

# On the ratifiability of efficient cartel mechanisms in first-price auctions with participation costs and information leakage

By Xiaoyong Cao<sup>a</sup>, Shao-Chieh Hsueh<sup>b</sup>, and Guoqiang Tian<sup>c</sup>

<sup>a</sup>Department of Economics, University of International Business and Economics, Beijing, 100029, China; e-mail: yongcx2000@uibe.edu.cn

<sup>b</sup>Institute of Economics, School of Economics and Wang Yanan Institute for Studies in Economics, Xiamen University, Xiamen, Fujian, China, 361005; e-mail: schsuechieh@gmail.com

<sup>c</sup>Department of Economics, Texas A&M University, College Station, TX 77843 and Institute for Advanced Studies in Finance and Economics, Hubei University of Economics, Wuhan, China, 430205; e-mail: tian@econmail.tamu.edu

## Abstract

This paper addresses the ratifiability of an efficient cartel mechanism in a first-price auction. When a seller uses a first-price sealed-bid auction, the efficient all-inclusive cartel mechanism will no longer be ratifiable in the presence of both participation costs and potential information leakage. A bidder whose value is higher than a cut-off in the cartel will have an incentive to leave the cartel, thereby sending a credible signal of his high value, which discourages other bidders from participating in the seller's auction. However, the cartel mechanism is still ratifiable where either the participation cost or information leakage is absent.

**JEL classifications:** D44, D61, D82.

## 1. Introduction

An auction is an effective way to increase allocative efficiency by extracting private information and by improving the competitiveness of potential buyers. Efficiency is diminished when collusion occurs among buyers. Fortunately, sellers may design rules to prevent the forming of cartels by revealing bidders' private information, which can have an adverse impact on the stability of collusion. Traditional literature treats the collusion in auctions as a mechanism design problem; it places particular focus on the feasibility of efficient collusion.<sup>1</sup> The main conclusion of the literature is that bidding rings can collude efficiently and

1 Much of the literature on collusion tries to overcome the problems associated with cartels, problem as mentioned in [Osborne \(1976\)](#), such as designing the rule, dividing the profit, detecting the cartel, and deterring cheating. For related literature, see [Cooper \(1977\)](#), [Roberts \(1985\)](#),

make all their members better off. This line of literature concentrates on the benefit to bidders. In this paper, which takes the seller's perspective, we study the stability of an efficient collusive mechanism with participation costs and potential information leakage through pre-auction knockouts by means of which bidders may update their beliefs.<sup>2</sup>

McAfee and McMillan (1992) is one of the papers investigating how to form a successful cartel. They state that bidders' collusion is possible if they use a strong cartel mechanism by implementing a prior auction before the seller's legitimate auction with a passive beliefs set-up. Following this set-up, Laffont and Martimort (1997) and Che and Kim (2006) study agents' collusion using mechanism design theory, in which collusive agents maintain their beliefs as priors.

However, two sources that may destabilize the cartel mechanism benefit the seller. One is information leakage, with which bidders may update their beliefs (for instance, other bidders' value distributions) from others' participation decisions after the cartel's prior auction. Cramton and Palfrey (1995) allow this updating of beliefs among bidders, so the participation decisions depend on strategic actions. The cartel mechanism is implemented when all bidders unanimously participate, which is defined as ratifiability; otherwise, the buyers bid competitively in the seller's auction under updated beliefs. They show that the participation decisions made by the vetoers in the prior auction disintegrate the optimal cartel mechanism in the general mechanism design problem. This is because the decision may reveal that vetoers have higher valuations for the auctioned item than other bidders, which discourages other bidders in the legitimate auction. The other means by which to destabilize the cartel mechanism is the participation costs that are incurred when bidders participate in an auction.<sup>3</sup> The existence of participation costs will further reduce the incentive of bidders with low valuations to enter the seller's auction.

Tan and Yilankaya (2007) apply the ratifiability concept introduced by Cramton and Palfrey (1995) to investigate whether efficient collusive mechanisms are affected by potential information leakage and participation costs, when the seller uses a second-price auction. They find that the standard efficient cartel mechanisms are not ratified by cartel members.

Similar questions on first-price auctions have received little attention in the literature, even though this type of auction is more common in practice. Hendricks *et al.* (2008) study the formation of the bidding rings in first-price sealed-bid auctions of common value assets, but they do not focus on the ratifiability problem. Tanno (2008) investigates the ratifiability of efficient cartels in first-price auctions with the assumptions of two potential bidders and uniform distributions on value, without bidders' participation costs. Costly participation typically induces asymmetric bidding strategies among bidders after entry occurs. Asymmetric bidding strategies in this situation are much more complicated than those in second-price auctions that result in truth-telling; this increases the technical difficulty of the analysis since it is difficult to find an explicit solution in this case. Besides, the analysis of such an extension is arguably more important because, in the real world, first-price

Graham and Marshall (1987), Cramton and Palfrey (1990), Graham *et al.* (1990), and Mailath and Zemsky (1991).

- 2 There are other branches of literature with diverse perspectives related to collusion. For research focusing on issues of bribery, see, for example, Esó and Schummer (2004) and Rachmilevitch (2013). For studies from the perspective of principals, see, for example, Balzer (2017).

- 3 See Cao and Tian (2010), for related literature.

auctions are much more common, and related empirical evidence about collusive problems is rich (see [Tan and Yilankaya, 2007](#), p. 392). Our results provide theoretical support for this evidence. In this paper, we address the ratifiability problem with information leakage and participation costs in a first-price auction with an independent private value setting, which is an important extension of the current literature.

Following [Cramton and Palfrey \(1995\)](#) and [Tan and Yilankaya \(2007\)](#), we consider a two-stage ratification game in a first-price sealed-bid auction format that allows for the presence of both participation costs and information leakage in the strong cartel mechanism studied in [McAfee and McMillan \(1992\)](#). We determine the veto set such that, if a bidder's value belongs to the set, then he will choose to opt out of the cartel by employing the following strategy: a bidder vetoes the cartel only when the expected revenue after the first-price auction is greater than from staying in the cartel. Other bidders can then update their beliefs on the vetoer's value distribution.

We show that, in a first-price auction, the efficient cartel mechanism will no longer be ratifiable when a bidder vetoes a pre-auction knockout in the presence of both participation costs and potential information leakage. A bidder with a value that is greater than a critical point will have an incentive to veto. By vetoing the mechanism, the bidder sends out a credible signal that he has a relatively high value for the item, which discourages other bidders from joining the seller's auction when there are positive participation costs. However, even if there is an information leakage problem, such that bidders can update their beliefs through a collusive mechanism, the efficient cartel mechanism is still ratifiable when there is no participation cost, since the bidder's vetoing signal is now not a credible threat. One implication of our results is that, in practice, the seller can charge an entry fee to the auction so as to decrease the likelihood of a cartel forming, thus enhancing the competition among bidders. This, in turn, increases the expected revenue of the seller.

The remainder of the paper is organized as follows. Section 2 describes the economic environment. Section 3 considers the benchmark case where no information leakage is allowed. Section 4 allows the presence of information leakage and investigates the ratifiability of efficient cartel mechanisms. Section 5 concludes. All proofs are included in the [online Appendix](#).

## 2. Economic environment

Consider a standard independent private value environment with one seller and  $n$  ( $n \geq 2$ ) potential risk-neutral bidders. The seller values their object at 0.<sup>4</sup> Bidder  $i$ 's value for the object being auctioned is  $v_i$ , which represents  $i$ 's willingness to pay for the object being sold in the auction, and  $v = (v_1, \dots, v_n)$  is the vector of  $n$  bidders' profiles. Variable  $v_i$  is private information, which is a random draw from the cumulative distribution function  $F(\cdot)$  with continuous and strictly positive density function  $f(\cdot)$  supported on  $[0, 1]$ .

In order to submit a bid, each bidder must pay a non-refundable participation cost that is common to all bidders and is denoted by  $c \in [0, 1]$ .<sup>5</sup> Bidders make the participation decisions simultaneously. When a bidder is indifferent about participating in the seller's

4 A positive reserve price only complicates our analysis without providing any additional insight into our main result. Although a positive reserve price may affect the seller's expected revenue, our main concern is the ratifiability of the mechanism of an efficient cartel.

5 Participation costs can be replaced by entry fees (charged by the seller), or by sunk costs, if we focus on the ratifiability of the mechanism of an efficient cartel (i.e. the behaviour of the bidders).

auction, we assume that they participate for convenience of the illustration. When a bidder submits a bid, they know the participation decisions of all other bidders. The seller uses a standard first-price sealed-bid auction—i.e. the bidder with the highest bid wins the auction and pays their bid. All participants in the seller’s auction must pay the participation costs.

The bidders may form a cartel. The seller is assumed to be passive—i.e. they do not know whether they face a cartel.

### 3. Auctions with no information leakage

In this section, we assume that there is no information leakage problem. Thus, we restrict our attention to the equilibria with passive beliefs—i.e. in the seller’s legitimate auction, the bidders have no updated belief on other bidders’ valuation distributions. To know whether the bidders prefer to form a cartel, we compare a bidder’s expected revenue between the non-collusive and collusive games without information leakage.

#### 3.1 Non-collusive first-price auction

Let  $v^*$  be the cut-off point for participation, which is determined by  $c = v^*F(v^*)^{n-1}$ . It is obvious that  $v^* > c$ . Following Cao and Tian (2010), when there are  $k \geq 2$  bidders participating in the seller’s auction, there exists a unique (up to changes for a measure zero set of values) symmetric (Bayesian-Nash) equilibrium where each bidder’s bidding function  $\lambda(v_i, v^*, k)$  is monotonically increasing when  $v_i \geq v^*$ , given by

$$\lambda(v_i, v^*, k) = v_i - \frac{\int_{v^*}^{v_i} [F(y) - F(v^*)]^{k-1} dy}{[F(v_i) - F(v^*)]^{k-1}} \tag{1}$$

with  $v_i \geq v^*$ . If  $v_i < v^*$ , then bidder  $i$  does not participate in the auction. When  $k = 1$ , then this bidder who participates in the auction bids zero. When  $v_i \geq v^*$ , the non-collusive expected profit  $\pi_i^s(v_i)$  for bidder  $i$  is as follows:

$$\pi_i^s(v_i) = v_i F(v^*)^{n-1} + \sum_{k=2}^n C_{n-1}^{k-1} F(v^*)^{n-k} [v_i - \lambda(v_i, v^*, k)] [F(v_i) - F(v^*)]^{k-1} - c \tag{2}$$

where  $C_{n-1}^{k-1}$  denotes the possible cases of choosing  $k-1$  out of the  $n-1$  bidders. On the right-hand side of Equation (2), the first term is the expected revenue when all others do not participate in the seller’s auction, and the second term gives the expected revenue when  $k \geq 2$ . With  $\lambda(v_i, v^*, k)$  and some simplifications, bidder  $i$ ’s expected profit from the non-collusive first-price auction is given as follows:

$$\pi_i^s(v_i) = \begin{cases} 0 & v_i < v^* \\ \int_{v^*}^{v_i} F(y)^{n-1} dy & v_i \geq v^* \end{cases} \tag{3}$$

#### 3.2 Efficient all-inclusive cartel mechanism

Now suppose that transfers among bidders are possible. Bidders may form a cartel and design an all-inclusive *ex post* efficient cartel mechanism outside the seller’s auction that maximizes the sum of each bidder’s expected profits with transfer payments.<sup>6</sup>

6 An efficient cartel mechanism such as this is called a strong cartel, which is shown to be feasible in McAfee and McMillan (1992). See Che and Kim (2006), for the existence of efficient cartel mechanisms.

An efficient cartel works as follows: in the first stage, all of the bidders vote on whether to join the cartel mechanism. In the second stage, if the cartel mechanism is unanimously accepted, the cartel can enforce its outcome—i.e. no member will ever disregard the mechanism when the mechanism dictates that he bids to lose.<sup>7</sup> If the cartel mechanism is vetoed by at least one bidder, collusion breaks down and bidders participate non-cooperatively in the seller's auction. The model in this paper endows the cartel with the ability to ensure obedience to the cartel mechanism's orders, so as to make honesty optimal.

After dropping the bidders' indices so as to simplify the notation, following [Tan and Yilankaya \(2007\)](#), the expected profit of a cartel member with value  $v$  can be written as follows:

$$\pi^m(v) = \begin{cases} \pi^m(0) & \text{if } v < c \\ \pi^m(0) + \int_c^v G(u)du & \text{if } v \geq c \end{cases} \quad (4)$$

where

$$G(u) = F(u)^{n-1} \quad (5)$$

and

$$\pi^m(0) = \int_c^1 \left[ y - \frac{1 - F(y)}{f(y)} - c \right] G(y) dF(y) \quad (6)$$

which is the transfer payment that each loser receives in the cartel.

Since  $v^* > c$  and  $\pi^m(0) > 0$ , it follows that  $\pi^m(v) > \pi^s(v)$ , which means that an efficient cartel mechanism satisfies the conditions of bidders' individual rationality and incentive compatibility. Bidders in the cartel earn more profit, whether they win the object being auctioned or not.

Compared to the non-collusive auction, all bidders prefer to form a cartel. This is true for all  $0 \leq c < 1$ . That is, it does not matter whether there is a participation cost in the seller's auction. Formally, we have the following proposition.

*Proposition 1* With no updating of beliefs, a strong cartel mechanism is efficient and interim individually rational with respect to symmetric equilibrium pay-offs in the seller's auction, regardless of whether there is a participation cost.

#### 4. Ratifiability of an efficient cartel mechanism

The assumption that there is no information leakage is unrealistic under certain circumstances; this is true where bidders will update their information, if any, from the cartel's prior

<sup>7</sup> Based on the enforcement methods in [McAfee and McMillan \(1992\)](#), the cartel could hire an enforcer to punish vetoers, or adopt a stringent trigger strategy in an infinitely repeated auction. A deviating bidder can be threatened with lower profit in all future auctions should he win the current auction when the mechanism dictates otherwise. In this paper, we focus on examining the constraints on the cartel that result from the privacy of the cartel members' information. Therefore, we directly assume that some punishment is available to the cartel, so that no member will ever disregard the mechanism when the mechanism dictates that he should bid to lose.

auction before they participate in the seller's auction.<sup>8</sup> In this section, we investigate the ratifiability of efficient cartel mechanisms with both participation costs and the information leakage issue, where bidders can update their beliefs through bidders' participation decisions. In this case, the bidding strategy of a bidder usually depends on his belief about others' values, and his belief, in turn, is affected by the ratifiability of the cartel.

Following [Cramton and Palfrey \(1995\)](#) and [Tan and Yilankaya \(2007\)](#), we consider the two-stage ratification game described in Section 3.2 to analyse the stability of an efficient cartel. Bidders decide whether to participate in the first-price auction with updated beliefs about vetoers' values.

In order to show that bidders may have an incentive to exit the cartel, we define a veto set  $A_i$ . If bidder  $i$  vetoes the cartel, then the others' belief in his valuation is updated to be in the set  $A_i$ .<sup>9</sup> This vetoing behaviour brings a higher profit to the vetoer than he would have otherwise gained in the collusive case. For illustrative simplicity, we assume that if a bidder is indifferent to remaining in the cartel or vetoing it, he will choose to veto the cartel. Let  $\pi_i^v(v_i, b^*)$  be the vetoer's expected pay-off at equilibrium  $b^*$  in the post-veto auction with updated beliefs on the vetoer's valuation, and denote  $\pi_i^m(v_i)$  as the pay-off when he stays in the cartel. Formally, we have the following definitions:

*Definition 1* A set  $A_i$  with  $\emptyset \neq A_i \subseteq [0, 1]$  for bidder  $i$  is said to be a credible veto set if there exists an equilibrium  $b^*$  in the post-veto auction with updated beliefs on the vetoer's valuation such that  $\pi_i^v(v_i, b^*) > \pi_i^m(v_i) \Rightarrow v_i \in A_i$  and  $v_i \in A_i \Rightarrow \pi_i^v(v_i, b^*) \geq \pi_i^m(v_i)$  and there is at least one  $v_i \in A_i$  such that  $\pi_i^v(v_i, b^*) > \pi_i^m(v_i)$ .

*Definition 2* The cartel mechanism is ratifiable, if there is no credible veto set for all  $i \in N$ .

With the same intuition developed in [Tan and Yilankaya \(2007\)](#), a bidder only vetoes the collusive mechanism when his value for the auctioned good exceeds a critical value. Indeed, low-value bidders gain more by participating in the cartel mechanism than high-value bidders do, and a low-value bidder is not able to gain very much from vetoing in the seller's auction. In the proof of Proposition 3, we show that there is a threshold above which the non-collusive pay-off exceeds the collusive pay-off. Thus, in the following, without loss of generality, we focus on the cut-off strategy.

Suppose that when bidder  $i$  vetoes the cartel, others believe that bidder  $i$ 's value is in the set  $[v_N, 1]$ , where  $N$  stands for refusing to engage in the cartel mechanism (the vetoer), and  $v_N$  is a threshold value at which bidder  $i$  is indifferent as to whether he vetoes or stays in the cartel. We will show that there is an asymmetric equilibrium of the auction with these updated beliefs, such that the vetoer's pay-off at equilibrium is greater than his pay-off in the cartel if his value is greater than  $v_N$ . As such, the vetoer has an incentive to leave the cartel, and  $A_i = [v_N, 1]$  is a credible veto set for bidder  $i$ .

When bidder  $i$  vetoes the cartel, his value is updated to be distributed on  $[v_N, 1]$  according to  $F_N(v) \equiv \frac{F(v) - F(v_N)}{1 - F(v_N)}$ , which is derived from  $F(\cdot)$  using the Bayesian rule. For all other bidders, the expected profit from participating in the seller's auction is a non-decreasing

8 Indeed, as recently emphasized by [Stiglitz \(2019\)](#), beliefs arise in part from strategic interactions and affect the behaviour of market participants and economic equilibrium.

9 Since we are looking for the possibility that all bidders unanimously ratify the cartel mechanism, it is sufficient to think only about unilateral deviation.

function of their true values. Thus, with participation costs, a bidder uses a cut-off strategy in which he submits a bid if, and only if, his value for the item is greater than or equal to a cut-off value.

Now suppose that all ratifiers use the same strategy, including the participation decision and bidding function—i.e. the cut-off points used by all collusive bidders to decide whether to participate in the seller's auction are the same and denoted by  $v_Y$ , where  $Y$  denotes agreement (ratifiers).<sup>10</sup> If any ratifier participates in the auction, then his value is updated to be distributed on  $[v_Y, 1]$  according to  $F_Y(v) \equiv \frac{F(v)-F(v_Y)}{1-F(v_Y)}$ . Thus, when both the vetoer and any ratifier participate in the seller's auction, we have an asymmetric first-price auction in which the bidders' valuation distributions are on different supports.

Since bidders can observe who else participates in the auction when they submit their bids, the bidding functions hinge on the number of other bidders. Thus, we can specify the strategy as a function of the number of ratifiers participating in the seller's auction,  $k$ . Let  $b_i^k(\cdot)$  and  $b_j^k(\cdot)$  be the optimal bidding function for the vetoer's and ratifiers' optimal bidding functions when the number of ratifiers participating in the seller's auction is  $k$ . When there is no ratifier participating, the vetoer bids zero—i.e. when  $k=0$ ,  $b_i^k(v_i) = 0$ .

Note that, from [Cao and Tian \(2010\)](#),  $v_Y \geq v_N$ , since otherwise a collusive bidder with value  $v_Y$  has no chance of winning in the seller's auction but still incurs the participation cost  $c$ . This makes a  $v_Y$  type collusive bidder's participation suboptimal. Let  $v_i^k(\cdot)$  and  $v_j^k(\cdot)$  be the optimal inverse bidding functions in the seller's auction. Then the maximization problem for the typical ratifier  $j$  is

$$\max_{b_j^k} (v_j - b_j^k) \left( \frac{F(v_i^k(b_j^k(v_j))) - F(v_Y)}{1 - F(v_Y)} \right)^{k-1} \frac{F(v_i^k(b_j^k(v_j))) - F(v_N)}{1 - F(v_N)} \quad (7)$$

Similarly, for vetoer  $i$ :

$$\max_{b_i^k} (v_i - b_i^k) \left[ \frac{F(v_j^k(b_i^k(v_i))) - F(v_Y)}{1 - F(v_Y)} \right]^k \quad (8)$$

Following the vast literature on first-price auctions—for example, see [Maskin and Riley \(2003\)](#) and [Cao and Tian \(2010\)](#)—we assume that a bidder with zero probability of winning will bid his true value when they participate. There is then a unique optimal bidding strategy in undominated strategies, where bidders with the same value distribution use the same bidding function, which is characterized in Lemma 1.<sup>11</sup>

**Lemma 1** Suppose  $k \geq 1$  bidders whose values are distributed on the interval  $[v_Y, 1]$  with cumulative distribution function  $F_Y(v) = \frac{F(v)-F(v_Y)}{1-F(v_Y)}$  and one bidder whose value is distributed on the interval  $[v_N, 1]$  with cumulative distribution function  $F_N(v) = \frac{F(v)-F(v_N)}{1-F(v_N)}$  participate in the first-price auction, where  $v_N \leq v_Y$ . Let  $\underline{b}_k = \operatorname{argmax}_b (F(b) - F(v_N))$

10 It should be noted that each collusive bidder may use a different cut-off value, as studied in [Cao and Tian \(2010\)](#), which may complicate our analysis considerably. However, if  $F(\cdot)$  is inelastic—i.e.  $F(v) \geq vf(v)$  for all  $v \in [0, 1]$ —then there is a unique equilibrium (see [Cao and Tian, 2010](#)).

11 See [Cao and Tian \(2010\)](#) and [Maskin and Riley \(2003\)](#), for justifications of the uniqueness of the equilibrium. The proof of Lemma 1 can also be found in [Cao and Tian \(2010\)](#), and is thus omitted in this paper.

$(F(b) - F(v_Y))^{k-1}(v_Y - b)$ . The optimal inverse bidding functions  $v_i^k(\cdot)$  and  $v_j^k(\cdot)$  are uniquely determined by the following:

$$v_i^k(b) = b \text{ for } v_N \leq b \leq \underline{b}_k \tag{9}$$

and for,

$$\begin{aligned} &\underline{b}_k < b \leq \bar{b}_k \\ &\left\{ \begin{aligned} &\frac{f(v_i^k(b))v_i^{k'}(b)}{F(v_i^k(b)) - F(v_N)} + \frac{(k-1)f(v_j^k(b))v_j^{k'}(b)}{F(v_j^k(b)) - F(v_Y)} = \frac{1}{v_i^k(b) - b} \\ &\frac{kf(v_j^k(b))v_j^{k'}(b)}{F(v_j^k(b)) - F(v_Y)} = \frac{1}{v_i^k(b) - b} \end{aligned} \right. \tag{10} \end{aligned}$$

with boundary conditions  $v_j(\underline{b}_k) = v_Y$ ,  $v_i(\underline{b}_k) = \underline{b}_k$ , and  $v_i(\bar{b}_k) = v_j(\bar{b}_k) = 1$ .

*Remark 1* It is relevant to make the following remarks on Lemma 1:

- i. By means of Maskin and Riley (2003), a unique solution exists for the system of differential equations in Lemma 1.
- ii. When  $k = 1$ , for the ratifier in the seller’s auction, his only rival is the vetoer, and thus  $\underline{b}_1 = \operatorname{argmax}_b (F(b) - F(v_N))(v_Y - b)$ . When  $k \geq 2$ , for any ratifier, he competes with the vetoer and other ratifiers. From  $\underline{b}_k = \operatorname{argmax}_b (F(b) - F(v_N))(F(b) - F(v_Y))^{k-1}(v_Y - b)$ , it can be checked that  $\underline{b}_k = v_Y$ . Thus, the vetoer with a value less than  $v_Y$  will not submit a meaningful bid if we rule out the dominated strategies, as suggested by Maskin and Riley (2003).
- iii. When  $v_N < v_i < \underline{b}_1$ , the vetoer wins the auction if, and only if, he is the only participant in the auction.
- iv.  $v_i^k(b_i^k(v_i)) \geq v_i$  for  $v_i \geq v_Y, \forall k$ —i.e. when  $v_i \geq v_Y$ —the vetoer bids more aggressively than the ratifiers. Indeed, the vetoer, with value distributed on  $[v_N, 1]$ —known as a weaker bidder since  $F_Y(v) \leq F_N(v)$  for  $v_i \geq v_Y$ —will be more aggressive, as suggested by Maskin and Riley (2000).

Since all the ratifiers use the same bidding strategy, if there are two or more ratifiers in the seller’s auction, then the probability for the ratifier with the value  $v_Y$  to win is zero. This ratifier has positive revenue only when he is the only ratifier in the seller’s auction and his bid  $\underline{b}$  is higher than the vetoer’s. Let  $\tilde{v}_Y$  be the solution to:

$$c = (\tilde{v}_Y - \underline{b})F(\tilde{v}_Y)^{n-2} \frac{F(\underline{b}) - F(v_N)}{1 - F(v_N)} \tag{11}$$

i.e. the pay-off of a  $\tilde{v}_Y$  bidder is equal to his participation cost whenever  $\tilde{v}_Y \leq 1$ . We then have  $v_Y = \min\{1, \tilde{v}_Y\}$ . Notice that  $v_Y$  is the cut-off point at which the ratifiers are indifferent to participating in the seller’s auction. Since  $v_Y > v_N$ , an increase in  $v_N$  leads to a higher  $v_Y$ . Thus, we have that  $v_Y$  is a strictly increasing function of  $v_N$  until it reaches 1 for some value of  $v_N$  and stays there for a greater value of  $v_N$ .

The expected revenue for the vetoer with a value  $v_i \in [v_N, 1]$  depends on the number of ratifiers participating in the seller’s auction. Let  $H_k(v_i)$  be the probability that all other bidders’ bids are lower than that of vetoer  $i$  when there are  $k$  ratifiers participating. The expected pay-off to vetoer  $i$  (given the ratifier’s belief that the vetoer’s value is in the

interval  $[v_N, 1]$  and the equilibrium in the asymmetric first-price auctions—i.e.  $b^*$ ) is as follows:

$$\pi_i^v(v_i, b^*) = \sum_{k=0}^{n-1} C_{n-1}^k [1 - F(v_Y)]^k F(v_Y)^{n-k-1} [v_i - b_i^k(v_i)] H_k(v_i) - c \tag{12}$$

where  $C_{n-1}^k [1 - F(v_Y)]^k F(v_Y)^{n-k-1}$  is the probability that there are  $k$  ratifiers participating in the seller's auction.

More specifically, when  $v_N < v_i < \underline{b}_1$ , a bidder can win the auction only when no other bidders participate in the auction, which happens with probability  $F(v_Y)^{n-1}$ . In this case, the vetoer bids zero. Thus:

$$\pi_i^v(v_i, b^*) = v_i F(v_Y)^{n-1} - c \tag{13}$$

We will show that there is a unique asymmetric equilibrium in the auction such that the vetoer's expected profit at equilibrium is greater than that obtained from staying in the cartel if his value is larger than  $v_N$ . Proposition 2 considers the case when  $c = 0$ .

*Proposition 2* An efficient cartel mechanism is ratifiable when  $c = 0$  and bidders can update their beliefs about the other participants' values.

The intuition is that when the participation cost is zero, if a bidder vetoes the cartel, we have  $v_Y = v_N = 0$  (i.e. all bidders enter the seller's auction symmetrically). In this case, the game becomes the basic non-collusive model in McAfee and McMillan (1992). It is obvious that  $\pi_i^m(v_i) \geq \pi_i^v(v_i, b^*) = \pi_i^s(v_i)$  for any  $v_i \in [0, 1]$ . Therefore, in this case, the bidder with the value  $v_i \in A_i = [0, 1]$  does not veto the cartel. This is because the bidder's vetoing signal becomes an incredible threat when there are no participation costs—now the losers in the cartel's auction can participate in the seller's auction without incurring a participation cost.

*Remark 2* Tanno (2008) reaches the same conclusion with a numerical example of government procurement auctions without participation costs, in which there are only two bidders and the values follow a uniform distribution. Our result is more general in the sense that it allows for any number of potential bidders and general distributions on values.

When participation costs are positive, considering the first-price auction with information leakage, we obtain the result presented in Proposition 3.

*Proposition 3* In a first-price sealed-bid auction, supposing  $c > 0$  and bidders can update their beliefs about the other bidders' values, the strong efficient cartel mechanism is no longer ratifiable.

*Remark 3* Here, we provide a basic intuition for the Proof of Proposition 3 (the formal closed form proof is in the online Appendix). Vetoer  $i$  has types of support  $[v_N, 1]$  with  $F_N(v)$ . By means of the envelope theorem, we can represent the pay-off of type  $v \in [v_N, 1]$  from the equilibrium characterized in Lemma 1 as  $\pi_i^v(v_i) = \int_{v_N}^{v_i} \tilde{G}_N(s) ds + \pi_i^v(v_N)$ , where  $\tilde{G}_N$  is the probability of winning the competitive auction. With  $\pi_i^m(v_i) = \pi_i^m(0) + \int_c^{v_i} G(y) dy$ , we need to show  $\pi_i^v(v_i) - \pi_i^m(v_i) > 0$ —i.e.  $\int_{v_N}^{v_i} \tilde{G}_N(s) ds + \pi_i^v(v_N) - \pi_i^m(0) - \int_c^{v_i} G(y) dy > 0$ —which gives the vetoer incentive to jump out of the cartel. In step 1 of the proof of Proposition 3, we show that there is a  $v_N$ , such that  $\pi_i^v(v_N) = \pi_i^m(v_N)$ , and Lemma 1 implies that  $\tilde{G}_N(v) > G(v)$  if  $v \geq v_N$ . Thus, it is sufficient to argue that every  $v \in [v_N, 1]$  wins against types that are larger when vetoing.

The intuition of Proposition 3 is based on the following two effects: the participation effect and the bidding effect. In the participation effect, having updated their beliefs that the vetoer's value belongs to  $A_i$ , other bidders with low values will not participate in the seller's auction, because they would have to pay the non-refundable participation costs. This, in turn, leads to a higher expected profit for the vetoer due to weakened competition. The bidding effect is induced by vetoing. Given  $v_N < v_Y$ , once a ratifier participates, the vetoer (known as the weak bidder) will bid more aggressively in the first-price auction (Maskin and Riley, 2000). Therefore, the vetoing bidder wins more often, as when he bids under efficient collusion he becomes the weaker bidder—i.e.  $\tilde{G}_N(v) > G(v)$ —which causes the ratifier to have less chance of winning.<sup>12</sup> Thus, he enjoys higher pay-offs when vetoing than when colluding. Note that the bidding effect works in the same direction as the participation effect, making a veto more desirable for someone with a value higher than  $v_N$ . As a result, the cartel mechanism is not ratifiable.

*Remark 4* As indicated by the proof of Proposition 3, if a bidder with the value  $v_i$  finds that it is more beneficial to leave than to stay in the cartel, then it must be the same case for any  $v'_i > v_i$ . Given that  $v_N$  is uniquely determined, the veto set is unique.

By combining the results of Propositions 2 and 3, one can see that an efficient cartel mechanism cannot be ratifiable when both the information leakage problem and positive participation costs are present. When participation costs exist without an information leakage problem, as in Proposition 1, a strong cartel mechanism is still ratifiable. Without the information leakage problem, bidders cannot update their beliefs about other bidders' valuations through the cartel's auction, so no one has an incentive to exit the cartel. On the other hand, when the information leakage problem exists without participation costs, a cartel mechanism is still ratifiable. Since bidders can submit a bid in the seller's auction at no cost, the vetoer cannot earn extra profit from vetoing the cartel. Thus, an efficient cartel mechanism is still ratifiable even if there is a participation cost or an information leakage problem, but it is not ratifiable when both are present. There is a discontinuity at  $c = 0$  for the ratifiability of the efficient cartel mechanism.

We should point out that the discontinuity at  $c = 0$  comes from the assumption that bidders participate in the auction when they are indifferent about participating. If this were not true, then the non-ratifiability of an efficient collusive scheme would continue to hold for  $c = 0$ .

To illustrate the main conclusion of this section, we present Example 1.

*Example 1* Suppose  $v$  is uniformly distributed on the interval  $[0, 1]$  and  $n = 2$ .  $v_N$  is determined by the indifference to remaining in or vetoing the cartel; thus, we have the first condition:

$$\pi^m(0) + \int_c^{v_N} F(u) du = v_N F(v_Y) - c \quad (14)$$

where the left-hand side of Equation (15) represents the expected pay-off for staying in the cartel and the right-hand side shows the expected pay-off from participating in the seller's auction (when  $v_i = v_N$ , a bidder can win the auction only if he is the sole bidder because, if

12 The bidding effect does not exist for the case considered in Tan and Yilankaya (2007), where the seller uses second-price auctions.

there is any other bidder, it must be the case that  $v_j \geq v_Y > v_N$  and thus a  $v_N$  vetoer has no chance of winning). With  $F(v) = v$ , Equation (15) is equivalent to:

$$v_N v_Y - c = \frac{1}{2} v_N^2 - \frac{c}{2} - \frac{1}{6} c^3 + \frac{1}{6} \quad (15)$$

where  $v_Y$  is determined by the zero net pay-off for a ratifier who participates in the seller's auction. For the ratifier  $j$  with  $v_j = v_Y$ , assuming that  $v_Y \leq 1$  since otherwise the ratifier never participates, their expected pay-off from participating in the auction is given by Equation (16):

$$\max_b \frac{F(b) - F(v_N)}{1 - F(v_N)} (v_Y - b) - c \quad (16)$$

This maximization problem gives  $b = \frac{v_Y + v_N}{2}$  and the zero net pay-off gives us the second condition, presented in Equation (17):

$$v_Y - v_N = \sqrt{4c(1 - v_N)} \quad (17)$$

Now we have two conditions on  $v_Y$  and  $v_N$ . Our numerical examples show that, when  $c$  is less than 0.1755,  $v_Y$  is less than 1 and the ratifier with value that is higher than  $v_Y$  has a chance to win in the seller's auction. When  $c \geq 0.1755$ , a ratifier never participates in the seller's auction. Thus, when  $c \geq 0.1755$ ,  $v_Y = 1$ , and  $v_N$  is determined by  $v_N - c = \frac{1}{2} v_N^2 - \frac{c}{2} - \frac{1}{6} c^3 + \frac{1}{6}$ . For example, when  $c = 0.1755$ ,  $v_N = 0.2980$ ; when  $c = 2/3$ ,  $v_N = 0.686$ .

Now consider an extreme case where  $c$  approaches zero. From the first condition, in Example 1, we have  $v_Y v_N = \frac{1}{2} v_N^2 + \frac{1}{6}$ , and from the second condition, in Example 2, we get  $v_Y - v_N = 0$ . These two conditions give  $v_N = 0.577$ , which implies that, even when  $c = 0$ , the cartel mechanism is not ratifiable if a ratifier with value of less than  $v_N$  does not participate—i.e. if we assume that a ratifier does not participate when he has no chance of winning.

However, if a ratifier still chooses to participate when he is indifferent about participating, then when  $c = 0$ , the vetoer must worry not only about a ratifier with a value in the interval  $[v_Y, 1]$ , but also about a ratifier with a value in the interval  $[0, v_Y]$ .<sup>13</sup> As such, bidders do not have any incentive to deviate. To see this, consider the first-price auction in which a vetoer's value is distributed on  $[v_N, 1]$  with  $F_{v_N}(v) = \frac{v - v_N}{1 - v_N}$  and a ratifier's value is distributed on  $[0, 1]$  with  $F(v) = v$ . A vetoer with the value  $v_N$  maximizes  $(v_N - b)F(b)$  in respect of  $b$ , which gives an expected pay-off of  $\frac{1}{4} v_N^2$ . The pay-off for staying in the cartel is  $\frac{1}{2} v_N^2 + \frac{1}{6}$ . Then, in this case, it is impossible to find a  $v_N \in [0, 1]$  such that  $\frac{1}{4} v_N^2 = \frac{1}{2} v_N^2 + \frac{1}{6}$ . So, when  $c = 0$ , no bidder has any incentive to veto the cartel. Thus, there is a discontinuity between  $c = 0$  and  $c > 0$ .

## 5. Conclusion

This paper studies the ratification of an efficient collusive mechanism in a setting where the seller uses a first-price sealed-bid auction. We find that a typically efficient cartel mechanism will no longer be ratifiable, in the sense that a bidder vetoes a pre-auction knockout in

13 A positive participation cost will eliminate a ratifier of this type.

the presence of both participation costs and potential information leakage. We also find that, in the absence of either the participation cost or the information leakage problem, an efficient cartel mechanism is still ratifiable. The implication of our results is that, in practice, the seller can charge an entry fee so as to reduce the possibility of bidders forming a cartel and, thus, enhance the competition among bidders.<sup>14</sup> This, in turn, increases the expected revenue of the seller.<sup>15</sup>

Several empirical studies show evidence of collusion, and these studies generally involve first-price sealed-bid auctions.<sup>16</sup> Our theoretical results shed light on why it becomes more difficult to form a cartel; it may be due to the presence of both participation costs and the potential for information leakage. In the real world, there may also be other factors that affect the formation of a cartel. For instance, members in a successful cartel may play repeated games where a vindictive strategy is typically used to enforce collusion.

Finally, it should be noted that, in our model, all bidders are assumed to be initially in an efficient cartel. However, it is possible that, in reality, not all bidders are in the cartel and there may be outside bidders. With participation costs and an outside threat, the cartel mechanism is not the same as an efficient cartel mechanism without an outside threat. [Marshall and Marx \(2007\)](#) investigate similar cases in the absence of participation costs and show that bidders in a cartel cannot satisfy the incentive compatible constraint. In addition, the bidders' expected profit would be affected by the bids and value distributions of both outsiders and members. When the seller uses a first-price auction with a participation cost; whether the not-all-inclusive cartel is ratifiable, or whether it exists, and how a cartel operates provide potentially interesting questions for future research.

## Supplementary material

[Supplementary material](#) is available on the OUP website. This is the [online appendix](#) containing the proofs.

## Funding

This work was supported by the National Natural Science Foundation of China [NSFC-71740016 to X.C., NSFC-71403223 to S.H., NSFC-71371117 to G.T.], Science Foundation of Ministry of Education of China [16YJC790116], and the Established Scholar programme at the University of International Business and Economics [18JQ05].

- 14 As a caveat, the existence of participation costs does not necessarily lead to more efficient competition. For example, as emphasized by [Tian \(2019\)](#), when analysing the main cause of the deceleration of China's economic growth, state-owned enterprises usually face much lower participation costs than private firms, thereby squeezing the private economy and hurting economic vitality.
- 15 This involves the optimal entry fee from the perspective of the seller. An important issue for our discussion would be how the seller should charge an optimal entry fee so as to deter collusion in a first-price auction, or to implement the optimal collusion-proof allocation mechanism as in [Laffont and Martimort \(1997\)](#). However, since our paper mainly focuses on the optimal behaviour of bidders (i.e. whether or not to deviate from an efficient cartel mechanism) rather than on the optimal behaviour of sellers, we leave these interesting questions for future research.
- 16 Examples can be found in, for example, [Porter and Zona \(1993\)](#) and [Baldwin et al. \(1997\)](#).

## Acknowledgments

We wish to thank Alan Beggs (the Associate Editor) and two anonymous referees for their helpful comments and valuable suggestions, which significantly improved the exposition of this paper. We would also like to thank the seminar participants of the Theory Workshop of the Department of Economics, Texas A&M University.

## References

- Baldwin, L., Marshall, R., and Richard, J.F. (1997) Bidder collusion at Forest Service Timber Sales, *Journal of Political Economy*, 105, 657–99.
- Balzer, B. (2017) *Collusion in auctions: an informed principal perspective*. University of Technology, Sydney.
- Cao, X., and Tian, G. (2010) Equilibria in first price auctions with participation costs, *Games and Economic Behavior*, 2, 258–73.
- Che, Y., and Kim, J. (2006) Robustly collusion-proof implementation, *Econometrica*, 74, 1063–107.
- Cooper, J.C. (1977) *Under the Hammer: The Auctions and Auctioneers of London*, Constable, London.
- Cramton, P.C. and Palfrey, T.R. (1990) Cartel enforcement with uncertainty about costs, *International Economic Review*, 31, 17–47.
- Cramton, P.C. and Palfrey, T.R. (1995) Ratifiable mechanisms: learning from disagreement, *Games and Economic Behavior*, 10, 255–83.
- Esó, P. and Schummer, J. (2004) Bribing and signaling in second price auctions, *Games and Economic Behavior*, 47, 299–324.
- Graham, D. and Marshall, R. (1987) Collusive bidder behavior at single-object second-price and English auctions, *Journal of Political Economy*, 95, 1217–119.
- Graham, D., Marshall, R., and Richard, J.F. (1990) Differential payments within a bidder coalition and the Shapley value, *American Economic Review*, 80, 493–510.
- Hendricks, K., Porter, R., and Tan, G. (2008) Bidding rings and the winner's curse, *The Rand Journal of Economics*, 39, 1018–41.
- Laffont, J. and Martimort, D. (1997) Collusion under asymmetric information, *Econometrica*, 65, 875–911.
- Mailath, G. and Zemsky, P. (1991) Collusion in second price auctions with heterogeneous bidders, *Games and Economic Behavior*, 3, 467–503.
- Marshall, R.C. and Marx, L.M. (2007) Bidder collusion, *Journal of Economic Theory*, 133, 374–402.
- Maskin, E.S. and Riley, J.G. (2000) Asymmetric auctions, *Review of Economic Studies*, 67, 413–38.
- Maskin, E.S. and Riley, J.G. (2003) Uniqueness of equilibrium in sealed high-bid auctions, *Games and Economic Behavior*, 45, 395–409.
- McAfee, R.P. and McMillan, J. (1992) Bidding rings, *American Economic Review*, 67, 579–99.
- Osborne, D.K. (1976) Cartel problems, *American Economic Review*, 66, 835–44.
- Porter, R. and Zona, D. (1993) Detection of bid rigging in procurement auctions, *Journal of Political Economy*, 101, 518–38.
- Rachmilevitch, S. (2013) Bribing in first-price auctions, *Games and Economic Behavior*, 77, 214–28.
- Roberts, K. (1985) Cartel behavior and adverse selection, *The Journal of Industrial Economics*, 33, 401–13.
- Stiglitz, J.E. (2019) An agenda for reforming economic theory, *Frontiers of Economics in China*, 14, 149–67.

- Tan, G. and Yilankaya, O.** (2007) Ratifiability of efficient collusive mechanisms in second-price auctions with participation costs, *Games and Economic Behavior*, **59**, 383–96.
- Tanno, T.** (2008) Ratifiable collusion and bidding systems in procurement, *SSRN Electronic Journal*.
- Tian, G.** (2019) Deceleration of China’s economic growth: causes and countermeasures, *Frontiers of Economics in China*, **14**, 3–25.