Optimal Interregional Redistribution and Local Budget Rules with Multidimensional Heterogeneity*

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Abstract

In this study we analyse optimal interregional redistribution and local budget rules in a two-region, two-period federation model. The two regions differ in privately observable discount factors and in publicly observable durability of local public goods. We address the question of whether the contributor region of redistribution should face a weaker borrowing constraint than the recipient region. The answer to this question is yes for two cases: (1) the patient region is the recipient and has public goods durability no greater than that of the impatient region; (2) the impatient region is the recipient with smaller public goods durability and the regional difference in discount factors is small. Otherwise, the recipient region may face a debt floor rather than a debt limit. These differentiated budget rules solve the self-selection problem under asymmetric information and decentralized borrowing and spending decisions, internalize the positive intergenerational externality durable public goods entail, and hence are constrained efficient. Moreover, optimal interregional redistribution schemes feature that the region with an undistorted intertemporal allocation of local public goods is always the contributor.

Keywords: Intergovernmental grants; debt limit; debt floor; durable public goods; heterogeneous time preferences; mechanism design.
JEL classification codes: D02, D82, H72, H73.

1 Introduction

For many countries that implement fiscal decentralization, especially those adopting fiscal federalism institutions, the central government (the "center" hereafter) undertakes income redistribution or intergovernmental grants across heterogeneous regions, and simultaneously imposes certain rules on local government borrowing. Interregional redistribution is mainly motivated by fiscal equalization while regulating local borrowing, such as imposing debt limits or debt ceilings,[1] and serves to address the fiscal profligacy problem faced by regional governments.

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[1]Such policies are implemented in various countries including Australia, Canada, Switzerland, Germany, Austria, and the United States (Bird and Slack 1983; Mathews 1984; Smekal 1984; Mahdavi and Westerlund 2011; Grembi, Nannicini, and Troiano 2016). Recent evidence shows that most states in the United States have a

Combining these two objectives together, we arrive at the following question concerning fiscal institution design under a normative criterion: should regions which contribute to interregional redistribution face weaker borrowing constraints than regions which benefit from interregional redistribution?

We use a two-period model of a federation composed of a center and two local governments in two regions to address this question. Whilst the local governments collect taxes, issue debt, and provide local public goods that exhibit a certain degree of durability,[2] the center is responsible for undertaking interregional income redistribution. We assume the two regions differ in discount factors (the rates of time preference of local politicians)[3] and the durability (or quality) of local public goods. As identified by most previous fiscal federalism literature, the center is subject to an informational constraint as it has cost disadvantages in the information acquisition of local conditions relative to local governments who are closer to their constituencies.[4] We assume that each region is privately informed about its rate of time preference.[5] Therefore, the benevolent center must offer incentive compatible and budget balanced contracts, which consist of debt and transfers to maximize the summation of the social welfare of the two regions.

Due to their relevance in causing regional welfare disparity, we focus on the following two dimensions of regional heterogeneity: discount factor and local public goods durability. In a dynamic setting, different rates of time preference or degrees of public goods durability, ceteris paribus, lead to different intertemporal allocations, thus leading to welfare disparity across regions. The discount factor can either be interpreted as measuring a local government's preference for future consumption, or as the degree to which it takes into account the welfare of future generations when making long-term policy decisions. Therefore, a patient region may provide more durable or higher quality public goods than those provided by an impatient region.[6]

When regions only differ in the privately observable discount factors, the asymmetric information optimum has two main features. Firstly, the impatient region is always allocated a higher level of debt issuance than the patient region. Secondly, the impatient region is the contributor and the patient region is the recipient of interregional redistribution if two conditions are satisfied: (1) the impatient region represents the top (or efficient) type, such that its incentive compatibility constraint is binding in the optimum and its intertemporal allocation is undistorted; and (2) the regional disparity in the rate of time preference is large. However, if the patient region is the top type and the regional disparity in the rate of time preference is small, then the patient region contributes towards redistribution whilst the impatient region is the beneficiary. To implement the asymmetric information optimum when borrowing and spending decisions are decentralized at the regional level, we obtain the following differentiated

balanced budget requirement (BBR) and the stringency of state requirements varies substantially. For example, some states face more stringent BBRs than others (National Conference of State Legislatures 2010; Smith and Hou 2013; Urban-Brookings Tax Policy Center 2021). Additonally, the increasing default risk of local government debt in the Chinese economy (The Economist 2015; Huang, Liu, and Tian 2020; Huang, Li, and Tian 2021) has led the central government to implement a series of strict rules on how local governments can issue and manage debt, the aim being to control financial risks.

[2]Following common practice in existing literature, we interpret public goods durability as a positive intergenerational externality (spillover). Therefore, the following three terms are interchangeable: public goods durability; intergenerational externality; and intergenerational spillover.

[3]Note that the following three terms are interchangeable: discount factor; rate of time preference; and the degree of patience.

[4]See, e.g., Oates (1972, 1999, 2005); Bucovetsky, Marchand, and Pestieau (1998); Cremer and Pestieau (1997); Raff and Wilson (1997); Lockwood (1999); Cornes and Silva (2000, 2002); Bordignon, Manasse, and Tabellini (2001); Huber and Runkel (2006, 2008); Breuillé and Gary-Bobo (2007); Kıbrıs and Tapkı (2014).

[5]As documented by Evans and Sezer (2004), asymmetric information with respect to the rate of time preference does exist in real world federations.

[6]As suggested by a referee, a positive correlation of the degree of patience and the degree of durability does not always reflect reality. Therefore, we consider both cases in the following formal analysis: either the patient region's public goods or the impatient region's public goods are more durable.

budget rules: only when the patient region is the recipient of redistribution should it face a stricter borrowing constraint than the contributor; if the impatient region is the recipient, then it should face a debt floor rather than a debt limit.[7]

We then characterize the asymmetric information optimum when regions differ in both discount factors and public goods durability; only discount factors remain private. In cases where the impatient region's public goods are less durable (of lower quality), we obtain the following results. Firstly, if the impatient region is the top type, the regional difference in public goods durability is small, and the regional difference in the degree of patience takes an intermediate value (bounded from below and above), and then the impatient region contributes to interregional redistribution and is allocated a lower level of debt issuance than the patient region. Secondly, the patient region contributes to interregional redistribution and is allocated a higher level of debt issuance than the impatient region if the patient region is the top type, and the regional difference in public goods durability is large. Truthful implementation of the asymmetric information optimum through decentralized debt decisions leads to the following differentiated budget rules: if the regional difference in the degree of patience is small, the impatient region should only face a stricter borrowing constraint than the contributor if the impatient region is the recipient; otherwise, the patient region should only face a debt floor rather than a debt limit if the patient region is the recipient.[8]

Under certain conditions placed on the regional difference in public goods durability, the asymmetric information optimum still features that the top type contributes to redistribution, but whenever the impatient region has more durable public goods (of higher quality), the impatient region is always allocated a higher level of debt issuance. Truthful implementation through decentralized debt decisions leads to differentiated budget rules: only when the patient region is the recipient should it face a stricter borrowing constraint than the contributor; otherwise, if the impatient region is the recipient, then it should face a debt floor rather than a debt limit.

The following five points summarize the key insights conveyed by these results.

Firstly, a joint consideration of the two dimensions of regional heterogeneity is significant in terms of solving the self-selection problem facing the center. For example, if regions only differ in the degree of patience, the asymmetric information optimum features that the impatient region is always allocated a higher level of debt issuance than the patient region, regardless of whether the impatient or patient region is identified as the undistorted top type. This result is reversed if the patient region has a higher public goods durability than the impatient region. Nevertheless, if the impatient region has durability higher than the patient region, the asymmetric information optimum still features that the impatient region is always allocated a higher level of debt issuance. This observation seems to suggest that the publicly observable regional heterogeneity on public goods durability dominates the privately observable time preferences in terms of determining which region should be allocated a higher level of debt issuance under asymmetric information.

Secondly, the top type with an undistorted intertemporal allocation always contributes to re-

[7] As pointed out by the reviewers, we do not seem to observe debt floors in reality. We argue that this is mainly due to political economy considerations. For instance, in economies with intense political competition, such as democracies with regular elections or the local government competition in China driven by political promotion incentives, rational and short-sighted politicians are prone to overspend and undertax in an effort to court current voters, who enjoy the benefits of expenditures funded, in part, by future taxpayers, improving their prospects of getting reelected or promoted while leaving debt repayment obligations to their successors. Therefore, future research may investigate the political feasibility of this normative result.

[8] The insight of this finding is consistent with the macroeconomic public finance literature. For example, using a model with both consumption and productive public goods, Bassetto and Sargent (2006) show that strict borrowing limits should be imposed on the provision of consumption public goods, and weak borrowing limits should be imposed on budgets for the provision of productive public goods. Once consumption public goods are relabelled as goods provided by the impatient region, and productive public goods as goods provided by the patient region; this finding echoes that of Bassetto and Sargent (2006).

distribution while extracting information rent from the center, regardless of whether the regions only differ in the degree of patience or in both dimensions.

Thirdly, it is not always true that the recipient region should face a stricter borrowing limit than the contributor, regardless of whether regions only differ in the degree of patience or in both discount factor and public goods durability. In cases where regions only differ in the degree of patience, a debt floor rather than a debt limit should be applied to the recipient if the impatient region is the recipient. Similarly, in the case of multidimensional regional heterogeneity, if the public goods of the recipient are more durable, then it should face a debt floor rather than a debt limit.

Fourthly, the key prerequisite for the recipient to face a strict debt limit is that its public goods durability is no higher than that of the contributor, regardless of whether the patient or impatient region is the recipient. This seems to suggests that the publicly observable regional heterogeneity on public goods durability dominates the privately observable time preferences in terms of determining whether the recipient should face a stricter borrowing constraint than the contributor. This insight yields a policy implication with a certain level of informational efficiency. Notably, if a region benefits from redistribution and provides local public goods with lower quality than the contributor, then this region should face a stricter borrowing rule than the contributor, regardless of whether or not it is patient.

Fifthly, departing from the political convention argument, as proposed by National Conference of State Legislatures (2010), we highlight the efficiency criterion of designing and enforcing differentiated budget rules, namely that economic efficiency requires a certain type of region to face a debt limit, whereas another type should face a weak borrowing constraint or even a debt floor. The logic of imposing debt floor is similar to that of imposing debt limit, which is to solve the self-selection problem under asymmetric information and simultaneously internalize the positive intergenerational externality created by durable public goods. It should be noted that in this context regional debt is issued for financing the provision of public goods that produce positive intergenerational spillovers. A strict borrowing limit may lead to an inefficient provision of public goods under certain conditions. Therefore, imposing a debt floor whenever necessary guarantees efficient provisions of durable public goods.[9]

This study closely relates to existing literature that examines the design of optimal interregional redistribution policy and budget institutions under asymmetric information, such as Huber and Runkel (2008) and Dai, Liu, and Tian (2019a,b) and references therein. Ignoring that local public goods are nondurable and regions differ only in the degree of patience, Huber and Runkel (2008) also justify the fiscal constitution with lax budget rules for contributors and strict budget rules for recipients of interregional redistribution. Dai, Liu, and Tian (2019a) consider a similar setting but assume that regions only differ in the privately observable durability of local public goods. They verify Huber and Runkel's conclusion when regions with higher durabilities contribute to redistribution and those regions with lower durabilities benefit from redistribution. Dai, Liu, and Tian (2019b) then modify Huber and Runkel's model by allowing for cross-region migrations, demonstrating that their conclusion holds if regional governments maximize the welfare of their respective residents, but will be reversed if the regional governments maximize the respective welfare of natives, migration intensity is high, and the regional difference in the degree of patience is large.[10]

[9]Generally, the center must address the problem of fiscal profligacy to avoid a deficit or a debt crisis in order to design efficient budget institutions for local governments; on the other hand, it must allow for sufficient budgets such that the local governments can serve residents and provide basic public goods such as roads, policing, and schools (Randall and Rueben 2017).

[10]However, neither Huber and Runkel (2008) nor Dai, Liu, and Tian (2019b) consider durable public goods that may produce intergenerational spillovers. Future research may extend our model to investigate how a joint consideration of intergenerational spillovers and interjurisdictional spillovers (or horizontal fiscal externalities) induced by migration (Schultz and Sjöström 2001, 2004; Breuillé and Gary-Bobo 2007; Conley, Driskill, and

Accounting for the intergenerational spillovers entailed by durable local public goods, our study enriches Huber and Runkel (2008)'s analysis in several ways. The intertemporal allocation of public goods plays an essential role in their analysis; the new factor of public goods durability modifies the tradeoff that governs a region's intertemporal decision-making. Public debt is issued in the present generation, but repayment occurs in future generations, and thus it will generate a negative forward intergenerational externality. It should also be noted that public debt will be used by the present generation for financing the provision of local public goods that produce a positive intergenerational externality. Thus, an optimal intertemporal allocation must internalize these two countervailing external effects, thereby leading to novel policy implications. Indeed, Huber and Runkel (2008) obtain the clear-cut prediction that impatient regions should contribute to redistribution and patient regions should be recipients. On the other hand, our findings demonstrate that this only represents one possibility. Even if the patient and impatient regions have the same public goods durability, our research provides reasonable conditions under which the patient region should rather be the contributor and the impatient region the recipient.

More importantly, our study not only subsumes their results as a special case. It also gains new insights into the design of optimal budget institutions for local governments in federations. For example, we identify the significance of public goods durability in characterizing asymmetric information welfare optima and determining optimal local budget rules. In the asymmetric information optima, the key prerequisite for the impatient region to be allocated a higher level of debt issuance is that its public goods durability is no smaller than that of the patient region; otherwise, it would be allocated a lower level of debt issuance. In addition, we establish that the recipient can only face a stricter borrowing constraint than the contributor when the recipient region's public goods are less durable than those of the contributor region, regardless of whether the recipient is patient or not.

Assuming that regions only differ in the privately observable durability of local public goods, Dai, Liu, and Tian (2019a) find that the recipient should only face a stricter borrowing constraint when it has a lower degree of durability. Whilst both those papers establish this budget rule for solving an adverse selection problem induced by asymmetric information between the center and regions, Dai, Liu, and Tian (2019a) consider the source of asymmetric information as public goods durability. However, we differ and apply time preference in this paper. Note that discount factor and public goods durability shape a region's intertemporal allocation in opposite directions: the opportunity cost of borrowing is increasing in discount factor, but is decreasing in public goods durability. Additionally, there are subtle differences regarding the conditions required for establishing this budget rule. We obtain this rule if either the impatient region has a lower degree of public goods durability and the regional difference in the degree of patience is small, or the patient region has a lower degree of public goods durability. In particular, combining the analyses in both papers yields an important policy implication: the factor of local public goods durability and quality is of critical importance in determining whether the redistribution recipient should face a strict borrowing limit, regardless of whether it is each region's private information.

In summary, our paper considers a more realistic circumstance of multidimensional regional heterogeneity, and demonstrates that the main conclusion of existing literature can be overturned. In terms of solving the self-selection problem faced by the center in the presence of asymmetric information regarding the exogenous characteristics of regions, the fiscal constitution characterized by weak borrowing constraints for contributors and strict borrowing constraints for recipients is not always efficient; hence, we may need to appeal to alternative more efficient local budget institutions.[11]

Wang 2019; Dai, Jansen, and Liu 2021) may shape the design of optimal fiscal constitutions.

[11] Intuitively, given that public debt is mainly used to finance the provision of public goods and services, strict budget requirements are not always efficient. For example, Chaney, Copley, and Stone (2002) show that fiscally

The remainder of the paper is organized as follows. Section 2 describes the economic environment. Section 3 derives the welfare optimum when regions only differ in the degree of patience. Section 4 derives the welfare optimum when regions differ in patience and public goods durability. Section 5 justifies differentiated budget rules that implement the asymmetric information welfare optima derived in Sections 3 and 4 based on the assumption that debt decisions are decentralized at the regional level. Section 6 concludes. All proofs are relegated to Appendix A, whilst Appendix B provides a brief discussion of the multidimensional screening issue when both discount factor and public goods durability are each region's private information.

2 The Model

We consider a two-period environment of a federation composed of a federal government (also referred to as the center) and two local governments in two regions, indexed A and B. Each region is populated by a cohort of identical individuals, the measure of which is assumed to be unity for notational simplicity, and they live for one period only. After the Period-1 cohort (called generation 1) has passed away, there is a new cohort of the same measure in Period 2 (generation 2). For Region $R \in \{A, B\}$, the present value of its welfare is given by

$$\overbrace{u(c_1^R) + g(G_1^R)}^{\text{utility of generation 1}} + \delta^R \cdot \overbrace{\left[u(c_2^R) + g(\theta^R G_1^R + G_2^R)\right]}^{\text{utility of generation 2}}, \tag{1}$$

in which c_t^R and G_t^R denote private consumption and the units of local public goods provision in period $t \in \{1, 2\}$, $\delta^R \in (0, 1)$ is Region R's discount factor (or the local politician's rate of time preference),[12] and $\theta^R \in (0, 1]$ is a parameter measuring the degree of intergenerational spillovers generated by the durable public goods G_1^R.[13] As usual, $u(\cdot)$ and $g(\cdot)$ in Equation (1) are strictly increasing and strictly concave, and satisfy the Inada conditions.

In reference to Huber and Runkel (2008) and Dai, Liu, and Tian (2019b), we impose the following:

Assumption 2.1 *The two regions differ in discount factors with* $\delta^A < \delta^B$.

In other words, Region A has a smaller discount factor and is therefore more impatient than Region B. Without loss of generality, we refer to Region A as the impatient region (L-region) and Region B as the patient region (H-region).

An individual of generation t in Region R faces the budget constraint $c_t^R + \tau_t^R = y_t$ for a given pre-tax income $y_t > 0$. The lump sum tax τ_t^R is collected for financing the provision of local public goods. In Period 1, Region R receives federal transfers, z^R, which could be negative, and issues debt b^R. If $z^R < 0$, then Region R pays a lump sum tax to the center. Debt and interest have to be repayed in Period 2 under a given interest rate $r > 0$. The public budget constraints of Region R in Periods 1 and 2 can be written as $G_1^R = \tau_1^R + b^R + z^R$ and $G_2^R = \tau_2^R - (1 + r)b^R$.[14]

stressed states in the United States both underfund their pensions and select discount rates which obscure the underfunding. As argued by Levinson (1998, 2007), strict budget rules may increase fiscal and economic volatility. Indeed, both during and after the Great Recession, states with strong balanced budget requirements cut their budgets and raised revenues precisely when the economy and residents would have benefited from states spending more and easing taxes, as shown by Rueben, Randall, and Boddupalli (2018).

[12] We let the discount factor be strictly smaller than one because the present generations that make local public policy decisions that will impact on the welfare of future generations are imperfectly altruistic (Altonji, Hayashi, and Kotlikoff 1992, 1997).

[13] This is the stylized way of introducing intergenerational spillovers into a formal model (Rangel 2003, 2005; and Dai, Liu, and Tian 2019a). Moreover, a non-depreciated portion of the public good is carried over from the first to the second period, and this specification is consistent with the concept of productive public goods, which frequently appears in the macroeconomic public finance literature (Chatterjee and Ghosh 2011).

[14] Note that issuing debt has the effect of redistributing resources intertemporally (or between generations), whilst levying tax redistributes resources intratemporally, they would be used as complementary fiscal tools.

Under the ad-hoc assumption of pure interregional redistribution, the budget constraint facing the center reads as follows:

$$\sum_{R\in\{A,B\}} z^R \leq 0. \tag{2}$$

In the remaining part of this section, the region index R is suppressed to simplify notation. Applying the private and public budget constraints to Equation (1), a region's value function (the maximized level of regional welfare) is given by

$$\begin{aligned} V(b,z,\theta,\delta) \ \equiv \ \max_{c_1,c_2} \ & u(c_1)+g(y_1+b+z-c_1) \\ & +\delta\left[u(c_2)+g(\theta(y_1+b+z-c_1)+y_2-b(1+r)-c_2)\right]. \end{aligned} \tag{3}$$

The first-order conditions are thus written as

$$u'(c_1)=g'(G_1)+\theta\delta g'(\theta G_1+G_2) \ \text{ and } \ u'(c_2)=g'(\theta G_1+G_2). \tag{4}$$

Departing from the Samuelson condition obtained by Huber and Runkel (2008) in a setting where public goods are nondurable, Equation (4) shows that the intratemporal provision of local public goods will be distorted by information frictions on the intertemporal margin.

3 The Benchmark Welfare Optimum with Unidimensional Heterogeneity

Using Equation (4), we write the optimal provisions of local public goods as functions of debt, transfers, the degree of intergenerational spillovers, and the degree of patience: $\hat{G}_1^R \equiv \phi(b,z,\theta,\delta^R)$ and $\hat{G}_2^R \equiv \psi(b,z,\theta,\delta^R)$. In the present section, we assume that $\theta^R \equiv \theta$ for any $R \in \{A,B\}$, i.e., regions differ only in one dimension as described in Assumption 2.1. To save on notation, θ will be suppressed from the value function V given by (3).

Lemma 3.1 *For the economic environment under consideration, the following statements are true:*

(i) The indifference curve in the (b,z)-space is U-shaped with the minimum at the point where the intertemporal rate of substitution (IRS) is equal to the intertemporal rate of transformation (IRT), namely,

$$\frac{g'\left(\hat{G}_1^R\right)}{\delta^R g'\left(\theta\hat{G}_1^R+\hat{G}_2^R\right)}=1+r-\theta.$$

(ii) The single-crossing property is satisfied in the sense that

$$\frac{\mathrm{d}}{\mathrm{d}\delta^R}\left(\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0}\right)>0 \text{ for all } \delta^R.$$

Proof. See Appendix A. ■

Using Lemma 3.1(i), the intertemporal rate of transformation, $1+r-\theta$, yields that the negative intergenerational spillover induced by debt repayment, denoted $1+r$, is partly offset by the positive intergenerational spillover produced by the durable local public goods, denoted θ. Under Assumption 2.1, Lemma 3.1(ii) yields that, in every point in the (b,z)-space, H-region's indifference curve has a larger slope than that of L-region. As is standard in the principal-agent theory (Laffont and Martimort 2002), the single-crossing property is crucial for solving adverse selection problems.

We assume that each region is privately informed about its degree of patience, but it is public knowledge that $\delta^R \in \{\delta^A, \delta^B\}$ with $\delta^A < \delta^B$, and that the two regions cannot be of the same type. Applying the revelation principle, the center offers each local government a contract stipulating the federal transfer and public debt. The timing of game reads as follows:[15]

- The two local governments privately observe δ^R.
- The federal government offers two contracts: $\{b^A, z^A\}$ and $\{b^B, z^B\}$ (or equivalently $\{b(\delta^R), z(\delta^R)\}$ for $\delta^R \in \{\delta^A, \delta^B\}$).
- The two local governments simultaneously pick a contract (or, equivalently, report their types), and then the game ends.

Formally, the benevolent center (namely the principal) solves the following maximization problem:

$$\max_{b^A, z^A, b^B, z^B} \quad V(b^A, z^A, \delta^A) + V(b^B, z^B, \delta^B)$$

subject to federal budget constraint (2) and the incentive-compatibility constraints:

$$\begin{array}{rcll} V(b^A, z^A, \delta^A) & \geq & V(b^B, z^B, \delta^A) & (\text{IC}_A); \\ V(b^B, z^B, \delta^B) & \geq & V(b^A, z^A, \delta^B) & (\text{IC}_B). \end{array}$$

The Lagrangian can thus be written as

$$\begin{aligned} \mathcal{L}(b^A, z^A, b^B, z^B; \mu^A, \mu^B, \lambda) = (1 + \mu^A) V(b^A, z^A, \delta^A) - \mu^A V(b^B, z^B, \delta^A) \\ + (1 + \mu^B) V(b^B, z^B, \delta^B) - \mu^B V(b^A, z^A, \delta^B) + \lambda(0 - z^A - z^B), \end{aligned} \tag{5}$$

in which μ^A, μ^B are nonnegative Lagrange multipliers associated to the truth-telling constraints. Given the center's pure redistribution goal, the federal budget constraint must be binding so that $\lambda > 0$, which thus gives a positive shadow price of federal funding.

Let us first consider the case of complete information. Throughout, we shall index the complete and asymmetric information optimum by the superscripts ^{FB} and *, respectively. By setting $\mu^A = \mu^B = 0$ in (5) and solving the maximization problem, the following lemma is obtained.

Lemma 3.2 *Under Assumption 2.1, the complete information optimum verifies:*

$$\frac{g'\left(G_1^{R,FB}\right)}{\delta^R g'\left(\theta G_1^{R,FB} + G_2^{R,FB}\right)} = 1 + r - \theta, \quad R \in \{A, B\},$$

with $c_1^{A,FB} = c_1^{B,FB}$, $G_1^{A,FB} = G_1^{B,FB}$, $c_2^{A,FB} < c_2^{B,FB}$, $G_2^{A,FB} < G_2^{B,FB}$, $b^{A,FB} > b^{B,FB}$, *and* $z^{A,FB} < 0 < z^{B,FB}$. *Moreover, it satisfies* IC_B *but violates* IC_A.

[15] In this study we invoke the assumption of federal policy commitment. Future research may relax this assumption and consider the context in which an ex-post bailout by the central government induces ex-ante adverse incentive consequences for local governments, as formulated by the literature on intergovernmental fiscal relations with soft budget constraints (e.g., Goodspeed 2002; Besfamille and Lockwood 2008; Akai and Sato 2008, 2011; Baskaran 2012).

The proof of Lemma 3.2 is straightforward and therefore omitted. The complete information optimum has three main features: (1) the intertemporal allocation is undistorted for both types so that IRS always equals IRT; (2) the impatient region contributes to interregional redistribution and is allocated a higher level of debt issuance than the patient region; (3) the first-best allocation is not incentive compatible if regions are privately informed about their types. This allocation gives the same Period-1 private consumption and public goods provision for both regions, whereas those given to the patient region in Period 2 are strictly larger than those given to the impatient region. Therefore, IC_A must be violated under this allocation. In other words, this allocation cannot induce the impatient region to truthfully report its type in the presence of private information.

We now proceed to the case in which each region is privately informed of its discount factor. As suggested by Lemma 3.2, either μ^A or μ^B in the Lagrangian (5) must be positive to enable the resulting allocation to be incentive compatible. Indeed, applying (5) again gives rise to the following characterization of the asymmetric information optimum.

Proposition 3.1 *Under Assumption 2.1, the asymmetric information optimum is characterized as follows:*

(i) If $\mu^A > \mu^B = 0$, then for $\delta^B/\delta^A > 1 + 2\mu^A$ we have

$$\frac{g'\left(G_1^{A*}\right)}{\delta^A g'\left(\theta G_1^{A*} + G_2^{A*}\right)} = 1 + r - \theta < \frac{g'\left(G_1^{B*}\right)}{\delta^B g'\left(\theta G_1^{B*} + G_2^{B*}\right)},$$

with $c_1^{A*} > c_1^{B*}$, $G_1^{A*} > G_1^{B*}$, $c_2^{A*} < c_2^{B*}$, $G_2^{A*} < G_2^{B*}$, $b^{A*} > b^{B*}$, and $z^{A*} < 0 < z^{B*}$.

(ii) If $\mu^B > \mu^A = 0$, then for $\delta^A/\delta^B > \mu^B$ we have

$$\frac{g'\left(G_1^{A*}\right)}{\delta^A g'\left(\theta G_1^{A*} + G_2^{A*}\right)} < 1 + r - \theta = \frac{g'\left(G_1^{B*}\right)}{\delta^B g'\left(\theta G_1^{B*} + G_2^{B*}\right)},$$

with $G_1^{A*} > G_1^{B*}$, $c_2^{A*} < c_2^{B*}$, $G_2^{A*} < G_2^{B*}$, $b^{A*} > b^{B*}$, and $z^{A*} > 0 > z^{B*}$. The relative magnitude of c_1^{A*} and c_1^{B*} is, nevertheless, ambiguous.

Proof. See Appendix A. ■

There are two key messages conveyed by Proposition 3.1. Firstly, if the impatient region's incentive compatibility constraint is binding and the regional difference in the degree of patience is large to the extent that $\delta^B/\delta^A > 1 + 2\mu^A$, then the asymmetric information optimum has two main features: (a) the impatient region is identified as the top type, and so the usual "no-distortion-at-the-top" property (Laffont and Martimort 2002) applies, whereas the intertemporal allocation of the patient region is distorted, such that IRS is greater than IRT; (b) optimal redistribution is from the impatient region to the patient region, whereby the impatient region is allocated with a higher level of debt issuance than the patient region. The asymmetric information optimum obtained by Huber and Runkel (2008), in a similar setting but with $\theta = 0$, has the same features as shown in Proposition 3.1(i). Secondly, if the patient region's incentive compatibility constraint is binding and the regional difference is small, such that $\delta^A/\delta^B > \mu^B$, then the patient region is identified as the top type, whereas the impatient region is distorted such that IRS is smaller than IRT.

We will now explain why it is always the top type that contributes to interregional redistribution, whilst the impatient region is allowed to issue more debt than the patient region. We will make use of the U-shape feature of region indifference curves in the (b, z)-space, as well as the single-crossing property established in Lemma 3.1.

Firstly, if impatient Region A is the top type, using Lemma 3.1(i) yields that (b^{A*}, z^{A*}) lies at the minimum of its indifference curve, and (b^{A*}, z^{A*}) and (b^{B*}, z^{B*}) must be indifferent for this region, and hence lie on the same indifference curve. Applying the single-crossing property, the indifference curve of the patient region must intersect once with that of the impatient region at (b^{B*}, z^{B*}). Additionally, given that IRS is greater than IRT, the patient region's intertemporal allocation is distorted such that $\mathrm{d}z/\mathrm{d}b|_{\mathrm{d}V=0} < 0$, thereby revealing that (b^{B*}, z^{B*}) must lie on the decreasing part of its indifference curve. Applying Lemma 3.1(ii) to these observations shows that $b^{A*} > b^{B*}$ and $z^{A*} < z^{B*}$.

Secondly, if patient Region B is the top type, then by the same token (b^{B*}, z^{B*}) lies at the minimum of its indifference curve, and so (b^{A*}, z^{A*}) and (b^{B*}, z^{B*}) must be indifferent for this region and hence lie on the same indifference curve. By the single-crossing property, the indifference curve of the impatient region must intersect with that of the patient region at (b^{A*}, z^{A*}). Additionally, given that IRS is smaller than IRT, the impatient region's intertemporal allocation is distorted such that $\mathrm{d}z/\mathrm{d}b|_{\mathrm{d}V=0} > 0$, namely, (b^{A*}, z^{A*}) must lie on the increasing part of its indifference curve. Using Lemma 3.1(ii) again, we thus have $b^{A*} > b^{B*}$ and $z^{A*} > z^{B*}$.

4 The Welfare Optimum with Multidimensional Heterogeneity

We will now proceed to the more realistic setting with multidimensional cross-region heterogeneity. We let the two regions differ in the durability of local public goods (e.g., Dai, Liu, and Tian 2019a) and consider the following two scenarios: either the patient region's public goods or the impatient region's public goods are more durable and of higher quality.

4.1 The Patient Region's Public Goods are More Durable

To consider the case in which the patient region's local public goods are more durable than those of the impatient region, we impose the following assumption:

Assumption 4.1 *The two regions differ in the durability of local public goods, and it is public knowledge that* $\theta^A < \theta^B$.

This assumption can be interpreted from two perspectives. Firstly, each region's public goods durability is public knowledge for the sake of avoiding a multidimensional screening problem that is generally complex, potentially leaving clear-cut predictions unattainable. Secondly, it assumes that the patient region provides durable public goods with higher quality than those provided by the impatient region. Section 4.2 considers the opposite case in which the impatient region provides durable public goods with higher quality.

The Lagrangian given by (5) is thus modified as follows:

$$\begin{aligned}\mathcal{L}(b^A, z^A, b^B, z^B; \mu^A, \mu^B, \lambda) = (1+\mu^A)V(b^A, z^A, \theta^A, \delta^A) - \mu^A V(b^B, z^B, \theta^A, \delta^A) \\ + (1+\mu^B)V(b^B, z^B, \theta^B, \delta^B) - \mu^B V(b^A, z^A, \theta^B, \delta^B) + \lambda(0 - z^A - z^B).\end{aligned} \tag{6}$$

For the case of complete information, setting $\mu^A = \mu^B = 0$ in (6) and solving the maximization program gives the following characterization of the full information optimum.

Lemma 4.1 *Under Assumptions 2.1 and 4.1, the complete information optimum verifies:*

$$\frac{g'\left(G_1^{R,FB}\right)}{\delta^R g'\left(\theta^R G_1^{R,FB} + G_2^{R,FB}\right)} = 1 + r - \theta^R, \quad R \in \{A, B\},$$

with $c_1^{A,FB} = c_1^{B,FB}$, $G_1^{A,FB} < G_1^{B,FB}$, $c_2^{A,FB} < c_2^{B,FB}$, *and* $\theta^A G_1^{A,FB} + G_2^{A,FB} < \theta^B G_1^{B,FB} + G_2^{B,FB}$. *Moreover, it satisfies* IC_B *but violates* IC_A *if one of the following conditions is met:*

(i) $g(\cdot) \equiv \ln(\cdot)$ *and* $\delta^B - \delta^A \geq (\theta^B - \theta^A)/(1 + r - \theta^B)$*;*

(ii) $g(\cdot) \equiv (\cdot)^\alpha$ *for parameter* $\alpha \in (0, 1)$ *and* $(\delta^B)^{1/(1-\alpha)} - (\delta^A)^{1/(1-\alpha)} \geq (\theta^B - \theta^A)(1 + r - \theta^B)^{1/(\alpha-1)}$*;*

(iii) $g(\cdot) \equiv -\beta^{-1}e^{-\beta(\cdot)}$ *for parameter* $\beta > 0$ *and* $\delta^B/\delta^A \geq \{(1 + r)/[\lambda(1 + r - \theta^B)]\}^{\theta^B - \theta^A}$.

Proof. See Appendix A. ∎

When comparing Lemma 4.1 to Lemma 3.2, the full information optimum under multidimensional heterogeneity differs from the full information optimum under unidimensional heterogeneity in two ways.

Firstly, under multidimensional heterogeneity the patient region provides more public good G_1 than the impatient region. However, they provide the same amount of public good G_1 when they only differ in the degree of patience. This departure lies in efficiency considerations. That is, if one public good produces more (positive) intergenerational spillovers than another, then, ceteris paribus, its provision should be larger because the social goal is to maximize the welfare of present and future generations. Secondly, in order to judge whether the full information optimum satisfies incentive compatibility conditions, both individual preferences and the relative magnitude of the difference in the degree of patience to the difference in the degree of intergenerational spillovers matter under multidimensional heterogeneity. Under these three types of utility function considered in Lemma 4.1, if the difference in the degree of patience is greater than the difference in the degree of intergenerational spillovers, then the full information optimum satisfies the patient region's incentive compatibility constraint but violates that of the impatient region. However, as shown in Lemma 3.2, this result always holds true if regions only differ in the degree of patience.

In particular, as the difference in the degree of intergenerational spillovers vanishes, namely as θ^A approaches θ^B, the conditions stated in Lemma 4.1(i)-(iii) are automatically satisfied under Assumption 2.1. Therefore, Lemma 4.1 subsumes Lemma 3.2 as the special case of $\theta^A = \theta^B$.

Given the interesting features of the optimal interregional redistribution and local debt policies under complete information and Assumptions 2.1 and 4.1, we give the characterization in the following proposition:

Proposition 4.1 *Under Assumptions 2.1 and 4.1, the local debt and interregional redistribution policies in the complete information optimum verify:*

(i) If $u(\cdot) = g(\cdot) = \ln(\cdot)$*, then we have* $z^{A,FB} < 0 < z^{B,FB}$*, and* $b^{A,FB} > b^{B,FB}$ *for* $\delta^B - \delta^A > \frac{(1+r)(\theta^B - \theta^A)}{2(1+r-\theta^A)(1+r-\theta^B)}$ *while* $b^{A,FB} \leq b^{B,FB}$ *for otherwise.*

(ii) If $u(\cdot) = g(\cdot) = (\cdot)^\alpha$ *for* $\alpha \in (0, 1)$*, then we have* $z^{A,FB} < 0 < z^{B,FB}$*, and* $b^{A,FB} > b^{B,FB}$ *for*

$$(\delta^B)^{1/(1-\alpha)} - (\delta^A)^{1/(1-\alpha)} > (1/2)\left[\theta^B(1 + r - \theta^B)^{1/(\alpha-1)} - \theta^A(1 + r - \theta^A)^{1/(\alpha-1)}\right]$$

while $b^{A,FB} \leq b^{B,FB}$ *for otherwise.*

(iii) If $u(\cdot) = g(\cdot) = -\beta^{-1}e^{-\beta(\cdot)}$ *for* $\beta > 0$*, then we have* $z^{A,FB} < 0 < z^{B,FB}$ *for* $\theta^B \leq 1 + r - e^{-1}$ *and* $\delta^B/\delta^A > \sqrt{[(1 + r)/\lambda]^{\theta^B - \theta^A}}$*, and* $z^{A,FB} > 0 > z^{B,FB}$ *for* $\theta^A \geq 1 + r - e^{-1}$ *and* $\delta^B/\delta^A < \sqrt{[(1 + r)/\lambda]^{\theta^B - \theta^A}}$*. In addition, we have* $b^{A,FB} > b^{B,FB}$ *for* $\delta^B/\delta^A > \sqrt{[(1 + r)/\lambda]^{\theta^B - \theta^A}\frac{(1+r-\theta^A)^{\theta^A}}{(1+r-\theta^B)^{\theta^B}}}$*; otherwise,* $b^{A,FB} \leq b^{B,FB}$.

Proof. See Appendix A. ■

When comparing Proposition 4.1 to Lemma 3.2 we observe an important difference regarding the optimal interregional redistribution and local debt policies under complete information. If regions only differ in the degree of patience, the impatient region contributes to redistribution and is allowed to issue more debt than the patient region, irrespective of individual preferences and the magnitude of cross-regional difference. In contrast, if regions also differ in the degree of intergenerational spillovers as specified in Assumption 4.1, then the optimal redistribution direction and the relative level of local debt issuance vary with individual preferences and the magnitude of cross-regional difference in both time preferences and intergenerational spillovers. As such, introducing regional difference in the degree of intergenerational spillovers does bring some novel features for the optimal fiscal policies.

If we elaborate further on a region's intertemporal allocation, we see that whilst the degree of intergenerational spillovers affects both IRS and IRT, the degree of patience affects only IRS. If everything else is equal, the patient region has a smaller IRT than the impatient region due to a higher degree of intergenerational spillovers under Assumption 4.1 and so has a smaller opportunity cost of borrowing. However, its IRS could be either larger or smaller than that of the impatient region because, on one hand, an increase of the degree of patience decreases IRS, while, on the other hand, an increase of the degree of intergenerational spillovers increases IRS, thereby making the net effect of these two opposite forces depend on preference specifications. In particular, we have identified the following conclusions.

Under logarithmic preferences, the impatient region always contributes to redistribution, and so the patient region always benefits from redistribution, whereas the relative level of local debt issuance depends on the relative magnitude of the difference in the degree of patience to the difference in the degree of intergenerational spillovers. In particular, if the relative magnitude is large such that $(\delta^B - \delta^A)/(\theta^B - \theta^A) > (1+r)/[2(1+r-\theta^A)(1+r-\theta^B)]$, then the impatient region can issue more debt than the patient region. A similar conclusion holds true for the case of power utility. However, under exponential utility, the optimal direction of redistribution also depends on the relative magnitude of the differences in the two dimensions. Specifically, if the relative magnitude is large to the extent that

$$\delta^B/\delta^A > \max\left\{\sqrt{[(1+r)/\lambda]^{\theta^B-\theta^A}}, \sqrt{[(1+r)/\lambda]^{\theta^B-\theta^A}\frac{(1+r-\theta^A)^{\theta^A}}{(1+r-\theta^B)^{\theta^B}}}\right\},$$

and the patient region's IRT (or opportunity cost of borrowing) is greater than $e^{-1} \approx 0.37$, then the impatient region contributes to redistribution and can issue more debt than the patient region; otherwise, if the relative magnitude is small to the extent that

$$\delta^B/\delta^A < \min\left\{\sqrt{[(1+r)/\lambda]^{\theta^B-\theta^A}}, \sqrt{[(1+r)/\lambda]^{\theta^B-\theta^A}\frac{(1+r-\theta^A)^{\theta^A}}{(1+r-\theta^B)^{\theta^B}}}\right\}$$

and the impatient region's IRT is smaller than 0.37, then the patient region contributes to redistribution and can issue more debt than the impatient region.

We now proceed to the case whereby each region is privately informed about its time preference. As suggested by Lemma 4.1, either μ^A or μ^B in Lagrangian (6) must be positive so that the resulting allocation is incentive compatible. Indeed, using (6) again gives the following characterization of the asymmetric information optimum in the presence of multidimensional cross-region heterogeneity.

Proposition 4.2 *Under Assumptions 2.1 and 4.1, the asymmetric information optimum with multidimensional heterogeneity satisfies:*

(i) If μ^A satisfies $1+[(\theta^B-\theta^A)/(1+r-\theta^B)] \leq \mu^A < \delta^B/\delta^A \leq 1+2\mu^A$ and $\mu^B = 0$, then we have

$$\frac{g'\left(G_1^{A*}\right)}{\delta^A g'\left(\theta^A G_1^{A*}+G_2^{A*}\right)} = 1+r-\theta^A \quad \text{and} \quad \frac{g'\left(G_1^{B*}\right)}{\delta^B g'\left(\theta^B G_1^{B*}+G_2^{B*}\right)} < 1+r-\theta^B,$$

with $G_1^{A*} < G_1^{B*}$, $c_2^{A*} > c_2^{B*}$, $G_2^{A*} > G_2^{B*}$, $\theta^A G_1^{A*}+G_2^{A*} > \theta^B G_1^{B*}+G_2^{B*}$, $b^{A*} < b^{B*}$, and $z^{A*} < 0 < z^{B*}$.

(ii) If $\mu^A = 0$ and $\mu^B \geq (1+r-\theta^B)/(1+r-\theta^A)$, then we have

$$\frac{g'\left(G_1^{A*}\right)}{\delta^A g'\left(\theta^A G_1^{A*}+G_2^{A*}\right)} > 1+r-\theta^A \quad \text{and} \quad \frac{g'\left(G_1^{B*}\right)}{\delta^B g'\left(\theta^B G_1^{B*}+G_2^{B*}\right)} = 1+r-\theta^B,$$

with $b^{A*} < b^{B*}$ and $z^{A*} > 0 > z^{B*}$.

Proof. See Appendix A. ■

If the impatient region's incentive compatibility constraint is binding and the shadow price and regional differences satisfy $\max\{1+[(\theta^B-\theta^A)/(1+r-\theta^B)], (1/2)[(\delta^B/\delta^A)-1]\} \leq \mu^A < \delta^B/\delta^A$, then the impatient region's intertemporal allocation is undistorted (i.e., IRS equals IRT), whereas IRS is smaller than IRT for the patient region. The optimal redistribution is from the impatient region to the patient region, and the impatient region is permitted to issue less debt than the patient region.

If the patient region's incentive compatibility constraint is binding with the shadow price greater than $(1+r-\theta^B)/(1+r-\theta^A)$, then IRS equals IRT for the patient region, but IRS is greater than IRT for the impatient region. The optimal redistribution is from the patient region to the impatient region, but the patient region is allowed to issue more debt than the impatient region.

Comparing Proposition 4.2 to Proposition 3.1, we can summarize the following two interesting insights. Firstly, regardless of whether regions only differ in the degree of patience under Assumption 2.1, or in both the degree of patience and the degree of intergenerational spillovers under Assumptions 2.1 and 4.1, it is always the top type (either H or L) whose intertemporal allocation is undistorted that contributes to redistribution. Under the center's goal of pure redistribution, this result implies that the opponent type (either L or H) benefits from redistribution by receiving a positive amount of federal transfers.

Secondly, the impatient region can always issue more debt than the patient region when the regions only differ in the degree of patience; in contrast, if they differ in both dimensions, then the patient region, which has a higher degree of intergenerational spillovers, can always issue more debt than the impatient region. Elaborating further, we can rewrite the IRT as $(1+r-\theta)\delta$, then for the case of the same θ, the region with a smaller δ faces a smaller IRT, and hence a lower opportunity cost of borrowing, which explains why the impatient region should be allowed to issue more debt. However, under multidimensional heterogeneity, the impatient region with a smaller δ also has a smaller θ that, ceteris paribus, implies a higher opportunity cost of borrowing. Consequently, Proposition 4.2 yields that the positive effect of decreasing IRT by a decrease of δ is dominated by the negative effect of increasing IRT by a decrease of θ, thereby yielding a higher opportunity cost of borrowing than that facing the patient region.

4.2 The Impatient Region's Public Goods are More Durable

To consider the case in which the impatient region's local public goods are more durable than those of the patient region, we impose the following assumption:

Assumption 4.2 *It is public knowledge that $\theta^A > \theta^B$.*

Under complete information, setting $\mu^A = \mu^B = 0$ in (6) and solving the maximization program gives the following characterization of the full information optimum.

Proposition 4.3 *Under Assumptions 2.1 and 4.2, the complete information optimum verifies:*

$$\frac{g'\left(G_1^{R,FB}\right)}{\delta^R g'\left(\theta^R G_1^{R,FB} + G_2^{R,FB}\right)} = 1 + r - \theta^R, \quad R \in \{A, B\},$$

with $c_1^{A,FB} = c_1^{B,FB}$, $G_1^{A,FB} > G_1^{B,FB}$, $c_2^{A,FB} < c_2^{B,FB}$, $G_2^{A,FB} < G_2^{B,FB}$, *and* $b^{A,FB} > b^{B,FB}$. *Moreover, the optimal interregional redistribution policy verifies:*

(i) If $u(\cdot) = g(\cdot) = \ln(\cdot)$, *then* $z^{A,FB} < 0 < z^{B,FB}$.

(ii) If $u(\cdot) = g(\cdot) = (\cdot)^\alpha$ *for parameter* $\alpha \in (0, 1)$, *then* $z^{A,FB} < 0 < z^{B,FB}$ *for*

$$\left(\frac{1}{1+r-\theta^A}\right)^{\alpha/(1-\alpha)} - \left(\frac{1}{1+r-\theta^B}\right)^{\alpha/(1-\alpha)} < 2\left[\left(\delta^B\right)^{1/(1-\alpha)} - \left(\delta^A\right)^{1/(1-\alpha)}\right],$$

and $z^{A,FB} > 0 > z^{B,FB}$ *for*

$$\left(\frac{1}{1+r-\theta^A}\right)^{\alpha/(1-\alpha)} - \left(\frac{1}{1+r-\theta^B}\right)^{\alpha/(1-\alpha)} > 2\left[\left(\delta^B\right)^{1/(1-\alpha)} - \left(\delta^A\right)^{1/(1-\alpha)}\right].$$

(iii) If $u(\cdot) = g(\cdot) = -\beta^{-1}e^{-\beta(\cdot)}$ *for parameter* $\beta > 0$, *then* $z^{A,FB} < 0 < z^{B,FB}$ *for* $\theta^B > 1 + r - e^{-1}$.

Proof. See Appendix A. ■

When the impatient region's public goods are more durable than those of the patient region, the full information optimum has three main features. Firstly, the impatient region should produce more Period-1 public goods than the patient region because its local public goods produce more intergenerational spillover than the patient region. Secondly, the impatient region is allocated a higher level of debt issuance than the patient region. Thirdly, whether the impatient or patient region should contribute to redistribution depends on the specification of individual preferences.

We now let each region be privately informed about its time preference, and then using (6) gives the following characterization of the asymmetric information optimum.

Proposition 4.4 *Under Assumptions 2.1 and 4.2, the asymmetric information optimum with multidimensional heterogeneity satisfies:*

(i) If μ^A *satisfies* $(1 + r - \theta^A)/(1 + r - \theta^B) \leq \mu^A < 1$ *and* $\mu^B = 0$, *then we have*

$$\frac{g'\left(G_1^{A*}\right)}{\delta^A g'\left(\theta^A G_1^{A*} + G_2^{A*}\right)} = 1 + r - \theta^A \quad \text{and} \quad \frac{g'\left(G_1^{B*}\right)}{\delta^B g'\left(\theta^B G_1^{B*} + G_2^{B*}\right)} > 1 + r - \theta^B,$$

with $b^{A*} > b^{B*}$ *and* $z^{A*} < 0 < z^{B*}$.

(ii) If $\mu^A = 0$ *and* $\mu^B \geq (1 + r - \theta^B)/(1 + r - \theta^A)$, *then we have*

$$\frac{g'\left(G_1^{A*}\right)}{\delta^A g'\left(\theta^A G_1^{A*} + G_2^{A*}\right)} < 1 + r - \theta^A \quad \text{and} \quad \frac{g'\left(G_1^{B*}\right)}{\delta^B g'\left(\theta^B G_1^{B*} + G_2^{B*}\right)} = 1 + r - \theta^B,$$

with $b^{A*} > b^{B*}$ *and* $z^{A*} > 0 > z^{B*}$.

Proof. See Appendix A. ■

When the public goods of the impatient region are more durable than those of the patient region, the asymmetric information optimum has three main features. Firstly, the top/efficient type, which extracts information rent from the center, faces an undistorted intertemporal allocation, namely, IRS equals IRT. Contrastingly, the bottom/inefficient type faces a distorted intertemporal allocation. If the impatient region is the top type, then IRS is greater than IRT in the patient region such that it borrows less than in the first-best (full information optimum); if the patient region is the top type, then IRS is smaller than IRT in the impatient region such that it borrows more than in the first-best. Secondly, the top type always contributes to redistribution, and hence the bottom type always benefits from redistribution. Thirdly, the impatient region is always allocated with a higher level of debt issuance than the patient region.

Table 1: Alternative welfare optima under asymmetric information

	Region A is top type	Region B is top type
$\delta^A < \delta^B$ $\theta^A = \theta^B$	$b^{A*} > b^{B*}$ and $z^{A*} < 0 < z^{B*}$	$b^{A*} > b^{B*}$ and $z^{A*} > 0 > z^{B*}$
$\delta^A < \delta^B$ $\theta^A < \theta^B$	$b^{A*} < b^{B*}$ and $z^{A*} < 0 < z^{B*}$	$b^{A*} < b^{B*}$ and $z^{A*} > 0 > z^{B*}$
$\delta^A < \delta^B$ $\theta^A > \theta^B$	$b^{A*} > b^{B*}$ and $z^{A*} < 0 < z^{B*}$	$b^{A*} > b^{B*}$ and $z^{A*} > 0 > z^{B*}$

5 Implementation of Welfare Optimum

In Sections 3 and 4, we let the center, which can be regarded as the social planner, determine both the interregional redistribution scheme and the level of debt issuance in the regions. Given that local governments usually have considerable autonomy in the choice of regional policies (Wildasin 2004), we now proceed to establish the conditions under which the welfare optima characterized in Propositions 3.1, 4.2 and 4.4 can be implemented through decentralized regional debt decisions. Table 1 presents the key features concerning local debt issuance and interregional redistribution of these asymmetric information optima.

Taking the federal transfer policy z^R as given, Region $R \in \{A, B\}$ solves the following maximization problem:

$$\max_{b^R} \; V(b^R, z^R, \theta^R, \delta^R).$$

Using (3), the first-order condition reads as follows:

$$g'(G_1^R) \;=\; (1 + r - \theta^R)\delta^R g'(\theta^R G_1^R + G_2^R), \tag{7}$$

which implies that IRS equals IRT, namely, the optimal decentralized borrowing decision is characterized by an undistorted intertemporal allocation.

Under asymmetric information, the center must design a redistribution scheme that guarantees incentive compatibility for both regions. Propositions 3.1, 4.2 and 4.4 clearly demonstrate that the intertemporal allocation of either H-region or L-region is distorted in the asymmetric information optimum, which can no longer be implemented through decentralized debt decisions characterized by condition (7) with the center simply setting $z^R = z^{R*}$ for $R \in \{A, B\}$. It is

Table 2: Alternative local budget rules for truthful implementation

	Region A is top type	Region B is top type
$\delta^A < \delta^B$ $\theta^A = \theta^B$	debt limit on Region B: $b^B \leq b^{B*}$	debt floor on Region A: $b^A \geq b^{A*}$
$\delta^A < \delta^B$ $\theta^A < \theta^B$	debt floor on Region B: $b^B \geq b^{B*}$	debt limit on Region A: $b^A \leq b^{A*}$
$\delta^A < \delta^B$ $\theta^A > \theta^B$	debt limit on Region B: $b^B \leq b^{B*}$	debt floor on Region A: $b^A \geq b^{A*}$

evident that certain institutional restrictions must be imposed on regional borrowing decisions (see Table 2), and these restrictions must be parts of the center's incentive compatible contracts.

To implement the asymmetric information optimum derived under unidimensional heterogeneity, the following result is obtained.

Proposition 5.1 *The asymmetric information optimum shown in Proposition 3.1 is implementable through decentralized debt decisions by setting $z^R = z^{R*}$ for $R \in \{A, B\}$ and simultaneously imposing the following local budget rules:*

(i) For the case of $\mu^A > \mu^B = 0$, an upper bound denoted $\bar{b} \equiv b^{B*}$ must be imposed on the debt issuance of Region B.

(ii) For the case of $\mu^B > \mu^A = 0$, a lower bound denoted $\underline{b} \equiv b^{A*}$ must be imposed on the debt issuance of Region A.

Proof. See Appendix A. ■

Note that these budget rules must be contained in the federal redistribution scheme to induce truth-telling under decentralized borrowing and spending decisions. As is obvious, Proposition 5.1(i) corresponds to Proposition 3.1(i), whilst Proposition 5.1(ii) corresponds to Proposition 3.1(ii). Since Proposition 3.1 shows that a positive Lagrange multiplier, $\mu^R > 0$, implies that Region R is the undistorted top type, which is indifferent between the two policies, (b^{A*}, z^{A*}) and (b^{B*}, z^{B*}), the debt floor, $\underline{b} \equiv b^{A*}$, under $\mu^B > 0$ applies only to L-region (impatient region), and the debt ceiling, $\bar{b} \equiv b^{B*}$, under $\mu^A > 0$ applies only to H-region (patient region). The fiscal constraint of a debt ceiling distorts the spending decision of H-region in favor of future public consumption, which makes the implicit borrowing constraint contained in the asymmetric information optimum explicit. Also, such level of debt ceiling renders the allocation of H-region unattractive for L-region, such that it voluntarily pays the lump-sum tax to the center instead of mimicking H-region. The fiscal constraint of a debt floor that distorts the spending decision of L-region in favor of present public consumption can be interpreted in a similar way. In other words, imposing these regional budget constraints helps to resolve the self-selection problem facing the center within the presence of privately observable time preferences.

Moreover, combining Proposition 5.1 with Proposition 3.1, we have the following observations. Firstly, if the impatient region is the top type that extracts information rent from the center, it will contribute to redistribution, and hence the patient region is the recipient of redistribution; truthful implementation requires the recipient to face a stricter borrowing constraint than the contributor region. Secondly, if the patient region is the top type, and hence the impatient region is the redistribution recipient, the recipient then faces a debt floor rather than a debt limit. We therefore conclude that only when the patient region is the redistribution recipient should it face a stricter borrowing constraint than the redistribution contributor.

Analogously, we obtain the following proposition to implement the asymmetric information optima derived under multidimensional heterogeneity.

Proposition 5.2 *The asymmetric information optima shown in Propositions 4.2 and 4.4 are implementable through decentralized debt decisions by setting $z^R = z^{R*}$ for $R \in \{A, B\}$ and simultaneously imposing the following local budget rules.*

(i) If the patient region's public goods are more durable than those of the impatient region, we have:

- *(i-a) For the case of $\mu^A > \mu^B = 0$, if the following condition is satisfied:*

$$\frac{\delta^B}{\delta^A} \leq \frac{(1+r-\theta^A)g'(\theta^A G_1^{B*} + G_2^{B*})}{(1+r-\theta^B)g'(\theta^B G_1^{B*} + G_2^{B*})},$$

  then a lower bound denoted $\underline{b} \equiv b^{B*}$ must be imposed on the debt issuance of Region B.

- *(i-b) For the case of $\mu^B > \mu^A = 0$, if the following condition is satisfied:*

$$\frac{\delta^B}{\delta^A} \leq \frac{(1+r-\theta^A)g'(\theta^A G_1^{A*} + G_2^{A*})}{(1+r-\theta^B)g'(\theta^B G_1^{A*} + G_2^{A*})},$$

  then an upper bound denoted $\bar{b} \equiv b^{A*}$ must be imposed on the debt issuance of Region A.

(ii) If the impatient region's public goods are more durable than those of the patient region, we have:

- *(ii-a) For the case of $\mu^A > \mu^B = 0$, then an upper bound denoted $\bar{b} \equiv b^{B*}$ must be imposed on the debt issuance of Region B.*
- *(ii-b) For the case of $\mu^B > \mu^A = 0$, then a lower bound denoted $\underline{b} \equiv b^{A*}$ must be imposed on the debt issuance of Region A.*

Proof. See Appendix A. ■

As above, these budget rules must be contained in the federal redistribution scheme to guarantee incentive compatibility. Proposition 5.2(i) corresponds to Proposition 4.2, while Proposition 5.2(ii) corresponds to Proposition 4.4. Note that if the public goods of the patient region are more durable than those of the impatient region, truthful implementation under these local budget rules requires a small regional difference in the degree of patience. Regardless of whether the more durable goods come from the patient or impatient region, the underlying intuition of Proposition 5.2 is the same as that of Proposition 5.1, namely that imposing these budget rules aims to solve the self-selection problem facing the center. Moreover, combining Proposition 5.2 with Propositions 4.2 and 4.4, we have the following observations.

Firstly, whenever the impatient region is the top type that extracts information rent from the center, the patient region always receives federal transfers (and hence the impatient region always contributes to redistribution) regardless of whether or not its public goods are more durable. However, the redistribution recipient is only constrained by a debt limit when the impatient region's public goods are more durable. Furthermore, the recipient faces a debt floor when the impatient region's public goods are less durable. Thus, in this case, only when the impatient region's public goods are more durable than those of the patient region should the recipient region face a stricter borrowing constraint than the contributor region.

Secondly, whenever the patient region is the top type that extracts information rent from the center, the impatient region always receives federal transfers, but is only constrained by a debt limit when the patient region's public goods are more durable; rather, it even faces a debt floor when the patient region's public goods are less durable. Thus, in this case, only when the patient region's public goods are more durable should the recipient face a stricter borrowing constraint than the contributor.

Therefore, regardless of whether the patient or impatient region is the recipient of redistribution, it should only face a stricter borrowing limit than the contributor region when its public goods produce smaller intergenerational spillovers than those of the contributor; otherwise, if the recipient's public goods turn out to produce greater intergenerational spillovers, it should face a debt floor rather than a debt limit. The main insight of truthful implementation under multidimensional regional heterogeneity is thus as follows: whilst a region's private information regarding time preferences determines which region should be the recipient of redistribution, only the recipient with smaller intergenerational spillovers should face a stricter borrowing constraint. We can thus conclude that the publicly observable regional heterogeneity on intergenerational spillovers seems to play a more critical role than the privately observable time preferences in terms of determining whether the redistribution recipient should face a stricter borrowing constraint.

6 Concluding Remarks

In a two-period model of a federation composed of a center and two regions, we addressed the question of whether the region which contributes to interregional redistribution should face a weaker borrowing constraint than the region which benefits from redistribution. Firstly, we considered the benchmark case with regions only differing in the privately observable discount factors. Secondly, we considered the more realistic case with regions differing in both discount factors and local public goods durability, but where only discount factors remain private. In this case, we firstly examined the scenario where the public goods provided by the patient region are more durable, namely a larger fraction of Period-1 public goods is still usable in Period 2, than those provided by the impatient region. We then proceeded to the scenario in which the impatient region's public goods are more durable. Following the mechanism design approach, we derived and characterized the welfare optima under both complete and asymmetric information. Furthermore, we justify the implementation of differentiated budget rules for the two regions such that the asymmetric information welfare optima can be truthfully implemented when borrowing and spending decisions are decentralized at the regional level. The multidimensional regional heterogeneity in discount factors and public goods durability enables us to provide reasonable conditions under which the budget institution — the patient region contributes to interregional redistribution and faces a weak borrowing constraint, whilst the impatient region benefits from redistribution and faces a strict borrowing constraint — emerges as an optimal arrangement.

The interpretation of our theoretical results could lead to interesting policy implications. For instance, we argue that the budget institution — namely, patient regions with higher quality of local public goods should contribute to redistribution and face lax budget rules, whereas impatient regions with lower quality of local public goods benefit from redistribution and face strict borrowing constraints — could have been an optimal choice for the European Union (EU) in the aftermath of the 2008 financial crisis. With everything else equal the European debt crisis between 2010 and 2011 could have thus potentially been avoided. The reasoning is as follows: EU members with strong economic growth, such as Germany, France, the United Kingdom (UK), Denmark, and Sweden, could be interpreted as patient regions that provide public goods of higher quality. On the other hand, EU member countries such as Greece, Ireland, Italy, Portugal, and Spain could be interpreted as impatient regions that provide public goods of lower quality. As such, instead of allowing fiscally weak countries like Greece to heavily borrow, ex-ante, followed by a huge amount of ex-post bailouts from the European Financial Stability Mechanism, fiscally sound countries like Germany could provide ex-ante bailouts to those fiscally weak countries. This would have helped them survive the 2008 financial crisis on the condition that they should follow strict budget rules. Such action could have resulted in much lower ex-post bailouts. As a caveat, this policy implication is relevant only from an efficiency perspective. This paper does not explore the more challenging political feasibility and time-consistency issues underpinning the implementability of the fiscal policy arrangement. Future research may investigate the design of political mechanisms to possibly enable the implementation of informationally efficient and time-consistent fiscal policies.

This paper highlights the point of accounting for intergenerational public goods provision, such as basic science, environmental protection, infrastructure and public capital, when designing socially optimal fiscal rules. And it shows that constrained efficiency may call for a debt floor rather than a debt limit under certain circumstances. This study could provide a theoretical basis for the recent debate on Germany's fiscal policy reform. The debt brake, which limits annual federal government borrowing (adjusted for the economic cycle) to no more than 0.35% of GDP, was written into the German constitution in 2009. It was supposed to spare future generations from being saddled with today's borrowing and to bolster confidence in the government's ability to repay its debts, sustaining public finances. But an influential younger generation of German economists argues that the debt brake probably created more problems than it solved. One supportive evidence is that Germany's public investment, at 2.3% of GDP, lags behind the euro-zone average and barely covers depreciation (The Economist 2019). Indeed, its public investment was squeezed, and between 2012 and 2017 was not high enough to stop the public capital stock from shrinking (The Economist 2021). They believe that there should be provisions allowing for greater investments to help Germany transform into a greener and more digitised economy. Therefore, optimal fiscal rules should avoid frivolous spending that saddled future generations with debt repayments and simultaneously provide sufficient public investments to benefit future generations.

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Appendix A: Proofs

Proof of Lemma 3.1. We complete the proof in 3 steps. To save on notation, region index R will be suppressed in the proof.

Step 1: By replacing the private consumptions in Equation (4) with public goods provision through application of budget constraints, we have the following:

$$\begin{aligned} u'\left(y_1 + b + z - \hat{G}_1\right) &= g'\left(\hat{G}_1\right) + \theta\delta g'\left(\theta\hat{G}_1 + \hat{G}_2\right), \\ u'\left(y_2 - b(1+r) - \hat{G}_2\right) &= g'\left(\theta\hat{G}_1 + \hat{G}_2\right). \end{aligned} \tag{8}$$

To simplify notations in the following derivation, let us denote the utilities for period $t \in \{1, 2\}$ by u_t and g_t. We then differentiate both sides of the two equations in (8) with respect to b, and apply the Implicit Function Theorem and the Cramer's Rule and find that

$$\begin{aligned} \phi_b &= \frac{u_1''(u_2'' + g_2'') + (1+r)\delta\theta u_2'' g_2''}{\Sigma} > 0 \text{ and} \\ \psi_b &= -\frac{(1+r)u_2''(u_1'' + g_1'' + \delta\theta^2 g_2'') + \theta u_1'' g_2''}{\Sigma} < 0, \end{aligned} \tag{9}$$

where $\Sigma \equiv (u_1'' + g_1'')(u_2'' + g_2'') + \delta\theta^2 u_2'' g_2'' > 0$. By the same token, we have

$$\phi_\delta = -\frac{\theta g_2'(u_2'' + g_2'')}{\Sigma} > 0 \quad \text{and} \quad \psi_\delta = \frac{\theta^2 g_2' g_2''}{\Sigma} < 0, \tag{10}$$

which immediately yields

$$\theta\phi_\delta + \psi_\delta = -\frac{\theta^2 g_2' u_2''}{\Sigma} > 0. \tag{11}$$

Step 2: Applying the Envelope Theorem to the value function (3), we obtain the slope of an indifference curve in the (b, z)-space as

$$\frac{\mathrm{d}z}{\mathrm{d}b}\Big|_{\mathrm{d}V=0} = -\frac{g'\left(\hat{G}_1\right) - (1+r-\theta)\delta g'\left(\theta\hat{G}_1 + \hat{G}_2\right)}{g'\left(\hat{G}_1\right) + \theta\delta g'\left(\theta\hat{G}_1 + \hat{G}_2\right)}. \tag{12}$$

Differentiating both sides of Equation (12) with respect to b and rearranging the algebra, we have

$$\frac{\mathrm{d}^2 z}{\mathrm{d}b^2}\Big|_{\mathrm{d}V=0} = \frac{(1+r)\delta[g_1' g_2''(\theta\phi_b + \psi_b) - g_2' g_1''\phi_b]}{(g_1' + \theta\delta g_2')^2} > 0$$

in which we have used (9) and

$$\theta\phi_b + \psi_b = -\frac{(1+r)u_2'' g_1'' + (1+r-\theta)u_1'' u_2''}{\Sigma} < 0. \tag{13}$$

The proof of part (i) is thus completed by setting $\mathrm{d}z/\mathrm{d}b|_{\mathrm{d}V=0} = 0$ in Equation (12).

Step 3: Differentiating both sides of Equation (12) with respect to δ and simplifying the algebra, we get

$$\frac{\mathrm{d}}{\mathrm{d}\delta}\left(\frac{\mathrm{d}z}{\mathrm{d}b}\Big|_{\mathrm{d}V=0}\right) = \frac{Q}{(g_1' + \theta\delta g_2')^2},$$

in which

$$\begin{aligned} Q \equiv\ & g_1' \cdot [g_1'' \cdot \phi_\delta + \theta g_2' + \theta\delta g_2'' \cdot (\theta\phi_\delta + \psi_\delta)] \\ & + (1+r)\{g_1' \cdot [g_2' + \delta g_2'' \cdot (\theta\phi_\delta + \psi_\delta)] - \delta g_2' \cdot g_1'' \cdot \phi_\delta\}. \end{aligned}$$

Making use of (10) and (11), we have

$$g_1'' \cdot \phi_\delta + \theta g_2' + \theta\delta g_2'' \cdot (\theta\phi_\delta + \psi_\delta) = \frac{\theta u_1'' \cdot (u_2'' + g_2'')g_2'}{\Sigma} > 0$$

and

$$g_2' + \delta g_2'' \cdot (\theta\phi_\delta + \psi_\delta) = \frac{(u_1'' + g_1'')(u_2'' + g_2'')g_2'}{\Sigma} > 0,$$

so we must have $Q > 0$ for any $\delta \in (0, 1)$, as predicted in part (ii). ■

Proof of Proposition 3.1. We complete the proof in 6 steps.

Step 1: Using Lagrangian (5) and assuming the existence of an interior solution, we get the following first-order conditions (FOCs) for Region A:

$$\begin{aligned} & (1+\mu^A)[g'(G_1^A) - (1+r-\theta)\delta^A g'(\theta G_1^A + G_2^A)] \\ = & \ \mu^B[g'(G_1^A) - (1+r-\theta)\delta^B g'(\theta G_1^A + G_2^A)]; \ \text{and} \\ & (1+\mu^A)[g'(G_1^A) + \theta\delta^A g'(\theta G_1^A + G_2^A)] \\ = & \ \mu^B[g'(G_1^A) + \theta\delta^B g'(\theta G_1^A + G_2^A)] + \lambda. \end{aligned} \tag{14}$$

By symmetry we get the following FOCs for Region B:

$$\begin{aligned}&(1+\mu^B)[g'(G_1^B)-(1+r-\theta)\delta^B g'(\theta G_1^B+G_2^B)]\\=\ &\mu^A[g'(G_1^B)-(1+r-\theta)\delta^A g'(\theta G_1^B+G_2^B)];\ \text{and}\\&(1+\mu^B)[g'(G_1^B)+\theta\delta^B g'(\theta G_1^B+G_2^B)]\\=\ &\mu^A[g'(G_1^B)+\theta\delta^A g'(\theta G_1^B+G_2^B)]+\lambda.\end{aligned} \tag{15}$$

Let us suppose IC_B is not binding such that $\mu^B=0$. It then follows from Lemma 3.2 that IC_A must be binding so that $\mu^A>0$. Applying these assumptions to the FOCs (14) and (15) gives rise to

$$\begin{aligned}g'(G_1^A)-(1+r-\theta)\delta^A g'(\theta G_1^A+G_2^A)\ &=0,\\ g'(G_1^A)+\theta\delta^A g'(\theta G_1^A+G_2^A)\ &=\frac{\lambda}{1+\mu^A};\end{aligned} \tag{16}$$

and

$$\begin{aligned}&g'(G_1^B)-(1+r-\theta)\delta^B g'(\theta G_1^B+G_2^B)\\&\quad=\mu^A[g'(G_1^B)-(1+r-\theta)\delta^A g'(\theta G_1^B+G_2^B)],\\&g'(G_1^B)+\theta\delta^B g'(\theta G_1^B+G_2^B)=\mu^A[g'(G_1^B)+\theta\delta^A g'(\theta G_1^B+G_2^B)]+\lambda.\end{aligned} \tag{17}$$

Step 2: Using (17) to solve for $g'(\theta G_1^B+G_2^B)$ gives rise to

$$g'(\theta G_1^B+G_2^B)=\frac{\lambda}{(1+r)(\delta^B-\mu^A\delta^A)}. \tag{18}$$

Thus, $\lambda>0$ yields $\delta^B>\mu^A\delta^A$. Rearranging the first equation of (17) shows that $\mu^A<1$ and

$$\frac{g'(G_1^B)}{\delta^B g'(\theta G_1^B+G_2^B)}=(1+r-\theta)\left(\frac{1}{1-\mu^A}\right)\left(1-\mu^A\cdot\frac{\delta^A}{\delta^B}\right)>1+r-\theta, \tag{19}$$

in which we have used Assumption 2.1. Using (16), Equation (18), and Assumption 2.1, we have

$$\begin{aligned}\delta^B g'(\theta G_1^B+G_2^B)&=\frac{\lambda}{(1+r)[1-\mu^A(\delta^A/\delta^B)]}\\&>\frac{\lambda}{(1+r)(1+\mu^A)}=\delta^A g'(\theta G_1^A+G_2^A).\end{aligned} \tag{20}$$

By comparing the first equation of (16) to (19), and using (20), it follows that $g'(G_1^B)>g'(G_1^A)$, which immediately produces $G_1^B<G_1^A$ given the strict concavity of $g(\cdot)$. Applying this result and (20) to (4) reveals that $u'(c_1^B)>u'(c_1^A)$, which then gives $c_1^B<c_1^A$. Moreover, $\mu^A>0$ implies that (b^A,z^A) and (b^B,z^B) lie on the same indifference U-shape curve of Region A, a property demonstrated by Lemma 3.1(i). By using the first equation of (16) we get that (b^A,z^A) lies at the minimum of this curve, thereby yielding that $z^A<0<z^B$. In addition, using (19) and Lemma 3.1 shows that $b^B<b^A$.

Step 3: We now proceed to establish the critical condition of $\delta^B/\delta^A>1+2\mu^A$. We shall do this by means of contradiction. Using (20), we have that $\delta^A+\mu^A\delta^A\geq\delta^B-\mu^A\delta^A$ is equivalent to $\delta^B/\delta^A\leq 1+2\mu^A$, under which we immediately arrive at $g'(\theta G_1^A+G_2^A)\leq g'(\theta G_1^B+G_2^B)$. Therefore, if $\mu^A<\delta^B/\delta^A\leq 1+2\mu^A$ holds, we must have $\theta G_1^A+G_2^A\geq\theta G_1^B+G_2^B$ and $c_2^A\geq c_2^B$ by using (4). Consequently, this result combined with the result established in Step 2 reveals that

$$\begin{aligned}&V(b^B,z^B,\delta^B)-V(b^A,z^A,\delta^B)\\=&\underbrace{u(c_1^B)-u(c_1^A)}_{-}+\underbrace{g(G_1^B)-g(G_1^A)}_{-}\\&+\delta^B\underbrace{[u(c_2^B)-u(c_2^A)+g(\theta G_1^B+G_2^B)-g(\theta G_1^A+G_2^A)]}_{-/0},\end{aligned}$$

which means IC_B is violated under this allocation. For Region A,

$$\begin{aligned}&V(b^A,z^A,\delta^A)-V(b^B,z^B,\delta^A)\\=&\underbrace{u(c_1^A)-u(c_1^B)}_{+}+\underbrace{g(G_1^A)-g(G_1^B)}_{+}\\&+\delta^A\underbrace{[u(c_2^A)-u(c_2^B)+g(\theta G_1^A+G_2^A)-g(\theta G_1^B+G_2^B)]}_{+/0},\end{aligned}$$

which means IC_A is not binding under this allocation so that $\mu^A=0$, which is an immediate contradiction. Therefore, we should instead have that $\delta^B/\delta^A>1+2\mu^A$, which combined with (20) and (4) shows that $\theta G_1^A+G_2^A<\theta G_1^B+G_2^B$ and $c_2^A<c_2^B$. The result of $G_1^B<G_1^A$ obtained in Step 2 implies that $G_2^A<G_2^B$, as predicted.

Step 4: We now proceed to the proof of part (ii). Applying $\mu^A=0$ and $\mu^B>0$ to (14) and (15) gives the following simplified FOCs:

$$\begin{aligned}&g'(G_1^A)-(1+r-\theta)\delta^A g'(\theta G_1^A+G_2^A)\\&\quad=\mu^B[g'(G_1^A)-(1+r-\theta)\delta^B g'(\theta G_1^A+G_2^A)],\\&g'(G_1^A)+\theta\delta^A g'(\theta G_1^A+G_2^A)=\mu^B[g'(G_1^A)+\theta\delta^B g'(\theta G_1^A+G_2^A)]+\lambda;\end{aligned}\tag{21}$$

and

$$\begin{aligned}g'(G_1^B)-(1+r-\theta)\delta^B g'(\theta G_1^B+G_2^B)&=0,\\g'(G_1^B)+\theta\delta^B g'(\theta G_1^B+G_2^B)&=\frac{\lambda}{1+\mu^B}.\end{aligned}\tag{22}$$

Solving for $g'(\theta G_1^A+G_2^A)$ using (21) shows that

$$g'(\theta G_1^A+G_2^A)=\frac{\lambda}{(1+r)(\delta^A-\mu^B\delta^B)}.\tag{23}$$

Thus, $\lambda>0$ implies that $\mu^B<\delta^A/\delta^B<1$ under Assumption 2.1. Rearranging the first equation of (21) reveals that

$$\frac{g'(G_1^A)}{\delta^A g'(\theta G_1^A+G_2^A)}=(1+r-\theta)\left(\frac{1}{1-\mu^B}\right)\left(1-\mu^B\cdot\frac{\delta^B}{\delta^A}\right)<1+r-\theta\tag{24}$$

under the condition $\mu^B<\delta^A/\delta^B<1$. If we solve $\delta^B g'(\theta G_1^B+G_2^B)$ from (22) and compare it to (23), we arrive at the following:

$$\begin{aligned}\delta^A g'(\theta G_1^A+G_2^A)&=\frac{\lambda}{(1+r)[1-\mu^B(\delta^B/\delta^A)]}\\&>\frac{\lambda}{(1+r)(1+\mu^B)}=\delta^B g'(\theta G_1^B+G_2^B),\end{aligned}\tag{25}$$

which then gives $\theta G_1^A + G_2^A < \theta G_1^B + G_2^B$ and $c_2^A < c_2^B$ by using Assumption 2.1, Condition (4), as well as the strict concavity of g and u.

Step 5: Moreover, applying $\mu^B > 0$, the first equation of (22), and inequality (24) to Lemma 3.1, we use the same reasoning in Step 2 to get $z^B < 0 < z^A$ and $b^B < b^A$. Then, applying these results to the budget constraints gives

$$G_2^A - G_2^B \;=\; (1+r)\underbrace{(b^B - b^A)}_{-} + \underbrace{c_2^B - c_2^A}_{+}$$

and

$$G_1^A - G_1^B \;=\; \underbrace{b^A - b^B}_{+} + \underbrace{z^A - z^B}_{+} + c_1^B - c_1^A. \tag{26}$$

We now show $G_2^A < G_2^B$ by means of contradiction. If, instead, we let $G_2^A \geq G_2^B$, then we must have $G_1^A < G_1^B$. Applying this result to (26) immediately reveals $c_1^B < c_1^A$. By using (4) and Equation (25) we thus arrive at

$$\begin{aligned} &0 < u'(c_1^B) - u'(c_1^A) \\ &\quad = g'(G_1^B) - g'(G_1^A) + \theta\underbrace{[\delta^B g'(\theta G_1^B + G_2^B) - \delta^A g'(\theta G_1^A + G_2^A)]}_{-}, \end{aligned}$$

by which we must have $G_1^B < G_1^A$, an immediate contradiction. Analogously, we shall prove $G_1^A > G_1^B$ by means of contradiction. If, instead, we suppose that $G_1^A \leq G_1^B$, then using (4) and Equation (25) again shows that

$$\begin{aligned} &u'(c_1^B) - u'(c_1^A) \\ &= \underbrace{g'(G_1^B) - g'(G_1^A)}_{-/0} + \theta\underbrace{[\delta^B g'(\theta G_1^B + G_2^B) - \delta^A g'(\theta G_1^A + G_2^A)]}_{-}, \end{aligned}$$

which yields $c_1^B > c_1^A$. Then by (25) we must have $V(b^B, z^B, \delta^B) > V(b^A, z^A, \delta^B)$, i.e., IC_B is not binding under this allocation so that $\mu^B = 0$, which is an immediate contradiction.

Step 6: Finally, we show that $\mu^A, \mu^B > 0$ cannot occur, namely that IC_A and IC_B cannot be simultaneously binding. We shall prove this by means of contradiction. Let us suppose instead that both IC_A and IC_B are binding. Under the complementary slackness conditions, we then get the following:

$$\begin{aligned} 0 &= V(b^B, z^B, \delta^B) - V(b^A, z^A, \delta^B) \\ &= u(c_1^B) - u(c_1^A) + g(G_1^B) - g(G_1^A) \\ &\quad + \delta^B[u(c_2^B) - u(c_2^A) + g(\theta G_1^B + G_2^B) - g(\theta G_1^A + G_2^A)], \end{aligned} \tag{27}$$

and

$$\begin{aligned} 0 &= V(b^A, z^A, \delta^A) - V(b^B, z^B, \delta^A) \\ &= u(c_1^A) - u(c_1^B) + g(G_1^A) - g(G_1^B) \\ &\quad + \delta^A[u(c_2^A) - u(c_2^B) + g(\theta G_1^A + G_2^A) - g(\theta G_1^B + G_2^B)]. \end{aligned} \tag{28}$$

Applying (27) to (28) and simplifying the algebra, we have under Assumption 2.1 that

$$u(c_2^A) - u(c_2^B) = g(\theta G_1^B + G_2^B) - g(\theta G_1^A + G_2^A),$$

which combined with the condition $u'(c_2) = g'(\theta G_1 + G_2)$ given in (4) as well as the strict concavity of u and g implies that

$$c_2^A = c_2^B \;\;\text{and}\;\; \theta G_1^B + G_2^B = \theta G_1^A + G_2^A. \tag{29}$$

It follows from rearranging the FOCs (14) and (15) that

$$\begin{aligned}(1+\mu^A-\mu^B)g'(G_1^A) &= [(1+\mu^A)\delta^A-\mu^B\delta^B](1+r-\theta)g'(\theta G_1^A+G_2^A),\\ \lambda &= [(1+\mu^A)\delta^A-\mu^B\delta^B](1+r)g'(\theta G_1^A+G_2^A);\end{aligned} \tag{30}$$

and

$$\begin{aligned}(1+\mu^B-\mu^A)g'(G_1^B) &= [(1+\mu^B)\delta^B-\mu^A\delta^A](1+r-\theta)g'(\theta G_1^B+G_2^B),\\ \lambda &= [(1+\mu^B)\delta^B-\mu^A\delta^A](1+r)g'(\theta G_1^B+G_2^B).\end{aligned} \tag{31}$$

Thus, applying $\lambda > 0$, (29) and Assumption 2.1 to (30) and (31) yields the following critical condition:

$$\frac{\mu^A}{1+\mu^B} < \frac{\delta^B}{\delta^A} = \frac{1+2\mu^A}{1+2\mu^B} < \frac{1+\mu^A}{\mu^B} \quad \text{with} \quad \mu^B < \mu^A < 1+\mu^B.$$

Under this critical condition and Assumption 2.1, we use (30) and (31) to obtain

$$\begin{aligned}\frac{g'(G_1^A)}{\delta^A g'(\theta G_1^A+G_2^A)} &= (1+r-\theta)\left[\frac{1+\mu^A-\mu^B(\delta^B/\delta^A)}{1+\mu^A-\mu^B}\right] < 1+r-\theta,\\ \frac{g'(G_1^B)}{\delta^B g'(\theta G_1^B+G_2^B)} &= (1+r-\theta)\left[\frac{1+\mu^B-\mu^A(\delta^A/\delta^B)}{1+\mu^B-\mu^A}\right] > 1+r-\theta,\end{aligned} \tag{32}$$

revealing that the intertemporal allocations of both regions are distorted relative to the complete information welfare optimum. Making use of Equation (32) yields:

$$\frac{g'(G_1^A)}{g'(G_1^B)} = \underbrace{\frac{1+\mu^B-\mu^A}{1+\mu^A-\mu^B}}_{<1} \cdot \underbrace{\frac{(1+\mu^A)\delta^A-\mu^B\delta^B}{(1+\mu^B)\delta^B-\mu^A\delta^A}}_{=1},$$

in which we have used Assumption 2.1 and the critical condition given above. We then immediately get $G_1^A > G_1^B$, and we apply this result to (29) to get $G_2^A < G_2^B$. Consequently, $b^B < b^A$ follows from the fact that

$$\underbrace{G_2^A - G_2^B}_{-} = (1+r)(b^B-b^A) + \underbrace{c_2^B - c_2^A}_{0}.$$

Moreover, applying $\mu^A, \mu^B > 0$ shows that (b^A, z^A) and (b^B, z^B) lie on the same indifference curves of both regions. Nevertheless, since (b^A, z^A) and (b^B, z^B) are two different points, the single-crossing property established in Lemma 3.1(ii) is violated. ■

Proof of Lemma 4.1. We complete the proof in 3 steps.

Step 1: Using (6), the FOCs read as follows:

$$\begin{aligned}g'(G_1^R) - (1+r-\theta^R)\delta^R g'(\theta^R G_1^R + G_2^R) &= 0,\\ g'(G_1^R) + \theta^R\delta^R g'(\theta^R G_1^R + G_2^R) &= \lambda,\end{aligned}$$

by which we obtain

$$\begin{aligned}g'(G_1^R) &= \frac{\lambda(1+r-\theta^R)}{1+r},\\ g'(\theta^R G_1^R + G_2^R) &= \frac{\lambda}{(1+r)\delta^R}.\end{aligned} \tag{33}$$

Applying Assumptions 2.1 and 4.1 to (33) shows that $\theta^A G_1^A + G_2^A < \theta^B G_1^B + G_2^B$ and $G_1^A < G_1^B$. By using (4) and (33) we then get $c_2^A < c_2^B$ and $c_1^A = c_1^B$, as anticipated.

Step 2: Note that

$$\begin{aligned}\theta^A G_1^A + G_2^A - (\theta^A G_1^B + G_2^B) &= \theta^A G_1^A + G_2^A - (\theta^B G_1^B + G_2^B) - (\theta^A - \theta^B) G_1^B, \\ \theta^B G_1^B + G_2^B - (\theta^B G_1^A + G_2^A) &= \theta^B G_1^B + G_2^B - (\theta^A G_1^A + G_2^A) - (\theta^B - \theta^A) G_1^A.\end{aligned} \tag{34}$$

Applying $g(\cdot) = \ln(\cdot)$ to (33) gives

$$\begin{aligned} G_1^R &= \frac{1+r}{\lambda(1+r-\theta^R)} \quad \text{and} \\ \theta^R G_1^R + G_2^R &= \frac{(1+r)\delta^R}{\lambda}, \end{aligned} \tag{35}$$

which by applying to (34) and simplifying the algebra gives rise to

$$\begin{aligned} \theta^A G_1^A + G_2^A - (\theta^A G_1^B + G_2^B) \leq 0 \;\Leftrightarrow\; \delta^B - \delta^A \geq \frac{\theta^B - \theta^A}{1+r-\theta^B}, \\ \theta^B G_1^B + G_2^B - (\theta^B G_1^A + G_2^A) \geq 0 \;\Leftrightarrow\; \delta^B - \delta^A \geq \frac{\theta^B - \theta^A}{1+r-\theta^A}. \end{aligned} \tag{36}$$

We note that from applying the results established in Step 1 to (3),

$$\begin{aligned} & V(b^A, z^A, \theta^A, \delta^A) - V(b^B, z^B, \theta^A, \delta^A) \\ &= \underbrace{g(G_1^A) - g(G_1^B)}_{-} + \delta^A \underbrace{[u(c_2^A) - u(c_2^B)]}_{-} \\ &\quad + \delta^A \cdot \underbrace{[g(\theta^A G_1^A + G_2^A) - g(\theta^A G_1^B + G_2^B)]}_{?} \end{aligned} \tag{37}$$

and

$$\begin{aligned} & V(b^B, z^B, \theta^B, \delta^B) - V(b^A, z^A, \theta^B, \delta^B) \\ &= \underbrace{g(G_1^B) - g(G_1^A)}_{+} + \delta^B \underbrace{[u(c_2^B) - u(c_2^A)]}_{+} \\ &\quad + \delta^B \cdot \underbrace{[g(\theta^B G_1^B + G_2^B) - g(\theta^B G_1^A + G_2^A)]}_{?}, \end{aligned} \tag{38}$$

applying (36) to (37) and (38) leads to the desired assertion (i) under Assumptions 2.1 and 4.1.

Step 3: Applying $g(\cdot) \equiv (\cdot)^\alpha$, for a constant parameter $\alpha \in (0, 1)$, to (33) gives

$$\begin{aligned} G_1^R &= \left[\frac{\alpha(1+r)}{\lambda(1+r-\theta^R)}\right]^{1/(1-\alpha)} \quad \text{and} \\ \theta^R G_1^R + G_2^R &= \left[\frac{\alpha(1+r)\delta^R}{\lambda}\right]^{1/(1-\alpha)}, \end{aligned} \tag{39}$$

which applying to (34) and simplifying the algebra reveals that

$$\begin{aligned} & \theta^A G_1^A + G_2^A - (\theta^A G_1^B + G_2^B) \leq 0 \\ \Leftrightarrow\; & (\delta^B)^{1/(1-\alpha)} - (\delta^A)^{1/(1-\alpha)} \geq (\theta^B - \theta^A)(1+r-\theta^B)^{1/(\alpha-1)}, \\ & \theta^B G_1^B + G_2^B - (\theta^B G_1^A + G_2^A) \geq 0 \\ \Leftrightarrow\; & (\delta^B)^{1/(1-\alpha)} - (\delta^A)^{1/(1-\alpha)} \geq (\theta^B - \theta^A)(1+r-\theta^A)^{1/(\alpha-1)}. \end{aligned} \tag{40}$$

Consequently, applying (40) to (37) and (38) leads to the desired assertion (ii) under Assumptions 2.1 and 4.1. Similarly, applying $g(\cdot) \equiv -\beta^{-1}e^{-\beta(\cdot)}$, for a constant parameter $\beta > 0$, to (33) shows that

$$\begin{aligned} G_1^R &= \frac{1}{\beta}\ln\left[\frac{1+r}{\lambda(1+r-\theta^R)}\right] \quad \text{and} \\ \theta^R G_1^R + G_2^R &= \frac{1}{\beta}\ln\left[\frac{(1+r)\delta^R}{\lambda}\right], \end{aligned} \tag{41}$$

which applying to (34) and simplifying the algebra reveals that

$$\begin{aligned} \theta^A G_1^A + G_2^A - (\theta^A G_1^B + G_2^B) \le 0 \;\Leftrightarrow\; \frac{\delta^B}{\delta^A} \ge \left[\frac{1+r}{\lambda(1+r-\theta^B)}\right]^{\theta^B-\theta^A}, \\ \theta^B G_1^B + G_2^B - (\theta^B G_1^A + G_2^A) \ge 0 \;\Leftrightarrow\; \frac{\delta^B}{\delta^A} \ge \left[\frac{1+r}{\lambda(1+r-\theta^A)}\right]^{\theta^B-\theta^A}. \end{aligned} \tag{42}$$

Therefore, applying (42) to (37) and (38) leads to the desired assertion (iii) under Assumptions 2.1 and 4.1. ■

Proof of Proposition 4.1. We complete the proof in 3 steps.

Step 1: Suppose u and g take the logarithmic utility functional form, then we get from (4) that $c_2^R = \theta^R G_1^R + G_2^R$. Then, using (35) and $(b^B - b^A)(1+r) = G_2^A - G_2^B + c_2^A - c_2^B$ gives rise to

$$b^B - b^A = \frac{1}{\lambda}\left[2(\delta^A - \delta^B) + \frac{\theta^B}{1+r-\theta^B} - \frac{\theta^A}{1+r-\theta^A}\right],$$

which when rearranged gives the cross-region debt comparison result in part (i). Applying this result and Lemma 4.1 to Period-1 budget constraints indicates that $z^A - z^B = G_1^A - G_1^B + b^B - b^A = (2/\lambda)(\delta^A - \delta^B) < 0$ under Assumption 2.1.

Step 2: Let us suppose u and g take the same power utility functional form, then it follows from applying (4) and (39) to $(b^B - b^A)(1+r) = G_2^A - G_2^B + c_2^A - c_2^B$ that

$$\begin{aligned} (b^B - b^A)(1+r)\left[\frac{\alpha(1+r)}{\lambda}\right]^{1/(\alpha-1)} &= 2\left[(\delta^A)^{1/(1-\alpha)} - (\delta^B)^{1/(1-\alpha)}\right] \\ + \theta^B\left(\frac{1}{1+r-\theta^B}\right)^{1/(1-\alpha)} - \theta^A\left(\frac{1}{1+r-\theta^A}\right)^{1/(1-\alpha)}, \end{aligned}$$

which when rearranged gives the cross-region debt comparison result in part (ii). Applying this result and Lemma 4.1 to Period-1 budget constraints shows that

$$\begin{aligned} (z^A - z^B)(1+r)\left[\frac{\alpha(1+r)}{\lambda}\right]^{1/(\alpha-1)} &= 2\underbrace{\left[(\delta^A)^{1/(1-\alpha)} - (\delta^B)^{1/(1-\alpha)}\right]}_{-} \\ + \underbrace{\left(\frac{1}{1+r-\theta^A}\right)^{\alpha/(1-\alpha)} - \left(\frac{1}{1+r-\theta^B}\right)^{\alpha/(1-\alpha)}}_{-} \end{aligned}$$

under Assumptions 2.1 and 4.1.

Step 3: If we suppose u and g take the same exponential utility functional form, it then follows from (41) that $1 + r > \lambda$. Then, applying (4) and (41) to $(b^B - b^A)(1+r) = G_2^A - G_2^B +$

$c_2^A - c_2^B$ gives

$$(b^B - b^A)(1+r)\beta = \underbrace{2\ln\left(\frac{\delta^A}{\delta^B}\right)}_{-} + \underbrace{\left(\theta^B - \theta^A\right)\ln\left(\frac{1+r}{\lambda}\right)}_{+} + \underbrace{\theta^B \ln\left(\frac{1}{1+r-\theta^B}\right) - \theta^A \ln\left(\frac{1}{1+r-\theta^A}\right)}_{+}$$

under Assumptions 2.1 and 4.1. Then, rearranging the algebra reveals that

$$b^B < b^A \;\Leftrightarrow\; \frac{\delta^B}{\delta^A} > \sqrt{\left(\frac{1+r}{\lambda}\right)^{\theta^B-\theta^A} \frac{(1+r-\theta^A)^{\theta^A}}{(1+r-\theta^B)^{\theta^B}}},$$

as predicted in part (iii). Applying the expression of $b^B - b^A$ given above, (41), and Lemma 4.1 to Period-1 budget constraints, we arrive at

$$\begin{aligned}&(z^A - z^B)(1+r)\beta\\ &= \underbrace{2\ln\left(\frac{\delta^A}{\delta^B}\right)}_{-} + \underbrace{\left(\theta^B - \theta^A\right)\ln\left(\frac{1+r}{\lambda}\right)}_{+}\\ &\quad + \underbrace{(1+r-\theta^B)\ln(1+r-\theta^B) - (1+r-\theta^A)\ln(1+r-\theta^A)}_{?}\end{aligned}$$

under Assumptions 2.1 and 4.1. If we note that

$$\frac{\partial(1+r-\theta)\ln(1+r-\theta)}{\partial\theta} \geq 0 \;\Leftrightarrow\; \theta \geq 1+r-e^{-1}$$

and

$$2\ln\left(\frac{\delta^A}{\delta^B}\right) + \left(\theta^B-\theta^A\right)\ln\left(\frac{1+r}{\lambda}\right) \geq 0 \;\Leftrightarrow\; \frac{\delta^B}{\delta^A} \leq \left(\frac{1+r}{\lambda}\right)^{(\theta^B-\theta^A)/2},$$

the predicted assertion in part (iii) is thus confirmed. ■

Proof of Proposition 4.2. We complete the proof in 3 steps.

Step 1: Differentiating Lagrangian (6) with respect to b^A and z^A and assuming the existence of an interior solution, we give the FOCs for Region A as follows:

$$\begin{aligned}&(1+\mu^A)[g'(G_1^A) - (1+r-\theta^A)\delta^A g'(\theta^A G_1^A + G_2^A)]\\ &= \mu^B[g'(G_1^A) - (1+r-\theta^B)\delta^B g'(\theta^B G_1^A + G_2^A)]; \text{ and}\\ &(1+\mu^A)[g'(G_1^A) + \theta^A\delta^A g'(\theta^A G_1^A + G_2^A)]\\ &= \mu^B[g'(G_1^A) + \theta^B\delta^B g'(\theta^B G_1^A + G_2^A)] + \lambda.\end{aligned} \tag{43}$$

By symmetry we get the following FOCs for Region B:

$$\begin{aligned}&(1+\mu^B)[g'(G_1^B) - (1+r-\theta^B)\delta^B g'(\theta^B G_1^B + G_2^B)]\\ &= \mu^A[g'(G_1^B) - (1+r-\theta^A)\delta^A g'(\theta^A G_1^B + G_2^B)]; \text{ and}\\ &(1+\mu^B)[g'(G_1^B) + \theta^B\delta^B g'(\theta^B G_1^B + G_2^B)]\\ &= \mu^A[g'(G_1^B) + \theta^A\delta^A g'(\theta^A G_1^B + G_2^B)] + \lambda.\end{aligned} \tag{44}$$

Let us suppose IC_B is not binding so that $\mu^B = 0$. Then, it follows from Lemma 4.1 that IC_A tends to be binding so that $\mu^A > 0$. Applying these assumptions to the FOCs (43) and (44) gives rise to

$$\begin{aligned} g'(G_1^A) - (1+r-\theta^A)\delta^A g'(\theta^A G_1^A + G_2^A) &= 0, \\ g'(G_1^A) + \theta^A \delta^A g'(\theta^A G_1^A + G_2^A) &= \frac{\lambda}{1+\mu^A}; \end{aligned} \tag{45}$$

and

$$\begin{aligned} & g'(G_1^B) - (1+r-\theta^B)\delta^B g'(\theta^B G_1^B + G_2^B) \\ & \quad = \mu^A[g'(G_1^B) - (1+r-\theta^A)\delta^A g'(\theta^A G_1^B + G_2^B)], \\ & g'(G_1^B) + \theta^B\delta^B g'(\theta^B G_1^B + G_2^B) = \mu^A[g'(G_1^B) + \theta^A \delta^A g'(\theta^A G_1^B + G_2^B)] + \lambda. \end{aligned} \tag{46}$$

Step 2: In (46), by plugging the expression of $g'(G_1^B)+\theta^B\delta^B g'(\theta^B G_1^B+G_2^B)$ from the second equation in the first equation and simplifying the algebra, we have

$$\frac{\lambda}{1+r} = \delta^B g'(\theta^B G_1^B + G_2^B) - \mu^A \delta^A g'(\theta^A G_1^B + G_2^B) > 0, \tag{47}$$

in which we have used the fact that $\lambda > 0$. Then, applying (47) to the first equation of (46) reveals that

$$(1-\mu^A) g'(G_1^B) > (1+r-\theta^B)\delta^B g'(\theta^B G_1^B + G_2^B)\left(\frac{\theta^A - \theta^B}{1+r-\theta^B}\right). \tag{48}$$

Considering the case of $\mu^A > 1$, we rearrange (48) to get

$$\frac{g'(G_1^B)}{\delta^B g'(\theta^B G_1^B + G_2^B)} < (1+r-\theta^B)\underbrace{\left(\frac{1}{\mu^A - 1}\right)\left(\frac{\theta^B - \theta^A}{1+r-\theta^B}\right)}_{+}$$

under Assumption 4.1. It is thus easy to verify the desired assertion:

$$\frac{g'(G_1^B)}{\delta^B g'(\theta^B G_1^B + G_2^B)} < 1+r-\theta^B \quad \text{for} \quad 1 + \frac{\theta^B - \theta^A}{1+r-\theta^B} \le \mu^A. \tag{49}$$

It follows from (45) that

$$\delta^A g'(\theta^A G_1^A + G_2^A) = \frac{\lambda}{(1+r)(1+\mu^A)}. \tag{50}$$

Applying $\lambda > 0$, Assumption 4.1 and the strict concavity of g to (47) gives rise to

$$\frac{\delta^B}{\mu^A \delta^A} > \frac{g'(\theta^A G_1^B + G_2^B)}{g'(\theta^B G_1^B + G_2^B)} > 1 \quad \text{and} \quad \frac{\lambda}{1+r} < g'(\theta^B G_1^B + G_2^B)(\delta^B - \mu^A \delta^A),$$

and from which we obtain

$$g'(\theta^B G_1^B + G_2^B) > \frac{\lambda}{(1+r)(\delta^B - \mu^A \delta^A)}. \tag{51}$$

As a consequence, comparing (51) to (50) reveals that

$$g'(\theta^B G_1^B + G_2^B) > g'(\theta^A G_1^A + G_2^A) \quad \text{for} \quad \frac{\delta^B}{\delta^A} \le 1 + 2\mu^A,$$

which combined with (4) shows that $c_2^B < c_2^A$, as predicted. Applying (45), (49), and $\mu^A > 0$ to Lemma 3.1 demonstrates that (b^A, z^A) lies at the minimum of, while (b^B, z^B) lies on the increasing part of, a U-shaped indifference curve of Region A. We, therefore, must have $z^A < 0 < z^B$ and $b^A < b^B$, as desired in Proposition 4.2(i). Applying these results to Period-1 budget constraints yields

$$G_1^A - G_1^B = \underbrace{b^A - b^B}_{-} + \underbrace{z^A - z^B}_{-} + c_1^B - c_1^A. \tag{52}$$

We now prove $G_1^A < G_1^B$ by means of contradiction. If, instead, we let $G_1^A \geq G_1^B$, then using (52) gives $c_1^B > c_1^A$. Moreover, using (50) and (51) reveals that

$$\begin{aligned}\theta^B\delta^B g'(\theta^B G_1^B + G_2^B) &> \frac{\lambda\theta^B}{(1+r)[1-\mu^A(\delta^A/\delta^B)]} \\ &> \frac{\lambda\theta^A}{(1+r)(1+\mu^A)} = \theta^A\delta^A g'(\theta^A G_1^A + G_2^A)\end{aligned}$$

under Assumptions 2.1 and 4.1. As such, we use (4) and get the following:

$$\begin{aligned}u'(c_1^B) - u'(c_1^A) = &\underbrace{g'(G_1^B) - g'(G_1^A)}_{+/0} \\ &+ \underbrace{\theta^B\delta^B g'(\theta^B G_1^B + G_2^B) - \theta^A\delta^A g'(\theta^A G_1^A + G_2^A)}_{+},\end{aligned}$$

which yields $c_1^B < c_1^A$, an immediate contradiction. By using Assumption 4.1, $G_1^A < G_1^B$, and $\theta^B G_1^B + G_2^B < \theta^A G_1^A + G_2^A$, we accordingly get $G_2^B < G_2^A$.

Step 3: We now proceed to the proof of Proposition 4.2(ii). Applying $\mu^A = 0$ and $\mu^B > 0$ to (43) and (44) gives the following FOCs:

$$\begin{aligned}g'(G_1^B) - (1+r-\theta^B)\delta^B g'(\theta^B G_1^B + G_2^B) &= 0, \\ g'(G_1^B) + \theta^B\delta^B g'(\theta^B G_1^B + G_2^B) &= \frac{\lambda}{1+\mu^B};\end{aligned} \tag{53}$$

and

$$\begin{aligned}&g'(G_1^A) - (1+r-\theta^A)\delta^A g'(\theta^A G_1^A + G_2^A) \\ &\quad = \mu^B[g'(G_1^A) - (1+r-\theta^B)\delta^B g'(\theta^B G_1^A + G_2^A)], \\ &g'(G_1^A) + \theta^A\delta^A g'(\theta^A G_1^A + G_2^A) = \mu^B[g'(G_1^A) + \theta^B\delta^B g'(\theta^B G_1^A + G_2^A)] + \lambda.\end{aligned} \tag{54}$$

We then get the following in light of (54) and $\lambda > 0$:

$$\frac{\lambda}{1+r} = \delta^A g'(\theta^A G_1^A + G_2^A) - \mu^B\delta^B g'(\theta^B G_1^A + G_2^A) > 0,$$

which by applying Assumption 4.1 to (54) shows that

$$(1-\mu^B)g'(G_1^A) > (1+r-\theta^A)\delta^A g'(\theta^A G_1^A + G_2^A)\left(\frac{\theta^B - \theta^A}{1+r-\theta^A}\right) > 0.$$

We must then have $\mu^B < 1$ and

$$\frac{g'(G_1^A)}{\delta^A g'(\theta^A G_1^A + G_2^A)} > (1+r-\theta^A)\left(\frac{1}{1-\mu^B}\right)\left(\frac{\theta^B - \theta^A}{1+r-\theta^A}\right),$$

which enables us to arrive at the expected conclusion:

$$\frac{g'(G_1^A)}{\delta^A g'(\theta^A G_1^A + G_2^A)} > 1 + r - \theta^A \quad \text{for} \quad \mu^B \geq \frac{1+r-\theta^B}{1+r-\theta^A}. \tag{55}$$

We now apply the first equation of (53), (55), and $\mu^B > 0$ to Lemma 3.1, which leads us to $z^B < 0 < z^A$ and $b^A < b^B$. ■

Proof of Proposition 4.3. Note that the FOCs are still given by (33). Using Assumption 4.2 and (33), we have $G_1^A > G_1^B$. Similarly, using Assumption 2.1 and (33), we have $\theta^A G_1^A + G_2^A < \theta^B G_1^B + G_2^B$. Then, when combined with Assumption 4.2 these two results reveal that $G_2^A < G_2^B$. We can also get that $c_2^A < c_2^B$ and $c_1^A = c_1^B$ by using (4). We therefore obtain $b^B < b^A$ by using $G_2^A - G_2^B = (1+r)(b^B - b^A) + c_2^B - c_2^A$.

We now proceed to the characterization of the optimal interregional redistribution policy. Firstly, we have by definition and the above results that $z^A - z^B = G_1^A - G_1^B + b^B - b^A$. We then consider the three utility functional forms. If $u = g = \ln$, then we get from Step 1 of the proof of Proposition 4.1 that $z^A - z^B = (2/\lambda)(\delta^A - \delta^B) < 0$ under Assumption 2.1, as predicted in part (i). For part (ii), we note from Step 2 of the proof of Proposition 4.1 that

$$(z^A - z^B)(1+r)\left[\frac{\alpha(1+r)}{\lambda}\right]^{1/(\alpha-1)} = 2\underbrace{\left[(\delta^A)^{1/(1-\alpha)} - (\delta^B)^{1/(1-\alpha)}\right]}_{-}$$
$$+\underbrace{\left(\frac{1}{1+r-\theta^A}\right)^{\alpha/(1-\alpha)} - \left(\frac{1}{1+r-\theta^B}\right)^{\alpha/(1-\alpha)}}_{+}$$

under Assumptions 2.1 and 4.2. For part (iii), we apply Assumptions 2.1 and 4.2, in view of Step 3 of the proof of Proposition 4.1, and then we have

$$\begin{aligned}&(z^A - z^B)(1+r)\beta \\ &= \underbrace{2\ln\left(\frac{\delta^A}{\delta^B}\right)}_{-} + \underbrace{\left(\theta^B - \theta^A\right)\ln\left(\frac{1+r}{\lambda}\right)}_{-} \\ &\quad + \underbrace{(1+r-\theta^B)\ln(1+r-\theta^B) - (1+r-\theta^A)\ln(1+r-\theta^A)}_{-/0}\end{aligned}$$

whenever $(1+r-\theta)\ln(1+r-\theta)$ is nondecreasing in θ. ■

Proof of Proposition 4.4. Firstly, applying Assumption 4.2 to (48) yields

$$(1-\mu^A)g'(G_1^B) > \underbrace{(1+r-\theta^B)\delta^B g'(\theta^B G_1^B + G_2^B)\left(\frac{\theta^A - \theta^B}{1+r-\theta^B}\right)}_{+}.$$

We thus have $\mu^A < 1$ and

$$\frac{g'(G_1^B)}{\delta^B g'(\theta^B G_1^B + G_2^B)} > (1+r-\theta^B)\left(\frac{1}{1-\mu^A}\right)\left(\frac{\theta^A - \theta^B}{1+r-\theta^B}\right) \geq 1 + r - \theta^B,$$

in which the second inequality follows from letting

$$\left(\frac{1}{1-\mu^A}\right)\left(\frac{\theta^A - \theta^B}{1+r-\theta^B}\right) \geq 1.$$

In so doing, we get the desired interregional redistribution and local borrowing policies. The proof of part (i) is thus complete.

For part (ii), using (54) and Assumption 4.2 yields

$$(1-\mu^B)g'(G_1^A) > (1+r-\theta^A)\delta^A g'(\theta^A G_1^A + G_2^A)\underbrace{\left(\frac{\theta^B-\theta^A}{1+r-\theta^A}\right)}_{-}.$$

Letting $\mu^B > 1$, we thus obtain

$$\frac{g'(G_1^A)}{\delta^A g'(\theta^A G_1^A + G_2^A)} < (1+r-\theta^A)\left(\frac{1}{1-\mu^B}\right)\left(\frac{\theta^B-\theta^A}{1+r-\theta^A}\right) \le 1+r-\theta^A$$

for

$$\underbrace{\left(\frac{1}{1-\mu^B}\right)\left(\frac{\theta^B-\theta^A}{1+r-\theta^A}\right)}_{+} \le 1,$$

which is expected. The remaining details of the proof are omitted to save space. ■

Proof of Proposition 5.1. We complete the proof in 2 steps.

Step 1: Applying the Envelope Theorem to value function (3), we have for $R \in \{A, B\}$ that

$$\begin{aligned} V_b(b^R, z^R, \delta^R) &= g'(G_1^R) - (1+r-\theta)\delta^R g'(\theta G_1^R + G_2^R), \\ V_{bb}(b^R, z^R, \delta^R) &= g_1'' \cdot \phi_b - (1+r-\theta)\delta^R g_2'' \cdot (\theta\phi_b + \psi_b) < 0, \end{aligned} \tag{56}$$

in which we have used (9), (13), and the strict concavity of g. We now prove Proposition 5.1(i). This is equivalent to showing that by imposing the upper bound denoted by $\bar{b} \equiv b^{B*}$ for regions of any type, setting the interregional redistribution scheme denoted by (z^{A*}, z^{B*}) implements the asymmetric information optimum when spending and borrowing decisions are decentralized at the regional level.

We first consider the case of $\mu^A > \mu^B = 0$. Let us suppose that Region A receives the federal transfer z^{B*} from the center, then its maximization problem becomes: $\max_{b^A} V(b^A, z^{B*}, \delta^A)$, subject to the constraint $b^A \le b^{B*}$. Using (56) and evaluating $V_b(b^A, z^{B*}, \delta^A)$ at $b^A = b^{B*}$ yields:

$$\begin{aligned} V_b(b^{B*}, z^{B*}, \delta^A) &= g'(G_1^{B*}) - (1+r-\theta)\delta^A g'(\theta G_1^{B*} + G_2^{B*}) \\ &> g'(G_1^{B*}) - (1+r-\theta)\delta^B g'(\theta G_1^{B*} + G_2^{B*}) > 0 \end{aligned}$$

in which we have used Assumption 2.1 and Proposition 3.1(i). Consequently, using $V_{bb} < 0$ leads to $0 < V_b(b^{B*}, z^{B*}, \delta^A) \le V_b(b^A, z^{B*}, \delta^A)$ for any $b^A \le b^{B*}$. Taking the federal transfer z^{B*} as given, Region A must choose local debt $b^A = b^{B*}$ accordingly; hence, allocation (b^{B*}, z^{B*}) is realized. Since Region A is indifferent between (b^{B*}, z^{B*}) and (b^{A*}, z^{A*}) under $\mu^A > 0$, it will report its type truthfully instead of mimicking Region B.

For Region B, in view of Proposition 3.1(i) we have $V_b(b^{B*}, z^{B*}, \delta^B) > 0$, thus using $V_{bb} < 0$ yields $0 < V_b(b^{B*}, z^{B*}, \delta^B) \le V_b(b^B, z^{B*}, \delta^B)$ for any $b^B \le b^{B*}$. This implies that (b^{B*}, z^{B*}) is also realized by Region B. Since we have $z^{A*} < 0 < z^{B*}$ according to Proposition 3.1(i) and that $V_z = g_1' + \theta g_2' > 0$ holds true for any type, we must have $V(b^B, z^{B*}, \delta^B) > V(b^B, z^{A*}, \delta^B)$ for any feasible $b^B \le b^{B*}$. Therefore, Region B has no incentive to misreport its type as A. In summary, the incentive compatibility constraints for both regions are fulfilled under this local budget rule.

Step 2: We now prove Proposition 5.1(ii). This is equivalent to showing that, by imposing the lower bound denoted by $\underline{b} \equiv b^{A*}$ for regions of any type, setting the interregional redistribution scheme denoted by (z^{A*}, z^{B*}) implements the asymmetric information optimum when spending and borrowing decisions are decentralized at the regional level.

We first consider truthful implementation in Region B. Let us suppose that Region B receives the federal transfer z^{A*} from the center, then its maximization problem becomes: $\max_{b^B} V(b^B, z^{A*}, \delta^B)$, subject to the constraint $b^B \geq b^{A*}$. Evaluating $V_b(b^B, z^{A*}, \delta^B)$ at $b^B = b^{A*}$ gives rise to

$$\begin{aligned} V_b(b^{A*}, z^{A*}, \delta^B) &= g'(G_1^{A*}) - (1 + r - \theta)\delta^B g'(\theta G_1^{A*} + G_2^{A*}) \\ &< g'(G_1^{A*}) - (1 + r - \theta)\delta^A g'(\theta G_1^{A*} + G_2^{A*}) < 0, \end{aligned}$$

in which we have used Assumption 2.1 and Proposition 3.1(ii). Thus using $V_{bb} < 0$ yields $V_b(b^B, z^{A*}, \delta^B) \leq V_b(b^{A*}, z^{A*}, \delta^B) < 0$ for any $b^B \geq b^{A*}$. If we take the federal transfer z^{A*} as given, Region B must choose local debt $b^B = b^{A*}$, and hence allocation (b^{A*}, z^{A*}) is realized. Noting that Region B is indifferent between (b^{B*}, z^{B*}) and (b^{A*}, z^{A*}) under $\mu^B > 0$, it will report its type truthfully.

For Region A, given $V_b(b^{A*}, z^{A*}, \delta^A) < 0$ according to Proposition 3.1(ii), using $V_{bb} < 0$ leads to $V_b(b^A, z^{A*}, \delta^A) \leq V_b(b^{A*}, z^{A*}, \delta^A) < 0$ for any $b^A \geq b^{A*}$. Therefore, (b^{A*}, z^{A*}) is also realized by Region A. Since we know that $z^{B*} < 0 < z^{A*}$ according to Proposition 3.1(ii) and that $V_z = g_1' + \theta g_2' > 0$ always holds true, we must have $V(b^A, z^{A*}, \delta^A) > V(b^A, z^{B*}, \delta^A)$ for any feasible $b^A \geq b^{A*}$. In other words, Region A has no incentive to misreport its type as B. In summary, the incentive compatibility constraints for both regions are guaranteed under this local budget rule. ■

Proof of Proposition 5.2. We complete the proof in 3 steps.

Step 1: We firstly consider the case of $\mu^A > \mu^B = 0$ corresponding to Proposition 4.2(i). Let us suppose that Region A receives the federal transfer z^{B*} from the center, then its maximization problem becomes: $\max_{b^A} V(b^A, z^{B*}, \theta^A, \delta^A)$, subject to the constraint $b^A \geq b^{B*}$. In light of (56) and evaluation of $V_b(b^A, z^{B*}, \theta^A, \delta^A)$ at $b^A = b^{B*}$ we have:

$$\begin{aligned} V_b(b^{B*}, z^{B*}, \theta^A, \delta^A) &= g'(G_1^{B*}) - (1 + r - \theta^A)\delta^A g'(\theta^A G_1^{B*} + G_2^{B*}) \\ &\leq g'(G_1^{B*}) - (1 + r - \theta^B)\delta^B g'(\theta^B G_1^{B*} + G_2^{B*}) < 0 \end{aligned}$$

under Proposition 4.2(i) and the following condition:

$$\underbrace{\frac{\delta^B}{\delta^A}}_{>1} \leq \underbrace{\frac{(1 + r - \theta^A)g'(\theta^A G_1^{B*} + G_2^{B*})}{(1 + r - \theta^B)g'(\theta^B G_1^{B*} + G_2^{B*})}}_{>1},$$

in which we have used Assumptions 2.1 and 4.1. Given $V_{bb} < 0$, we thus have $V_b(b^A, z^{B*}, \theta^A, \delta^A) \leq V_b(b^{B*}, z^{B*}, \theta^A, \delta^A) < 0$ for any $b^A \geq b^{B*}$. Taking the federal transfer z^{B*} as given, Region A must choose local debt $b^A = b^{B*}$, and hence allocation (b^{B*}, z^{B*}) is realized. Since Region A is indifferent between (b^{B*}, z^{B*}) and (b^{A*}, z^{A*}) under $\mu^A > 0$, it has no incentive to mimic Region B.

For Region B, in view of Proposition 4.2(i) we have $V_b(b^{B*}, z^{B*}, \theta^B, \delta^B) < 0$. Thus using $V_{bb} < 0$ yields $V_b(b^B, z^{B*}, \theta^B, \delta^B) \leq V_b(b^{B*}, z^{B*}, \theta^B, \delta^B) < 0$ for any $b^B \geq b^{B*}$. Therefore, (b^{B*}, z^{B*}) is also realized by Region B. Given $z^{A*} < 0 < z^{B*}$ according to Proposition 4.2(i) and that $V_z = g_1' + \theta g_2' > 0$ always holds true, we must have $V(b^B, z^{B*}, \theta^B, \delta^B) > V(b^B, z^{A*}, \theta^B, \delta^B)$ for any feasible $b^B \geq b^{B*}$. As a result, Region B has no incentive to mimic Region A. In summary, the incentive compatibility constraints for both regions are guaranteed under this budget rule.

Step 2: The proof of Proposition 5.2(i-b) is similar to that of Proposition 5.2(i-a), so we only show the main difference. For Region B, we now use (56) and evaluate $V_b(b^B, z^{A*}, \theta^B, \delta^B)$

at $b^B = b^{A*}$, which gives rise to

$$\begin{aligned} V_b(b^{A*}, z^{A*}, \theta^B, \delta^B) &= g'(G_1^{A*}) - (1 + r - \theta^B)\delta^B g'(\theta^B G_1^{A*} + G_2^{A*}) \\ &\geq g'(G_1^{A*}) - (1 + r - \theta^A)\delta^A g'(\theta^A G_1^{A*} + G_2^{A*}) > 0 \end{aligned}$$

under Proposition 4.2(ii) and the following condition:

$$\underbrace{\frac{\delta^B}{\delta^A}}_{>1} \leq \underbrace{\frac{(1 + r - \theta^A)g'(\theta^A G_1^{A*} + G_2^{A*})}{(1 + r - \theta^B)g'(\theta^B G_1^{A*} + G_2^{A*})}}_{>1},$$

in which we have used Assumptions 2.1 and 4.1.

Step 3: For Proposition 5.2(ii-a), note that for Region A,

$$\begin{aligned} V_b(b^{B*}, z^{B*}, \theta^A, \delta^A) &= g'(G_1^{B*}) - (1 + r - \theta^A)\delta^A g'(\theta^A G_1^{B*} + G_2^{B*}) \\ &> g'(G_1^{B*}) - (1 + r - \theta^B)\delta^B g'(\theta^B G_1^{B*} + G_2^{B*}) > 0, \end{aligned}$$

in which we have used Assumptions 2.1 and 4.2, and Proposition 4.4(i). Considering this region's maximization problem: $\max_{b^A} V(b^A, z^{B*}, \theta^A, \delta^A)$ subject to the constraint $b^A \leq b^{B*}$. By using $V_{bb} < 0$, we thus have $V_b(b^A, z^{B*}, \theta^A, \delta^A) \geq V_b(b^{B*}, z^{B*}, \theta^A, \delta^A) > 0$ for any $b^A \leq b^{B*}$. Taking the federal transfer z^{B*} as given, Region A must choose local debt $b^A = b^{B*}$ accordingly, and so allocation (b^{B*}, z^{B*}) is realized. Since Region A is indifferent between (b^{B*}, z^{B*}) and (b^{A*}, z^{A*}) under $\mu^A > 0$, it has no incentive to mimic Region B. For Region B, in view of Proposition 4.4(i) we have $V_b(b^{B*}, z^{B*}, \theta^B, \delta^B) > 0$. Thus using $V_{bb} < 0$ yields $V_b(b^B, z^{B*}, \theta^B, \delta^B) \geq V_b(b^{B*}, z^{B*}, \theta^B, \delta^B) > 0$ for any $b^B \leq b^{B*}$. Therefore, (b^{B*}, z^{B*}) is also realized by Region B. Given $z^{A*} < 0 < z^{B*}$ according to Proposition 4.4(i) and that $V_z = g'_1 + \theta g'_2 > 0$ always holds true, we must have $V(b^B, z^{B*}, \theta^B, \delta^B) > V(b^B, z^{A*}, \theta^B, \delta^B)$ for any feasible $b^B \leq b^{B*}$. Put differently, Region B has no incentive to mimic Region A. Consequently, the incentive compatibility constraints for both regions are fulfilled under this budget rule.

The proof of Proposition 5.2(ii-b) is analogous to that of Proposition 5.2(ii-a), and hence one just needs to note that for Region B, we now have

$$\begin{aligned} V_b(b^{A*}, z^{A*}, \theta^B, \delta^B) &= g'(G_1^{A*}) - (1 + r - \theta^B)\delta^B g'(\theta^B G_1^{A*} + G_2^{A*}) \\ &< g'(G_1^{A*}) - (1 + r - \theta^A)\delta^A g'(\theta^A G_1^{A*} + G_2^{A*}) < 0, \end{aligned}$$

in which we have used Assumptions 2.1 and 4.2, and Proposition 4.4(ii). The remaining details are omitted to save space. ■

Appendix B: Discussion on Multidimensional Screening

We now consider the case in which both the degree of patience and the degree of intergenerational spillovers are privately observable by each region. As is well known, it is analytically intractable to obtain interesting results in the presence of multidimensional private information (Rochet and Choné 1998; Armstrong and Rochet 1999). For analytical tractability, we thus impose the following assumption.

Assumption 6.1 *Let $\theta^R \equiv \varphi(\delta^R)$ for $R \in \{A, B\}$ and $\varphi(\cdot)$ be a continuously differentiable function satisfying:*

(a) φ is publicly observable;

(b) $\varphi'(\cdot) > 0$;

and (c) $\delta^R \varphi'(\delta^R)/\varphi(\delta^R) \geq \max\{g''_1/u''_1, [1 + (g''_2/u''_2)][-\varphi(\delta^R)G_1 g''_2/g'_2]\}$.

Assumption 6.1(a) states that there is a publicly observable functional relationship between the degree of patience and the degree of intergenerational spillovers. Although the function φ connecting the two parameters is public knowledge, θ^R is still each region's private information because δ^R is privately observable. Assumption 6.1(b) is in line with Assumptions 2.1 and 4.1. It assumes that the region with a greater discount factor would entail a higher degree of intergenerational spillovers. Assumption 6.1(c) states that there is a lower bound for the elasticity of θ^R with respect to δ^R, which is a technical requirement for establishing the following lemma.

Lemma 6.1 *Suppose Assumptions 2.1 and 6.1 hold, then the single-crossing property is satisfied in the sense that*

$$\frac{\mathrm{d}}{\mathrm{d}\delta^R}\left(\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0}\right) < 0 \text{ for all } \delta^R.$$

Proof. To simplify notations, we will suppress the region index, R, throughout the proof. Under Assumption 6.1, the FOCs (8) are rewritten as follows:

$$\begin{aligned} u'\left(y_1 + b + z - \hat{G}_1\right) &= g'\left(\hat{G}_1\right) + \varphi(\delta)\delta g'\left(\varphi(\delta)\hat{G}_1 + \hat{G}_2\right), \\ u'\left(y_2 - b(1+r) - \hat{G}_2\right) &= g'\left(\varphi(\delta)\hat{G}_1 + \hat{G}_2\right), \end{aligned} \tag{57}$$

in which $\hat{G}_1 \equiv \Phi(b, z, \delta)$ and $\hat{G}_2 \equiv \Psi(b, z, \delta)$. Differentiating both sides of the two equations in (57) with respect to δ, we then apply the Implicit Function Theorem and the Cramer's Rule and find that

$$\begin{aligned} \Phi_\delta &= -\frac{[\varphi(\delta) + \delta\varphi'(\delta)]g_2'(u_2'' + g_2'') + \delta\varphi(\delta)\varphi'(\delta)\Phi \cdot u_2''g_2''}{\Gamma} \quad \text{and} \\ \Psi_\delta &= \frac{\varphi(\delta)[\varphi(\delta) + \delta\varphi'(\delta)]g_2'g_2'' - \varphi'(\delta)\Phi \cdot (u_1'' + g_1'')g_2''}{\Gamma}, \end{aligned} \tag{58}$$

where $\Gamma \equiv (u_1'' + g_1'')(u_2'' + g_2'') + \delta\varphi(\delta)^2 u_2''g_2'' > 0$. Additionally, Equation (12) can be rewritten as follows:

$$\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0} = -\frac{g'\left(\hat{G}_1\right) - [1 + r - \varphi(\delta)]\delta g'\left(\varphi(\delta)\hat{G}_1 + \hat{G}_2\right)}{g'\left(\hat{G}_1\right) + \varphi(\delta)\delta g'\left(\varphi(\delta)\hat{G}_1 + \hat{G}_2\right)}.$$

Then by rearranging the algebra, we arrive at

$$\begin{aligned} \frac{\Xi}{\Gamma} &\equiv \frac{\mathrm{d}}{\mathrm{d}\delta}\left(\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0}\right) \cdot \frac{[g_1' + \varphi(\delta)\delta g_2']^2}{1+r} \\ &= (g_1' - \delta\Phi_\delta g_1'')g_2' - \delta^2\varphi'(\delta)(g_2')^2 + \delta g_1'g_2''[\varphi'(\delta)\Phi + \varphi(\delta)\Phi_\delta + \Psi_\delta]. \end{aligned} \tag{59}$$

Using (58) and simplifying the algebra, we get

$$\begin{aligned} &\Gamma[\varphi'(\delta)\Phi + \varphi(\delta)\Phi_\delta + \Psi_\delta] \\ &= \left\{(u_1'' + g_1'')\varphi'(\delta)\Phi - \varphi(\delta)[\varphi(\delta) + \delta\varphi'(\delta)]g_2'\right\} u_2'' > 0. \end{aligned} \tag{60}$$

Then, applying (60), (58) and Assumption 6.1 to (59) and simplifying the algebra, we have

$$\begin{aligned} 0 > \Xi = &\underbrace{\delta^2\varphi(\delta)\varphi'(\delta)u_2''g_2''g_2'(\Phi g_1'' - g_1') - \delta^3\varphi(\delta)^2\varphi'(\delta)(g_2')^2u_2''g_2''}_{-} \\ &+ \underbrace{(u_1'' + g_1'')g_1'}_{-} \cdot \underbrace{[(u_2'' + g_2'')g_2' + \delta\varphi'(\delta)\Phi u_2''g_2'']}_{+/0} \\ &+ \underbrace{\delta(u_2'' + g_2'')(g_2')^2}_{-} \cdot \underbrace{[\varphi(\delta)g_1'' - \delta\varphi'(\delta)u_1'']}_{+/0} \end{aligned} \tag{61}$$

due to the fact that

$$\varphi(\delta)g_1'' \geq \delta\varphi'(\delta)u_1'' \ \Leftrightarrow\ \frac{\delta\varphi'(\delta)}{\varphi(\delta)} \geq \frac{g_1''}{u_1''}$$

and

$$(u_2''+g_2'')g_2' + \delta\varphi'(\delta)\Phi u_2''g_2'' \geq 0 \ \Leftrightarrow\ \frac{\delta\varphi'(\delta)}{\varphi(\delta)} \geq -\frac{(u_2''+g_2'')g_2'}{\varphi(\delta)\Phi u_2''g_2''}$$

under Assumption 6.1. Therefore, the desired assertion follows from applying (61) to (59). ■

In contrast to Lemma 3.1(ii), Lemma 6.1 yields that, in every point in the (b, z)-space, the indifference curve of H-region (or patient region) has a smaller slope than the indifference curve of L-region (or impatient region). We can then apply Lemma 6.1 and the reasoning used in the proof of Proposition 4.2 to establish the corresponding redistribution and local debt policies.

Similarly, when the public goods of the impatient region are more durable we can impose the following assumption:

Assumption 6.2 *Let $\theta^R \equiv \varphi(\delta^R)$ for $R \in \{A, B\}$ and $\varphi(\cdot)$ be a continuously differentiable function satisfying: (a) φ is publicly observable; (b) $\varphi'(\cdot) < 0$; and (c)*

$$\frac{\delta^R\varphi(\delta^R)g_2'}{\delta^R\varphi(\delta^R)g_2' - (u_1''+g_1'')\Phi} \leq -\frac{\delta^R\varphi'(\delta^R)}{\varphi(\delta^R)} \leq 1.$$

Assumption 6.2(b) yields that the impatient region's public goods turn out to be more durable and is in line with Assumptions 2.1 and 4.2. Assumption 6.2(c) is a technical requirement for establishing the following lemma, which means that the absolute value of the elasticity of the degree of durability with respect to the degree of patience is no greater than 1 and no smaller than a positive threshold.

Lemma 6.2 *Suppose Assumptions 2.1 and 6.2 hold, then the single-crossing property is satisfied in the sense that*

$$\frac{\mathrm{d}}{\mathrm{d}\delta^R}\left(\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0}\right) > 0 \textit{ for all } \delta^R.$$

Proof. The proof is analogous to that of Lemma 6.1, one just needs to note that, under Assumption 6.2, we have $\Phi_\delta > 0$, $\Gamma[\varphi'(\delta)\Phi + \varphi(\delta)\Phi_\delta + \Psi_\delta] \leq 0$, and hence

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}\delta}\left(\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{\mathrm{d}V=0}\right)\cdot\frac{[g_1'+\varphi(\delta)\delta g_2']^2}{1+r}\\ =\ &\underbrace{(g_1'-\delta\Phi_\delta g_1'')g_2'}_{+} - \underbrace{\delta^2\varphi'(\delta)(g_2')^2}_{-} + \underbrace{\delta g_1'g_2''[\varphi'(\delta)\Phi+\varphi(\delta)\Phi_\delta+\Psi_\delta]}_{+/0}.\end{aligned}$$

Thus, the desired assertion follows immediately. ■

We therefore conclude that, as shown in Lemma 3.1(ii), Lemma 6.2 yields that, in every point in the (b, z)-space, the indifference curves of the patient region have a greater slope than those of the impatient region. Using Lemma 6.2, one can characterize the corresponding asymmetric information optimum, as shown in Proposition 4.4. Due to paper length limitations, those details will be omitted.