$. Let $TTC_i(P)$ denote the house assigned to agent i under P in the Top Trading Cycles (TTC) mechanism. If $\tau(P_2, H) = h_2$, then $f_2(P) = h_2$. By efficiency, we have $f_i(P) = TTC_i(P)$ for all i. If $\tau(P_2, H) \neq h_2$, then all agents can point to any other agent, leading to $f_i(P) = TTC_i(P)$ for all i. Consequently, mechanism f is outcome equivalent to the TTC mechanism and is therefore strategy-proof.

A.6 Introduction of the NTTC Mechanism

Liu (2022) introduces the class of mechanisms known as **Neighborhood Top Trading Cycles (NTTC) mechanisms**. An NTTC mechanism determines the final allocation through a series of steps. At each step, a set of houses is chosen exogenously. Each agent currently occupying a chosen house points to the owner of her most-preferred house within this set. Agents forming cycles (including self-cycles) trade and exit the step with their new houses. This process is repeated until all chosen houses exit the step. The NTTC mechanism concludes after a step where all houses have been selected. The chosen houses are constrained to be adjacent to each other at each step, and such a set may be determined path-dependently based on allocations in previous steps. NTTC mechanisms are shown to be individually rational, Pareto efficient, and strategy-proof with single-peaked preferences.

For each NTTC mechanism, Liu (2022) specifies a tree to determine the sets available for exchange throughout the iteration process. An *available tree* is a function $T : \Gamma \to 2^H \setminus \{\emptyset\}$ satisfying the following conditions:

- For any infinite history $\bar{\gamma} \in \bar{\Gamma}$, there exists a non-terminal sub-history $\gamma \succ \bar{\gamma}$ such that $T(\gamma') = H_m$, where m is the last sub-allocation in γ.
- For any finite history $\bar{\gamma} \in \Gamma \setminus \bar{\Gamma}$, $T(\gamma') \subseteq H_m$, where m is the last sub-allocation in γ.

An available tree T is referred to as a *neighborhood tree* if it meets an additional condition: for any $h_i, h_j \in T(\gamma)$ and $h_i < h_k < h_j$, then $h_k \in T(\gamma)$ for any $h_k \in H_m$ and any $\gamma \in \Gamma \setminus \bar{\Gamma}$, where m is the last sub-allocation in γ.

Given a neighborhood tree $T : \Gamma \to 2^H \setminus \{\emptyset\}$, an NTTC mechanism operates as follows:

Step 1**. Preparation sub-step:** Check for each agent if her current house is her most-preferred among the remaining houses. If so, allow the agent to exit with her current house. This process is repeated until no such agent exists. If all agents exit, the iteration terminates. Denote the resulting sub-allocation by $\bar{e}$.

Exchange sub-step: Each agent occupying a house in the set $T(\bar{e})$ points to the owner of her most preferred house within $T(\bar{e})$. There must be at least one cycle (including self-cycles) and these cycles do not intersect. Allow agents in cycles to trade and exit the step with their new houses. This process is repeated until all houses in $T(\bar{e})$ exit this step. Denote the allocation after all trades in this step by m_1.

Step $k, k \geq 2$. **Preparation sub-step:** Check for each agent if her current house is her most-preferred one among the remaining houses. If so, allow the agent to exit with her current house. Repeat this process until no such agent exists. If all agents exit, the iteration terminates. Otherwise, denote the resulting sub-allocation by $\bar{m}_{k-1}$.

Exchange sub-step: Each agent who owns a house in the set $T(\bar{e}, \bar{m}_1, \bar{m}_2, \cdots, \bar{m}_{k-1})$ points to the owner of her most preferred house within $T(\bar{e}, \bar{m}_1, \bar{m}_2, \cdots, \bar{m}_{k-1})$. There must be at least one cycle (including self-cycles), and these cycles do not intersect. Allow agents in cycles to trade and exit the step with their new houses. Repeat this process until all houses in

$T(\bar{e}, \bar{m}_1, \bar{m}_2, \cdots, \bar{m}_{k-1})$ have been involved in trades during this step. Denote the allocation after all trades in this step by m_k.

The neighborhood tree ensures that, given a tree and a preference profile, the entire set of houses becomes available after a finite number of steps, at which point the algorithm terminates. Liu (2022) demonstrates that NTTC mechanisms are individually rational, Pareto efficient, and strategy-proof within the context of single-peaked preferences.

A.7 The Examples of the First Two Statements in Proposition 2

Consider $I = \{1, 2, 3, 4\}$ and $H = \{h_1, h_2, h_3, h_4\}$. We examine three preference profiles. In the first case, agents 1, 2, and 3 most prefer h_4, while agent 4 prefers h_1. The preferences for case 1 are as follows:

P_1	P_2	P_3	P_4
h_4	h_4	h_4	h_1
h_3	h_3	h_3	h_2
h_2	h_2	h_2	h_3
h_1	h_1	h_1	h_4

Preferences for case 2 are illustrated in the following table:

P_1	P_2	P_3	P_4
h_1	h_4	h_2	h_2
h_2	h_3	h_1	h_1
h_3	h_2	h_3	h_3
h_4	h_1	h_4	h_4

Preferences for case 3 are shown in the table below:

P_1	P_2	P_3	P_4
h_4	h_1	h_1	h_1
h_3	h_2	h_2	h_2
h_2	h_3	h_3	h_3
h_1	h_4	h_4	h_4

First, we show that the 2-neighborhood mechanism f is not equivalent to any NTTC mechanism. Suppose by way of contradiction that an NTTC mechanism f^{NTTC} is equivalent to f. Let T be the corresponding available tree. Consider the infinite history $\gamma_e = eee\cdots$. By definition, there exists a history $\gamma_0 \succ \gamma_e$ where $|T(\gamma_0)| \geq 2$ and $|T(\gamma)| = 1$ for all $\gamma \succ \gamma_0$. This means $T(\gamma_0)$ is the first set containing at least two houses in the sequence $T(e), T(ee), T(eee), \cdots$. Examining elements of $T(\gamma_0)$, if $T(\gamma_0) = \{h_1, h_2, h_3, h_4\}$, the NTTC mechanism selects $(4h_1, 2h_2, 3h_3, 1h_4)$ in case 1, differing from the 2-neighborhood mechanism f which selects $(4h_1, 1h_2, 3h_3, 2h_4)$. Thus, $T(\gamma_0) \neq \{h_1, h_2, h_3, h_4\}$. If the set $T(\gamma_0) = h_1, h_2, h_3$ or $T(\gamma_0) = h_2, h_3$, then in case 2, agent 3 receives h_2 in the NTTC mechanism, whereas she receives h_3 in the 2-neighborhood mechanisms. Consequently, $T(\gamma_0) \neq h_2, h_3$ and $T(\gamma_0) \neq h_1, h_2, h_3$, and by symmetry, $T(\gamma_0) \neq h_2, h_3, h_4$. If $T(\gamma_0) = h_1, h_2$, then in case 3, agent 2 receives h_1 in the NTTC mechanism but obtains h_2 in the 2-neighborhood mechanisms. Therefore, $T(\gamma_0) \neq h_1, h_2$ holds, and by symmetry, $T(\gamma_0) \neq h_3, h_4$. In conclusion, no such history $\gamma_0 \succ \gamma_e$ exists where $|T(\gamma_0)| \geq 2$, leading to a contradiction. Thus, the 2-neighborhood mechanism f is not equivalent to any NTTC mechanism.

The following NTTC (Neighborhood Top Trading Cycles) mechanism, denoted as g, does not have an outcome equivalent within any neighborhood mechanism framework. Initially, agents 1, 2, and 3 are set to trade, followed by trading among all agents. Specifically, the neighborhood tree T is defined such that $T(\gamma) = h_1, h_2, h_3$ for all histories γ of length 1, and for histories longer than one, $T(\gamma) = H_m$, where m represents the last sub-allocation within γ. Under mechanism g, the outcome for Case 1 is the allocation $(4h_1, 2h_2, 3h_3, 1h_4)$, and for Case 3, it selects $(3h_1, 2h_2, 4h_3, 1h_4)$. In any neighborhood mechanism that results in the allocation $(4h_1, 2h_2, 3h_3, 1h_4)$ for Case 1, agents

1 and 4 must be able to point to each other in the initial step, implying a neighborhood size of at least 3 for both ($r_1^1 = l_4^4 \geq 3$). Consequently, such a neighborhood mechanism would also select $(4h_1, 2h_2, 3h_3, 1h_4)$ for Case 3, illustrating that the NTTC mechanism g does not align with the neighborhood mechanisms category.

A.8 Proof of the Third Statement in Proposition 2

Proof. If f is an NTTC mechanism with a neighborhood tree $T : \Gamma \setminus \bar{\Gamma}$, we aim to identify a generalized size function such that the corresponding generalized neighborhood mechanism is outcome equivalent to f and satisfies the available set expanding condition.

Define an indirect size function as a mapping $d : I \times M \times 2^H \to \mathbb{N} \times \mathbb{N}$ as follows:

$$d(i, m, \bar{H}) = \begin{cases} (l, r), & \text{if } i \in I_m \text{ and } m(i) \in \bar{H}, \\ (0, 0), & \text{if } i \in I_m \text{ and } m(i) \notin \bar{H}, \\ (\bar{n}, \bar{n}), & \text{if } i \notin I_m, \end{cases}$$

where $l = |\{h \in \bar{H} : h < m(i)\}|$ and $r = |\{h \in \bar{H} : h > m(i)\}|$.

Given a sub-allocation history $\gamma = em_1m_2 \cdots m_p$ and a self-cycle history $\phi = I_0 I_1 \cdots I_p$, define a sequence of $t+1$ positive integers $1 = k_1(\gamma) < k_2(\gamma) < \cdots < k_t(\gamma) \leq p < k_{t+1}(\gamma)$, where $k_1(\gamma) = 1$ and $k_\alpha(\gamma) = k_{\alpha-1}(\gamma) + 2|T(m_{k_1(\gamma)} m_{k_2(\gamma)} \cdots m_{k_{\alpha-1}(\gamma)})|$ for $\alpha = 2, 3, \cdots, t+1$. Then, define a set of houses $G(\gamma, \phi)$ recursively as:

$$G(\gamma, \phi) = \begin{cases} \emptyset, & \text{if } \gamma = e, \\ T(m_{k_1(\gamma)} m_{k_2(\gamma)} \cdots m_{k_t(\gamma)}), & \text{if } p = k_t(\gamma), \\ G(\tilde{\gamma}, \phi) \setminus \tilde{H}_{p-1}, & \text{otherwise}, \end{cases}$$

where $\tilde{\gamma} = em_1m_2 \cdots m_{p-1}$. Then, define $S(i, \gamma, \phi) = d(i, m_p, G(\gamma, \phi))$ for each i.

Let g be the corresponding generalized neighborhood mechanism. We now show that g is outcome equivalent to f and that g satisfies the available set expanding condition.

Given a preference profile P, suppose the neighborhood mechanism g terminates after step p and generates a sub-allocation history $\gamma = em_1m_2\cdots m_p$. Define $t+1$ positive integers $1 = k_1(\gamma) < k_2(\gamma) < \cdots < k_t(\gamma) \leq p < k_{t+1}(\gamma)$ such that $k_\alpha = k_{\alpha-1}(\gamma) + 2|T(m_{k_1(\gamma)}m_{k_2(\gamma)}\cdots m_{k_{\alpha-1}(\gamma)})|$, for all $\alpha = 2,3,\cdots,t+1$. Suppose the NTTC mechanism f terminates after step q and generates a sub-allocation history $\bar{\gamma} = \bar{e}\bar{m}_1\bar{m}_2\cdots\bar{m}_q$.

For convenience, define the set of houses $G_k = G(em_1m_2\cdots m_k, I_0I_1\cdots I_k)$ for each $k = 1,2,\cdots,p$. By the construction of the indirect size function d, if each agent i can point to the most-preferred house in the neighborhood with size $S(i,\gamma,\phi) = d(i,m_p,G_k)$ for some k, then each agent endowed with a house in G_k is able to point to an agent in G_k, and each agent who is not endowed with a house in G_k can only point to herself.

In step 1 of g, since $G(e,\phi) = \emptyset$, each agent can only point to herself. Then all the agents i such that $\tau(P_i) = h_i$ are removed with h_i. Thus, $m_{k_1(\gamma)} = m_1 = \bar{e}$ by definition.

Since $k_1(\gamma) = 1$, we have $G_1 = T(\bar{e})$. Only agents endowed with a house in $T(\bar{e})$, denoted by I_{G_1}, can point to each other. If an agent i is initially endowed with the most preferred house in G_1, she is either removed or in a self-cycle in the next step. Suppose there are x houses in G_1, then in the following $2x$ steps, the set of houses G_k sequentially exclude the houses which are removed or whose owners are in self-cycles. Agents point to the most-preferred agent who is endowed with the most preferred house in $G_k, k = 2,3,\cdots,2x+1 = k_2(\gamma)$. By the definition of the TTC mechanism, each agent i in I_{G_1} is endowed with the same house in the second step of the NTTC mechanism f, i.e., $\bar{m}_2(i) = m_{k_2(\gamma)}(i)$. Agents not in I_{G_1} do not move in the $2x$ steps of g. Thus, we have $\bar{m}_2 = m_{k_2(\gamma)}$. The same argument holds for any further step, and we have $m_{k_t(\gamma)} = \bar{m}_q$. Since the NTTC mechanism f terminates in step q, the generalized neighborhood mechanism g terminates in step $k_t(\gamma)$. Therefore, $p = k_t(\gamma)$, and mechanisms f and g are outcome equivalent.

Next, we show that the generalized neighborhood mechanism g satisfies the available set expanding condition. As discussed, we divide the algorithm into t rounds, each replicating a step in the NTTC mechanism. In the last step of each round, the neighborhood contains only one house by definition. Thus, in the cross-round step (i.e., the first step of a round), the available set expanding condition is not violated. By definition, the neighborhoods progressively exclude houses involved in self-cycles, ensuring the available set expanding condition is not violated in the in-round steps. In conclusion, the mechanism g satisfies the available set expanding condition. $\square$

A.9 An example of a NTTC mechanism that is not group strategy-proof

Suppose $I = \{1, 2, 3, 4\}$ and $H = \{h_1, h_2, h_3, h_4\}$. Consider the NTTC mechanism corresponding to the following neighborhood tree T:

$$T(\gamma) = \begin{cases} \{h_2, h_3, h_4\}, & \text{if } |\gamma| = 1, h_1 \in H_\gamma, \\ \{h_3, h_4\}, & \text{if } |\gamma| = 1, h_1 \notin H_\gamma, \\ \{h_1, h_2, h_3, h_4\}, & \text{if } |\gamma| \geq 2. \end{cases}$$

Consider the following preference profile P and preference P_1':

P_1	P_2	P_3	P_4	P_1'
h_4	h_4	h_4	h_2	h_1
h_3	h_3	h_3	h_3	h_2
h_2	h_2	h_2	h_4	h_3
h_1	h_1	h_1	h_1	h_4

If each agent i reports P_i, then $h_1 \in H_{\bar{e}}$ holds. Agents 2, 3, and 4 are chosen, and agent 2 (with h_2) trades with agent 4 (with h_4). Then, the algorithm terminates in the preparation sub-step in step 2, and the matching $(1h_1, 4h_2, 3h_3, 2h_4)$ is selected.

If (P_1', P_2, P_3, P_4) is reported, h_1 is not in $H_{\bar{e}}$. Only agents 3 and 4 are chosen, and they trade with each other. In step 2, agent 2 (with h_2) trades with agent 4 (with h_3). The algorithm terminates in the preparation sub-step in step 3. The matching $(1h_1, 4h_2, 2h_3, 3h_4)$ is selected.

Then, consider the coalition $C = \{1, 3\}$. When misreporting to (P_1', P_3), agent 3 receives h_4 instead of h_3 and is strictly better off, while agent 1 receives the same house h_1. Therefore, this NTTC mechanism is not group strategy-proof.

References

Alva, S. (2017), When is manipulation all about the ones and twos?, *working paper*.

Andersson, M., J. Klaesson and J. Larsson (2014), How Local are Spatial Density Externalities? Neighbourhood Effects in Agglomeration Economies, *Regional Studies*, 50(6), 1-14.

Bade, S. (2019), Matching with single-peaked preferences, Journal of Economic Theory, 180, 81-99.

Bayer, P., F. V. Ferreira, and R. McMillan(2007), A Unified Framework for Measuring Preferences for Schools and Neighborhoods, *Journal of Political Economy*, 115(4), 588-638.

Black, S. (1999), Do Better Schools Matter? Parental Valuation of Elementary Education, *The Quarterly Journal of Economics*, 114(2), 577-599.

Damamme, A., A. Beynier, Y. Chevaleyre and N. Maudet (2015), The Power of Swap Deals in Distributed Resource Allocation. *The 14th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2015), May 2015, Istanbul, Turkey.* pp.625-633, 2015.

Ergin, H., T. Sönmez and M. U. Ünver (2020), Efficient and Incentive-Compatible Liver Exchange *Econometrica*, 88(3), 965-1005.

Gale,D. and L. Shapley(1962), College admissions and the stability of marriage, *American Mathematical Monthly*, 69, 9-15.

He, M., G. He, J. Chen and Y. Wang (2019), Sense of control matters: A long spatial distance leads to a short-term investment preference, *Judgment and Decision Making*, 14(3), 299-308.

Hotelling, H. (1929), Stability in Competition, *The Economic Journal*, 153, 41-57.

Ioannides, Y. (2019), Market allocation through search: equilibrium adjustment and price dispersion, *Journal of Economic Theory*, 11(2), 247-262.

Li, S. (2017), Obviously Strategy-Proof Mechanisms, *American Economic Review*, 107, 3257-87.

Liu, P. (2022), Neighborhood Top Trading Cycles, *working paper*.

Ma, J. (1994), Strategyproofness and the Strict Core in a Market with Indivisibilites, *International Journal of Game Theory*, 23, 75-83

Mandal, P. and S. Roy (2021), Strategy-proof allocation of indivisible goods when preferences are single-peaked, *working paper*.

Pal, D. (1998), Does Cournot competition yield spatial agglomeration?, *Economics Letters*, 60(1998), 49-53.

Pápai, S. (2000), Strategyproof assignment by hierarchical exchange, *Econometrica*, 68, 1403-1433.

Pycia, M. and P. Troyan (2022), A Theory of Simplicity in Games and Mechanism, *working paper*.

Pycia, M. and M. U. Ünver, (2017), Incentive compatible allocation and exchange of discrete resources , *Theoretical Economics* , 12, 287-329.

Robert, M. and C. Goh (2011), Density, distance and division: the case of Chongqing municipality, China, *Cambridge Journal of Regions, Economy and Society*, 4(2), 189-204.

Rosenthal, S. and W. Strange (2008), Agglomeration and Hours Worked, *The Review of Economics and Statistics*, 90(1), 105-118.

Roth, A. (1982), Incentive compatibility in a market with indivisible goods, *Economics letters*, 9, 127-132.

Roth, A. and A. Postlewaite (1977), Weak versus strong domination in a market with indivisible goods, *Journal of Mathematical Economics*, 4, 131-137.

Roth, A., T. Sönmez and M. U. Ünver (2004), Kidney Exchange, *The Quarterly Journal of Economics*, 119(2), 457-488.

Schummer, J. and S. Serizawa (2019), The Role of Characterizations in Market Design, *The Future of Economic Design: The Continuing Development of a Field as Envisioned by Its Researchers* , 201-211.

Sethuraman, J. (2016), An alternative proof of a characterization of the TTC mechanism, *Operations Research Letters* , 44, 107-108.

Shapley, L. and H. Scarf (1974), On cores and indivisibility, *Journal of mathematical economics*, 1, 23-37.

Sivey, P. (2012), The effect of waiting time and distance on hospital choice for English cataract patients, *Healthy Economics*, 21(4), 444-456.

Stigle, G. (1961), The economics of information, *The Journal of Political Economy*, 69(3), 213-225.

Tamura, Y. (2022), Object reallocation problems under single-peaked preferences: two characterizations of the Crawler, *International Journal of Game Theory*, 51, 537-565.

Tamura, Y. (2023), Obviously Strategy-proof Rules for Object Reallocation, *Working paper*.

Tamura, Y. and H. Hosseini (2022), The cawler: three equivalence results for object (re)allocation problems when preferences are single-peaked, *Journal of Economic Theory*, 203, 105466.

Troyan, P. (2019), Obviously strategy-proof implementation of top trading cycles, *International Economic Review*, 60(3), 127-132.