Optimal Regional Insurance Provision: Do Federal Transfers Complement Local Debt?*

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Abstract

We study optimal regional insurance provision in federations with regionally privately observable shocks to the degree of intergenerational externality (DIE) induced by, or to the degree of technological progress (DTP) for producing, local intergenerational public goods (IPGs). Federal transfers provide interregional insurance while local debt provides intergenerational insurance. If optimal federal transfers increase (decrease) with a region's debt level, we say the two insurance policies are complements (substitutes). We address such questions as whether it is efficiency-enhancing to adopt both schemes for regional insurance provision and how the answer varies with alternative regional economic shocks. The main results of the paper are the following. Under the DIE shocks, federal transfers and local debt act as complements in the course of implementing the asymmetric information optimum when borrowing and spending decisions are decentralized at the regional level. Under the DTP shocks, they act as complements with observable output of the IPGs, but act as substitutes with observable expenditure on the IPGs.

Keywords: Intergovernmental transfer; local government debt; intergenerational spillover; regional economic shocks; asymmetric information; mechanism design.

JEL Codes: H41, H74, H77, D82.

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1 Introduction

Intergovernmental grants implemented by the central government of a federal fiscal system are justified on the grounds that they internalize interregional spillovers generated by local public goods provision (Oates 1972) or by inter-jurisdictional migrations,[1] redistribute income between regions,[2] and, in particular, serve as a risk-sharing device against region-specific shocks.[3] As shown by Sala-i-Martin and Sachs (1992), even policies aimed at income redistribution may have an effect on the degree of interregional risk sharing. Indeed, there is empirical evidence showing that fiscal transfers from the federal government provide substantial insurance against regional economic fluctuations in the United States, Canada, Japan, and Norway.[4]

On the other hand, local government debt serves as a public contract for sharing risks between generations or over life-cycle in a given region. For instance, in an infinite-horizon economy where individuals face uninsurable risks to their human capital accumulation, Gottardi, Kajii, and Nakajima (2015) show that the benefits of government debt increase with the magnitude of risks and the degree of risk aversion. Since present generations are imperfectly altruistic (e.g., Altonji, Hayashi, and Kotlikoff 1992, 1997), however, the design of optimal public debt that accounts for intergenerational conflicts turns out to be a nontrivial mechanism design task (e.g., Dai, Liu, and Tian 2019a; Huber and Runkel 2008; Rangel 2003, 2005).

Given the insurance role played by both federal transfers and local debt, the following questions arise. Is it socially optimal or welfare-enhancing to adopt both schemes for regional insurance provision? Under decentralized borrowing decisions made by local governments, how would interregional insurance provided by the central government interact with intergenerational insurance provided locally? In addition, how might the answers to these questions vary with alternative sources of regional economic shocks? The goal of this paper is to address these questions via tackling the optimal design and implementation of risk-sharing contracts consisting of both intergovernmental grants and regional public debt along space and time dimensions, respectively.

Indeed, whether federal grants and local debt act as complements or substitutes matters greatly in regional insurance design. Specifically, if the two insurance schemes complement each other, then a joint implementation for regional insurance provision is justified on efficiency grounds. If they act as substitutes, on the other hand, then efficiency considerations require using either federal grants or local debt but not both of them simultaneously in the provision of regional insurance. In particular, identifying the case with substitutability creates a sort of policy flexibility, that is, they can be adopted simultaneously while targeting alternative policy goals facing the national government. For example, federal transfers are used for interregional income redistribution or for hori-

[1]See, e.g., Breuillé and Gary-Bobo (2007), Cremer, Marchand, and Pestieau (1997), Dai, Liu, and Tian (2019b), Figuieres and Hindriks (2002), and Hercowitz and Pines (1991).

[2]See, e.g., Bordignon, Manasse, and Tabellini (2001), Cornes and Silva (2000), Cremer and Pestieau (1997), and Raff and Wilson (1997).

[3]See, e.g., Bucovetsky (1998), Cornes and Silva (2000), Jüßen (2006), Lockwood (1999), and Persson and Tabellini (1996a,b).

[4]See, e.g., Asdrubali, Sørensen, and Yosha (1996), Athanasoulis and Wincoop (2001), Atkeson and Bayoumi (1993), Borge and Matsen (2004), Evers (2015), Kalemli-Ozcan, Sørensen, and Yosha (2003), and Mélitz and Zumer (1999). Even in the presence of complete markets, Farhi and Werning (2017) provide a rationale for government intervention in terms of public risk sharing.

zontal externality correction, whereas local debt is used for regional insurance provision. Or, federal transfers are used for regional insurance provision, whereas local debt can be strictly constrained to defuse local government debt bomb (e.g., The Economist 2015), which is practically relevant for countries with high risk of local government debt default such as China (e.g., Huang, Liu, and Tian 2020).

We consider a country that consists of a central government and many sub-national governments located in geographically decentralized regions. The center is in charge of revenue transfers across regions whereas local governments are responsible for collecting taxes used for the provision of local public goods. Each region is populated by a continuum of identical residents who live for one period only. The economy lasts for two periods, thus enabling us to incorporate intergenerational concerns into the model. The current generation chooses how much debt to pass to the future generation and how much to invest in intergenerational public goods (IPGs), namely durable public goods which entail positive intergenerational spillovers, such as basic science, environmental protection, and public capital. Initially, as a centralization benchmark to which we refer, we let the center jointly determine the amount of public debt a region can issue as well as the transfers it can receive. We then move to the more realistic situation with decentralized leadership in which local governments have the autonomy in choosing the level of regional public debt. We therefore characterize the federal grants scheme which implements the asymmetric information welfare optimum through decentralized regional borrowing decisions.

Regions are assumed to be ex ante identical but are subject to stochastic shocks to either the degree of intergenerational externality induced by, or the degree of technological progress for producing, the IPGs. In this context, while regional heterogeneity in shock realizations creates a natural role for interregional insurance represented by transfers from the center to the regions, a potential role of intergenerational insurance played by government debt is also easy to understand because both types of shocks primarily affect the future generations. Firstly, the present generation incurs the cost of IPG investment that generates a positive externality on the future generation. Secondly, it is well recognized that the progress made in fields like basic science, space exploration and environmental protection benefits from standing on the shoulder of giants, and hence a high degree of technological progress to be realized in the future appeals to knowledge accumulation and R&D investments in the present.

As is customarily assumed in the fiscal federalism literature, regional governments are better informed about the shocks than the federal government.[5] We are thus interested in incentive compatible insurance provision schemes. That is, intergovernmental grants and regional public debt form the risk-sharing contracts designed by the center, subject to the federal fiscal budget balance and truth-telling constraints. From solving the mechanism design problem facing the center, conducting the comparison with the full-information optimum (or the first-best allocation), and implementing the optimal allocation through regional debt issuance decisions, we obtain the following four results, regardless of the source of shocks. While the first two results characterize the asymmetric information welfare optimum, the last two results show the key features of the implementation scheme over the entire shock distribution.

First, the intertemporal allocation is undistorted only for the bottom and top types, i.e., the intertemporal rate of substitution between current and future consumption equals

[5]See, for example, Bucovetsky (1998), Cornes and Silva (2000, 2002), Dai, Liu, and Tian (2019a,b), Huber and Runkel (2008), Lockwood (1999), and Oates (1972).

the intertemporal rate of transformation only at the endpoints of shock distribution. As such, relative to the full-information optimum in which intertemporal rate of substitution equals intertemporal rate of transformation for all types, regions of types between the bottom and top may borrow too much or too little in the asymmetric information optimum. So, regions of some types are likely to face welfare losses under asymmetric information. Also, full insurance is not achievable for any type of regions. That is, the amount of public/private goods consumption in the future is not the same regardless of the realizations of economic shocks. Since the informational asymmetry between the center and regions prevents complete public insurance from happening in this setting, thereby providing a rationale for the usefulness of other insurance provision schemes such as private insurance.

Second, relative to the full-information optimal allocation, if the intergovernmental grant received by the bottom type is distorted upward (or is large), then the amount of public debt it can issue must be distorted downward (or be small), and vice versa; meanwhile, the direction of distortion is qualitatively reversed between the bottom and top types. Let us explain this result for the case of shocks to intergenerational externality. Recall that the intertemporal rate of transformation is the rate at which savings in the first period can be transformed into consumption in the second period, and an increase in which implies an increase in the opportunity cost of borrowing. Note that the positive intergenerational spillovers of IPGs partly offset the negative intergenerational externality of debt issuance, the bottom-type regions face the largest opportunity cost of borrowing. In consequence, imposing an upward distortion on transfers while a downward distortion on debt provides appropriate incentives such that the bottom-type regions reveal their type truthfully under asymmetric information.

Third, for all but the bottom and top types, to truthfully implement the welfare optimum through decentralized regional debt decisions, the intergovernmental grant scheme enforced by the center must be a nonlinear, almost everywhere differentiable and monotonic function of local debt. Since the intertemporal allocation is distorted for these types in the asymmetric information optimum, the amounts of public debt allocated to these regions are different than the ones determined by maximizing their respective regional goals. As a result, if borrowing decisions are decentralized to the regional governments, the grant scheme enforced by the center must depend on regional debt such that regions have incentives to truthfully reveal their types.

And fourth, for the bottom and top types, the grant scheme that decentralizes the welfare optimum is, however, independent of the local government debt. The reason is that the intertemporal allocations desired by regions of the extreme types are not distorted in the asymmetric information welfare optimum, namely, the relationship of intertemporal rate of substitution and intertemporal rate of transformation characterized by regional welfare maximization coincides with that in the asymmetric information welfare optimum determined by the central government. Consequently, incentive compatibility can be guaranteed for such types by directly setting the grants established in the asymmetric information optimum.

Moreover, concerning the potentially different effects of different sources of regional shocks on the center's optimal allocation of federal transfers across heterogeneous regions, it would be interesting to analyze the interactive relationship of the two insurance instruments for decentralizing the asymmetric information welfare optimum. We just need to focus on regional types between the top and bottom of shock distribution because only for these types nontrivial interactions between federal transfers and local debt arise from the task of optimal decentralization.

When regions differ in the degree of intergenerational externality induced by IPGs, federal transfers and local debt act as complements for regional insurance provision. The immediate implication is that it is socially optimal to use the two insurance schemes simultaneously when facing this sort of shocks. The intuition for this result is the following. Given that the asymmetric information optimum under such shocks features intertemporal rate of substitution being smaller than intertemporal rate of transformation for these regions, they thus borrow too much relative to the first-best allocation and, hence, welfare losses may emerge. Indeed, the higher the level of debt a regional government issues, ceteris paribus, the greater the welfare loss it may face. As such, to implement the asymmetric information optimum with the feature given above, this region should receive more federal transfers as a sort of compensation; otherwise, it may misreport its type by mimicking those regions issuing less debt than it issues. That is, local debt and federal transfers act as complements to guarantee self-selection through regional debt decisions in the course of decentralizing the asymmetric information optimum.

When regions differ in the degree of technological progress for producing the IPGs, federal transfers and local debt act as complements if it is the physical output of public goods that is observable, whereas they act as substitutes if it is the regional expenditure on public goods that is observable. The intuition for this latter result is the following.

The asymmetric information optimum under observable expenditure features intertemporal rate of substitution being greater than intertemporal rate of transformation, implying that these regions borrow too little relative to the first-best allocation. The lower the level of debt a regional government issues, ceteris paribus, the greater the welfare loss it may face. As such, this region should receive more federal transfers as a sort of compensation; otherwise, it may mimic those regions issuing more debt than it issues. This explains why local debt and federal transfers act as substitutes for implementing this asymmetric information optimum. The result under observable physical output can be explained analogously. Note that the corresponding asymmetric information optimum features intertemporal rate of substitution being smaller than intertemporal rate of transformation, implying that these regions borrow too much relative to the first-best allocation. Due to the opposing features of the asymmetric information optima, therefore, the endogenous interaction between local debt and federal transfers reverses from the case of observable expenditure to the case of observable output to guarantee desirable self-selection under asymmetric information, even though under the same source of economic shocks.

This distinction between the case of observable output and the case of observable expenditure is practically relevant as well. For example, the physical output of some IPGs such as parks, public schools and highways is observable, whereas the output of some IPGs such as environmental protection, basic science and R&D is unobservable, at least in the short run, by the center who is generally not involved in the process of producing these public goods. Consequently, whether it is the input or output of IPGs that is observable makes a nontrivial difference in determining whether the insurance provided by federal grants and the insurance provided by local debt should be used jointly or in isolation. In terms of identifying the effect of alternative observability on the implementation of information constrained optima, this finding contributes to the public finance and regional science literature. In sum, these results characterize the connections between the public risk-sharing schemes and the underlying economic environment, thereby helping us understand how they *should* be adopted in a federation.

The present study is related to the literature that examines theoretically the design of regional insurance provision in a federation, such as Bucovetsky (1998), Cornes and Silva (2000), Lockwood (1999), and Persson and Tabellini (1996a,b). A comparison with these studies reveals three distinctive features of our study. Firstly, rather than adopting a one-period static setting, we consider a two-period setting which accounts for intergenerational concerns and gives a natural insurance provision role to public debt. Secondly, the sources of regional economic shocks considered in the paper are novel for analyzing optimal regional insurance provision. In particular, these types of shocks have nontrivial effects on intergenerational resource allocation. Thirdly, instead of treating local debt and federal grants as unrelated policy variables, we study the *joint design* of the two risk-sharing schemes along space and time dimensions, namely intergovernmental grants along the interregional dimension and public debt along the intertemporal/intergenerational dimension, and further analyze their interaction in the course of optimal decentralization. Indeed, this paper represents the first attempt on the joint design of two public insurance schemes widely used by governments in both developed and underdeveloped economies. For these features, our paper extends and complements existing literature.

The remainder of the paper is organized as follows. Section 2 describes the economic environment. Section 3 derives the welfare optimum and discusses its implementation when regions differ in the degree of intergenerational externality. Section 4 conducts a similar analysis when regions differ in the degree of technological progress for producing local IPGs. Section 5 concludes. Proofs are relegated to the Appendix. Appendix B further examines the case in which regions differ in both DIE and DTP, and details the conditions under which the two insurance policies act as complements, regardless of whether output or expenditure is observable by the central government.

2 Environment

We consider a two-period economy of a federation consisting of a federal government (also referred to as the center) and n regions, each of which is inhabited by a representative immobile resident in each period.[6] That is, each resident lives for one period only. The social welfare of region i, for $i = 1, 2, ..., n$, is given by

$$\underbrace{u_1(c_1^i) + g_1(G_1^i)}_{\text{utility of generation 1}} + \underbrace{u_2(c_2^i) + g_2(\theta^i G_1^i + G_2^i)}_{\text{utility of generation 2}}, \tag{1}$$

in which c_1^i and c_2^i are private consumptions, G_1^i and G_2^i are public goods, and $\theta^i \in (0, 1]$ is a parameter measuring the degree of intergenerational externality of the IPG, G_1^i.[7] All four functions in (1) are strictly increasing and concave, twice continuously differentiable, and satisfy the usual Inada conditions.

The representative resident of generation t, for $t = 1, 2$, in region i has private budget

[6] We leave the interesting case with horizontal fiscal externalities induced by cross-region labor mobility to future research project.

[7] IPG is a kind of public good produced in generation 1 and still (partially) usable in generation 2 (e.g., Conley, Driskill, and Wang 2019; Dai, Jansen, and Liu 2021; Dai and Tian 2020; Rangel 2005). To guarantee tractability, we focus on the effect of intergenerational externalities and assume away the external effect a region's local public goods may exert on its neighboring regions.

constraint $c_t^i + \tau_t^i = y_t$, where y_t is the commonly given income across all regions.[8] The lump sum tax τ_t^i is collected by the local government to finance the provision of local public goods. In period 1, the local government receives a transfer z^i from the center ($z^i < 0$ means that the local government pays a tax to the center) and issues debt b^i. Debt and interest payments must be repayed in period 2, taking as given the common interest rate $r > 0$.[9] The fiscal budget constraints of region i in periods 1 and 2 can be written as $G_1^i = \tau_1^i + b^i + z^i$ and $\mathcal{G}_2^i = \tau_2^i - (1+r)b^i$, respectively. We let $\mathcal{G}_2^i = \xi^i G_2^i$, in which the parameter $\xi^i \in (0, 1]$ measures the per unit cost of period-2 public goods provision. The case of $\xi^i < 1$ captures the effect of technological progress, which as argued by Rangel (2005) is important for IPGs such as infrastructure, space exploration and environmental capital. In addition, as shown by Maskin and Riley (1985), whether the expenditure $\mathcal{G}_2^i$ or the physical output G_2^i is observable to the mechanism designer generally makes a difference for implementation. If it is the expenditure that is observable, then we need to express the output as a function of expenditure, namely $G_2^i = \mathcal{G}_2^i/\xi^i \equiv \rho^i \mathcal{G}_2^i$, where $\rho^i \equiv 1/\xi^i$.

For expositional convenience, the region index i is suppressed in the remainder of the set-up. Combining the private budget constraints with the public budget constraints and applying them to equation (1), a region's social welfare maximization problem is given by

$$\begin{aligned}\max_{c_1, c_2} \quad & u_1(c_1) + g_1(y_1 + b + z - c_1) \\ & + u_2(c_2) + g_2\left(\theta(y_1 + b + z - c_1) + \rho(y_2 - b(1+r) - c_2)\right),\end{aligned} \tag{2}$$

where $\rho \geq 1$. Note that in problem (2), choosing c_1 and c_2 is equivalent to choosing τ_1 and τ_2. The first-order conditions are thus written as

$$u_1'(c_1) = g_1'(G_1) + \theta g_2'(\theta G_1 + G_2) \quad \text{and} \quad u_2'(c_2) = \rho g_2'(\theta G_1 + G_2), \tag{3}$$

which represent the Samuelson conditions for the optimal provision of public goods.

We allow regions to differ in two dimensions in terms of privately observable shocks: the degree of intergenerational externality measured by θ and the degree of technological progress for producing IPGs measured by ξ (or, equivalently, by ρ). Both θ and ξ (or ρ) are treated as private information of local governments. The per unit cost of public goods provision has been previously treated as the private information of local governments in Boadway, Horiba, and Jha (1999), Cornes and Silva (2002), and Lockwood (1999). Interpreted as a measure of the quality or durability of local IPGs, it is reasonable to assume that θ is only privately observable by local governments for the following two reasons.

Firstly, the quality of the physical output of some IPGs, such as basic science, local environmental protection and R&D, is objectively unobservable, at least in the short run, by the center who is in general not involved in the process of producing these public goods. Secondly, the local politicians have subjective incentives to hide/misreport such information for the sake of either getting more transfers, getting personal promotions, or

[8]Instead of considering income heterogeneity across regions, which has been well studied in terms of designing optimal intergovernmental grants, we consider some novel dimensions of cross-region heterogeneity. Also, one can interpret our model as restricting attention to regions of similar personal incomes, such as California and Texas in the United States, or Jiangsu and Zhejiang provinces in China.

[9]Assuming that there is a common capital market within a federation, there is a unique rental price level of capital such that arbitrage opportunities are eliminated.

avoiding punishments. For example, local politicians in China may get promoted to higher levels because of doing a good job in public infrastructure investment or establishing a business friendly environment, or may get punished for being responsible for *tofu-dreg projects*[10] in the provision of local IPGs, such as public schools, bridges and dams, that end up in very low quality or even tragic failures.

As is well known, it is analytically intractable to obtain interesting results in the presence of multidimensional private information (e.g., Armstrong and Rochet 1999; Rochet and Choné 1998). We thus consider two separate cases with the first one featuring privately observable shocks to the degree of intergenerational externality (DIE) and the second one featuring privately observable shocks to the degree of technological progress (DTP). The random variables are assumed to be continuously distributed in intervals $[\underline{\theta}, \bar{\theta}] \equiv \Theta$ and $[\underline{\xi}, \bar{\xi}] \equiv \Xi$ (or $[\underline{\rho}, \bar{\rho}] \equiv \Upsilon$), and also are identically and independently distributed across regions. We denote by $f = F' > 0$ and F, respectively, the density and distribution functions, which are public knowledge throughout.

3 Welfare Optimum and Implementation when Regions Differ in Intergenerational Externality

In this section, we focus on the optimal provision of regional insurance against shocks to the degree of intergenerational externality θ^i. For this purpose, we assume that all regions have the same degree of technological progress, and let ρ^i (or ξ^i) $= 1$ for all i without loss of generality. We introduce first the problem of the center. Then, we proceed to derive welfare optimum in cases of complete and asymmetric information between the center and regions, and discuss the implementation issue.

3.1 The Problem of the Center

The center is responsible for determining time-consistent (or credible) policies of regional debt and cross-region transfers as non-market insurances against shocks to the intergenerational externality.[11] Assuming it treats all regions equally and the realization of shocks can be privately observed by each region, it thus maximizes the expectation of the value function (2) of any region, subject to federal fiscal budget balance and incentive-compatibility constraints.

We follow the mechanism design approach and apply the direct revelation principle. The center offers each region i a contract stipulating the federal transfer and the region's debt conditional only on its report of type θ^i. The reported type also belongs to set Θ. Since all regions are ex ante identical, the insurance contract can be thought of being signed between the principal (the center) and an agent (a region) whose type θ belongs to set Θ following the distribution F.[12]

The timing of the underlying game is given as follows:

[10] This is a well-known phrase coined by Zhu Rongji, the former premier of the People's Republic of China, on a visit to Jiujiang City, Jiangxi Province to describe a jerry-built dam.

[11] This study will not consider the issue of soft budget constraints which may emerge when the central government cannot commit to the transfers it sets ex-ante.

[12] In fact, the center offers each region i a contract stipulating the federal transfer and the region's debt conditional only on its report of type θ^i, denoted by $\hat{\theta}^i$, i.e., $b^i = b(\hat{\theta}^i)$ and $z^i = z(\hat{\theta}^i)$. To formulate the constraints facing the mechanism designer, we consider the limiting case with the number of regions

- Shock occurs, i.e., nature moves first.
- Local governments privately observe shock realizations.
- The federal government offers the contract menu, $\{b(\theta), z(\theta)\}$, for all $\theta \in \Theta$.
- The local governments simultaneously pick a contract (or equivalently report their types), and the game ends.

We write the value function generated by the maximization problem (2) as $V(b, z, \theta)$. As all regions are ex ante identical, the objective function of the center can be written as:

$$EU = \int_{\underline{\theta}}^{\bar{\theta}} V(b(\theta), z(\theta), \theta) f(\theta) d\theta. \tag{4}$$

The truth-telling constraints require that any region with shock realization θ prefers to report θ rather than some θ'; formally

$$V(b(\theta), z(\theta), \theta) \geq V(b(\theta'), z(\theta'), \theta) \;\; \forall \theta' \neq \theta, \;\; \theta', \theta \in \Theta. \tag{5}$$

The federal budget balance constraint for large n reads as

$$\int_{\underline{\theta}}^{\bar{\theta}} z(\theta) f(\theta) d\theta \leq 0, \tag{6}$$

which implies that in aggregate transfers must sum to at most zero.

The problem facing the center is thus to choose $\{b(\theta), z(\theta)\}_{\theta \in \Theta}$ to maximize (4) subject to (5) and (6). As is common in the mechanism design literature, we let b and z be piecewise continuously differentiable functions, and let $b(\theta)$ be everywhere continuous.

3.2 Welfare Optimum

As a standard benchmark result to which we can refer, we start our analysis by deriving the full-information (first-best) allocation that maximizes (4) subject to (6) only. We index the first-best optimum by the superscript ^{FB}.

Lemma 3.1 *In the full-information case, the welfare optimum $\{b^{FB}(\theta), z^{FB}(\theta)\}_{\theta \in \Theta}$ satisfies:*

(i) The intertemporal rate of substitution between current and future public goods consumption equals intertemporal rate of transformation, namely

$$\frac{g_1'(G_1^{FB}(\theta))}{g_2'(\theta G_1^{FB}(\theta) + G_2^{FB}(\theta))} = 1 + r - \theta \;\; \text{for any } \theta \in \Theta.$$

(ii) Full insurance is achievable, namely

$$V_z\left(b^{FB}(\theta), z^{FB}(\theta), \theta\right) = \gamma \;\; \text{for any } \theta \in \Theta,$$

in which $\gamma > 0$ denotes the Lagrange multiplier on the budget constraint (6).

being large, i.e., $n \to \infty$. Making use of the weak law of large numbers, the empirical distributions of b^i and z^i across regions approximate the theoretical distributions generated by $b^i = b(\hat{\theta}^i)$, $z^i = z(\hat{\theta}^i)$ and F.

Proof. Straightforward and omitted. ■

Part (i) yields that the intertemporal allocation of any type of regions in the first-best optimum features that intertemporal rate of substitution equals intertemporal rate of transformation. Part (ii) gives the standard insurance condition which states that the consumption of period-2 public goods is the same, irrespective of the shock realization on the degree of intergenerational spillovers.

We now turn to the more interesting case with asymmetric information between the center and regions. In this case, the realization of the random variable measuring the degree of intergenerational externality is private information so that regions of one type could mimic regions of another type in order to obtain (more) insurance transfers. We now index the second-best allocation by the superscript *.

We shall need the following assumption:

Assumption 3.1 $-\theta G_1 g_2'' \leq g_2'$ *for all* $\theta \in (\underline{\theta}, \bar{\theta})$*, namely the absolute value of the elasticity of generation 2's marginal utility from* G_1 *is no greater than one for all but the endpoints of the type distribution.*

This is a technical restriction imposed on generation 2's preference on public goods. It is easy to verify that this assumption is satisfied for log and power utility functions. In fact, Assumption 3.1 can be interpreted as follows. Applying the Envelope Theorem to (2), we get

$$\begin{aligned} V_z(b, z, \theta) &= g_1'(G_1) + \theta g_2'(\theta G_1 + G_2) > 0, \\ V_{z\theta}(b, z, \theta) &= g_2'(\theta G_1 + G_2) + \theta G_1 g_2''(\theta G_1 + G_2). \end{aligned}$$

As such, Assumption 3.1 guarantees that $V_{z\theta}(b, z, \theta) \geq 0$ for all $\theta \in (\underline{\theta}, \bar{\theta})$. Loosely speaking, in terms of enhancing these regions' welfare, federal transfers and intergenerational spillovers act as complements. In other words, the higher the degree of intergenerational spillovers realized in a region is, the greater the effect of federal transfers in enhancing this region's welfare. Or, the larger the amount of federal transfers received by a region is, the greater the effect of a positive shock to intergenerational spillovers in enhancing this region's welfare.

Proposition 3.1 (Asymmetric Information Optimum under DIE Shocks) *In the asymmetric-information case without bunching, namely* $\dot{b}(\theta) > 0$*, the welfare optimum* $\{b^*(\theta), z^*(\theta)\}_{\theta \in \Theta}$ *satisfies:*

(i) Concerning the relationship between the intertemporal rate of substitution between current and future public goods consumption and the intertemporal rate of transformation, we have:

$$\frac{g_1'(G_1^*(\theta))}{g_2'(\theta G_1^*(\theta) + G_2^*(\theta))} \begin{cases} = \ 1 + r - \theta & \text{for } \theta \in \{\underline{\theta}, \bar{\theta}\}; \\ < \ 1 + r - \theta & \text{for } \theta \in (\underline{\theta}, \bar{\theta}). \end{cases}$$

(ii) Suppose Assumption 3.1 holds. Let $\mu_1(\theta) > 0$ *be the Lagrange multiplier on the value constraint* $v(\theta) \equiv V(b(\theta), z(\theta), \theta)$ *of any type-*θ *region who is reporting truthfully, then we have:*

$$V_z\left(b^*(\theta), z^*(\theta), \theta\right) \begin{cases} = \ \gamma/\mu_1(\theta) & \text{for } \theta \in \{\underline{\theta}, \bar{\theta}\}; \\ < \ \gamma/\mu_1(\theta) & \text{for } \theta \in (\underline{\theta}, \bar{\theta}). \end{cases}$$

Proof. See the Appendix. ■

The key finding of this proposition is the following. First, the intertemporal allocation under asymmetric information is not distorted, relative to the first-best allocation given by Lemma 3.1, only at the endpoints of shock distribution. That is, only for regions of the highest and lowest degrees of intergenerational externality, the intertemporal rate substitution still equals intertemporal rate of transformation. For regions of any types in between, however, their intertemporal allocations are distorted relative to their respective first-best allocations in the sense that the intertemporal rate substitution is now smaller than the intertemporal rate of transformation. In fact, given that $V_{bb}(b, z, \theta) = g_1''(G_1) + [\theta - \rho(1+r)]^2 g_2''(\theta G_1 + G_2) < 0$, we must have $b^*(\theta) > b^{FB}(\theta)$ for any $\theta \in (\underline{\theta}, \bar{\theta})$, namely, in the asymmetric information optimum regions of these types should borrow more than they would borrow in the full-information optimum. Second, given that the multiplier $\mu(\theta)$ is type-dependent, there is incomplete insurance under asymmetric information.

As informational friction is what we focus on in this study, it is interesting to identify the effect of asymmetric information between the center and regions on optimal debt and intergovernmental grants policies. To this end, it is worthwhile providing a detailed characterization of the Lagrange multiplier $\mu_1(\theta)$.

Lemma 3.2 *For the current economic environment, the following statements are true.*

(i) If $\mu_1(\theta)$ is decreasing in θ, then there exists some $\tilde{\theta} \in (\underline{\theta}, \bar{\theta})$ such that $\mu_1(\theta) > 1$ for $\theta \in [\underline{\theta}, \tilde{\theta})$, $\mu_1(\theta) = 1$ for $\theta = \tilde{\theta}$, and $\mu_1(\theta) < 1$ for $\theta \in (\tilde{\theta}, \bar{\theta}]$.

(ii) If $\mu_1(\theta)$ is increasing in θ, then there exists some $\breve{\theta} \in (\underline{\theta}, \bar{\theta})$ such that $\mu_1(\theta) < 1$ for $\theta \in [\underline{\theta}, \breve{\theta})$, $\mu_1(\theta) = 1$ for $\theta = \breve{\theta}$, and $\mu_1(\theta) > 1$ for $\theta \in (\breve{\theta}, \bar{\theta}]$.

Proof. See the Appendix. ■

Although $\mu(\theta)$ could be a complex nonlinear function of θ, here we focus on the case of monotonicity for obtaining some clear-cut results. Indeed, in light of Lemma 3.2, the following proposition is established by comparing Lemma 3.1 to Proposition 3.1.

Proposition 3.2 (Policy Effect of Asymmetric Information on DIE) *Under Assumption 3.1, the following statements are true.*

(i) If $\mu_1(\theta)$ is decreasing in θ, then (i-a) $z^*(\theta) > z^{FB}(\theta)$ for all $\theta \in [\underline{\theta}, \tilde{\theta}]$; (i-b) $z^*(\bar{\theta}) < z^{FB}(\bar{\theta})$; and (i-c) $b^*(\underline{\theta}) < b^{FB}(\underline{\theta})$ and $b^*(\bar{\theta}) > b^{FB}(\bar{\theta})$.

(ii) If $\mu_1(\theta)$ is increasing in θ, then (ii-a) $z^*(\theta) > z^{FB}(\theta)$ for all $\theta \in [\breve{\theta}, \bar{\theta}]$; (ii-b) $z^*(\underline{\theta}) < z^{FB}(\underline{\theta})$; and (ii-c) $b^*(\underline{\theta}) > b^{FB}(\underline{\theta})$ and $b^*(\bar{\theta}) < b^{FB}(\bar{\theta})$.

Proof. See the Appendix. ■

If $\mu_1(\theta)$ is decreasing in θ, then the shadow price of the value constraint under truth-telling is larger for low types of regions (i.e., regions of low degrees of intergenerational externality) than for high types. In other words, this is the case wherein the incentive compatibility constraints of low types are binding. As a result, as shown in claim (i-a), low types *extract* the information rent and receive more transfers under asymmetric information than they would receive under complete information. Claim (ii-a) can be interpreted in a similar way.

Regardless of whether $\mu_1(\theta)$ is decreasing or increasing in θ, the optimal policy mix of federal transfers and local debt under asymmetric information is distorted for both top and bottom types. Importantly, the distortion is qualitatively reversed between the two extreme types. For instance, if $\mu_1(\theta)$ is decreasing in θ, then, relative to their respective federal transfers received and local debt issued in the full-information optimum, the bottom type receives more transfers and issues less debt while the top type receives less transfers and issues more debt in the asymmetric information optimum. The intuition for this result is the following.

Recall first that the intertemporal rate of transformation is the rate at which savings in the first period can be transformed into consumption in the second period, and an increase in which implies an increase in the opportunity cost of borrowing. Note that the *positive intergenerational spillovers* of the IPGs partly offset the *negative intergenerational externality* caused by local government borrowing, the bottom-type regions have the largest opportunity cost of borrowing because $1 + r - \underline{\theta} > 1 + r - \theta$ for any $\theta > \underline{\theta}$. In consequence, when $\mu_1(\theta)$ is decreasing in θ, imposing an upward distortion on transfers while a downward distortion on debt provides appropriate incentives such that the bottom-type regions reveal their type truthfully under asymmetric information. The other case, in which the shadow price of the value constraint under truth-telling is larger for high types than for low types, can be analyzed analogously.

3.3 Implementation

We have established the welfare optimum under both complete and asymmetric information in the previous subsection, we now proceed to consider how to implement it via regionally decentralized debt decisions. That is, both regions choose a level of public debt to maximize their regional welfare, taking as given the intergovernmental grants scheme provided by the center. Formally, the maximization problem of regions of type-θ is

$$\max_{b(\theta)} \; V(b(\theta), z(\theta), \theta),$$

taking as given the federal transfers received, $z(\theta)$. Rewriting private consumptions as $c_1 = \tilde{\phi}(b(\theta), z(\theta), \theta)$ and $c_2 = \tilde{\psi}(b(\theta), z(\theta), \theta)$ and applying the Envelope Theorem, the first-order condition is thus written as

$$\frac{g_1'\left(y_1 + b(\theta) + z(\theta) - \tilde{\phi}(b(\theta), z(\theta), \theta)\right)}{g_2'\left(\theta\left(y_1 + b(\theta) + z(\theta) - \tilde{\phi}(b(\theta), z(\theta), \theta)\right) + y_2 - (1+r)b(\theta) - \tilde{\psi}(b(\theta), z(\theta), \theta)\right)} = 1 + r - \theta, \tag{7}$$

showing that the intertemporal rate of substitution must be equal to the intertemporal rate of transformation at the regional welfare optimum.

Making use of (7) and Lemma 3.1, we immediately have the result: The full-information optimum is attained by simply setting $z(\theta) = z^{FB}(\theta)$ for all $\theta \in \Theta$. The reason is that the center can observe the type of each region and also the full-information optimum does not distort the intertemporal allocation desired by each region.

Under asymmetric information, the center must design intergovernmental grants scheme that guarantees incentive compatibility for all regions. It follows from Proposition 3.1 that the intertemporal allocation of regions of all but top and bottom types is distorted, so the asymmetric-information optimum can no longer be implemented through decentralized debt decisions characterized by (7) with the center simply setting $z(\theta) = z^*(\theta)$. Indeed, we have established the following proposition.

Proposition 3.3 (Optimal Decentralization under DIE Shocks) *Suppose the second-order sufficient condition for incentive compatibility is not binding, namely $\dot{b}(\theta) > 0$ is fulfilled. Then, the grant scheme $z^*(b)$ that decentralizes $\{b^*(\theta), z^*(\theta)\}_{\theta\in\Theta}$ is a nonlinear nondecreasing function of b, almost everywhere differentiable, with the slope*

$$\frac{dz^*}{db} \begin{cases} = \;\; 0 & \text{for } b \in \{b^*(\underline{\theta}), b^*(\bar{\theta})\}; \\ > \;\; 0 & \text{for } b \in (b^*(\underline{\theta}), b^*(\bar{\theta})). \end{cases}$$

Proof. See the Appendix. ■

The monotonicity constraint derived from guaranteeing incentive compatibility, namely $\dot{b}(\theta) > 0$, means that regions of high degrees of intergenerational spillovers (H-regions) are allowed to issue more debt than regions of low degrees (L-regions). This condition also makes sense in terms of efficiency because H-regions face smaller opportunity costs of borrowing than those facing L-regions. Elaborating further, given that the intertemporal rate of transformation is given by $1 + r - \theta$, relative to L-regions with small θ, H-regions with large θ have stronger ability to mitigate the negative intergenerational externality induced by borrowing, namely, to reduce the amount of repayment of debt plus interest placed on future generations.

In view of Proposition 3.1 we see that the intertemporal allocation under asymmetric information is not distorted at the endpoints of type distribution, thus the socially optimal levels of local debt for regions of bottom and top types, namely $b^*(\underline{\theta})$ and $b^*(\bar{\theta})$, can be realized by simply setting $z(\underline{\theta}) = z^*(\underline{\theta})$ and $z(\bar{\theta}) = z^*(\bar{\theta})$, respectively. This explains why we have $dz^*/db = 0$ at the endpoints of type distribution in Proposition 3.3.

For regions of types between $\underline{\theta}$ and $\bar{\theta}$, their intertemporal allocations are indeed distorted relative to the first-best. As such, if H-regions are allowed to issue more debt than L-regions, then the grant scheme that decentralizes the asymmetric information optimum should be designed such that more grants are allocated to H-regions than to L-regions. The intuition of this result is the following. We get from Proposition 3.1 that the asymmetric information optimum for these types features that intertemporal rate of substitution is smaller than intertemporal rate of transformation, which thus implies that these types borrow too much relative to the first-best and, hence, welfare loss emerges in the presence of asymmetric information. In particular, the more debt a region issues, the greater the welfare loss it will face. Given that the monotonicity constraint shows that H-regions issue more debt than L-regions, H-regions thus face greater welfare losses than L-regions, ceteris paribus. In consequence, the center must allocate more transfers to H-regions to prevent them from mimicking L-regions under asymmetric information.

The implication for the optimal funding structure of IPGs when facing this kind of shocks is the following: more (respectively less) federal transfers plus more (respectively less) local borrowing for regions of higher (respectively lower) degrees of intergenerational externality generated by the IPGs. Therefore, to guarantee incentive compatibility when the leadership of local borrowing is decentralized to each region, federal transfers and local debt exhibit *complementarity* in the case of shocks to the DIE.

4 Welfare Optimum and Implementation when Regions Differ in Technological Progress

To analyze the optimal regional insurance provision when regions differ in the realization of shocks to the degree of technological progress, we assume that all regions have the same degree of intergenerational externality, denoted θ. Now, ρ^i (or ξ^i) is a random variable, the realization of which is region i's private information. As shown by Lockwood (1999) and Maskin and Riley (1985), whether the expenditure, $\mathcal{G}_2^i$, or the physical output, G_2^i, is observable to the mechanism designer generally makes a difference for implementing the asymmetric information optimum. So, we shall discuss both possibilities in what follows.

4.1 The First-Best Benchmark

For the case with observable expenditure on the IPGs, the first-order conditions of problem (2) are now written as

$$u_1'(c_1) = g_1'(G_1) + \theta g_2'(\theta G_1 + \rho \mathcal{G}_2) \quad \text{and} \quad u_2'(c_2) = \rho g_2'(\theta G_1 + \rho \mathcal{G}_2), \tag{8}$$

and the corresponding regional value function is written as $V(b, z, \rho)$.

For the case with observable physical output of the IPGs, the value function of regions of type-ξ reads as follows:

$$\begin{aligned} V(b, z, \xi) \equiv \max_{G_1, G_2} \; & u_1(y_1 + b + z - G_1) + g_1(G_1) \\ & + u_2(y_2 - b(1+r) - \xi G_2) + g_2(\theta G_1 + G_2). \end{aligned} \tag{9}$$

The first-order conditions are given by

$$\begin{aligned} u_1'(y_1 + b + z - G_1) &= g_1'(G_1) + \theta g_2'(\theta G_1 + G_2) \quad \text{and} \\ \xi u_2'(y_2 - b(1+r) - \xi G_2) &= g_2'(\theta G_1 + G_2). \end{aligned} \tag{10}$$

Applying the Envelope Theorem, the first-best allocation can be characterized as stated in Lemma 4.1. The proof is straightforward and omitted.

Lemma 4.1 *In the full-information case, the welfare optimum verifies:*

- *If expenditure on the IPGs is observable, the first-best policy mix, $\{b^{FB}(\rho), z^{FB}(\rho)\}_{\rho \in \Upsilon}$, satisfies:*

  (i) The intertemporal rate of substitution between current and future public goods consumption equals intertemporal rate of transformation, namely

  $$\frac{g_1'(G_1^{FB}(\rho))}{g_2'(\theta G_1^{FB}(\rho) + \rho \mathcal{G}_2^{FB}(\rho))} = \rho(1+r) - \theta \quad \text{for any } \rho \in \Upsilon.$$

  (ii) Full insurance is achievable, namely

  $$V_z\left(b^{FB}(\rho), z^{FB}(\rho), \rho\right) = \gamma \quad \text{for any } \rho \in \Upsilon,$$

  in which $\gamma > 0$ denotes the Lagrange multiplier on the budget constraint $\int_{\underline{\rho}}^{\bar{\rho}} z(\rho) f(\rho) d\rho \leq 0$.

- *If output of the IPGs is observable, the first-best policy mix, $\{b^{FB}(\xi), z^{FB}(\xi)\}_{\xi\in\Xi}$, satisfies:*

  (i) The intertemporal rate of substitution between current and future private goods consumption equals intertemporal rate of transformation, namely

$$\frac{u_1'(c_1^{FB}(\xi))}{u_2'(c_2^{FB}(\xi))} = 1+r \quad \textit{for any } \xi \in \Xi.$$

  (ii) Full insurance is achievable, namely

$$u_1'\left(c_1^{FB}(\xi)\right) = \gamma \quad \textit{for any } \xi \in \Xi,$$

  in which $\gamma > 0$ denotes the Lagrange multiplier on the budget constraint $\int_{\underline{\xi}}^{\bar{\xi}} z(\xi)f(\xi)d\xi \leq 0$.

In the presence of complete information, there is no difference in the first-best welfare optimum of the two cases stated in Lemma 4.1, which can be easily verified by comparing the FOCs given by (8) and (10). Therefore, just for the sake of comparing the full-information optimum to the corresponding asymmetric information optimum, the first-best optimum is characterized in terms of public goods consumption in the case of observable expenditure while it is characterized in terms of private goods consumption in the case of observable output. The key features of the full-information optimum are the following. Part (i) shows that the intertemporal allocation of any type of regions is not distorted in the sense that intertemporal rate of substitution equals intertemporal rate of transformation, and part (ii) gives the standard full insurance condition.

4.2 Asymmetric Information Optimum

To derive the asymmetric information optimum, we shall need the following assumptions:

Assumption 4.1 *For the case of observable expenditure, we have $-\rho\mathcal{G}_2 g_2'' \leq g_2'$ for all $\rho \in (\underline{\rho}, \bar{\rho})$. That is, the absolute value of the elasticity of marginal utility from consuming public good $G_2 = \rho\mathcal{G}_2$ is no greater than one for all but the endpoints of the type distribution.*

Assumption 4.2 *For the case of observable output, we have $-G_2 g_2'' \leq g_2'$ for all $\xi \in (\underline{\xi}, \bar{\xi})$. That is, the absolute value of the elasticity of marginal utility from G_2 for generation 2 is no greater than one for all but the endpoints of the type distribution.*

These are technical restrictions imposed on the preferences of public goods consumption and they can be interpreted analogously to Assumption 3.1.

For the case of observable expenditure, the center under Assumption 4.1 is thought of as solving the following maximization problem:

$$\begin{aligned} &\max \int_{\underline{\rho}}^{\bar{\rho}} v(\rho) f(\rho) d\rho \\ s.t. \quad v(\rho) &= V(b(\rho), z(\rho), \rho); \\ \int_{\underline{\rho}}^{\bar{\rho}} z(\rho) f(\rho) d\rho &\leq 0; \\ \dot{v}(\rho) &= g_2' \left(\theta \phi(b(\rho), z(\rho), \rho) + \rho \psi(b(\rho), z(\rho), \rho)\right) \psi(b(\rho), z(\rho), \rho); \\ \dot{b}(\rho) &\leq 0 \end{aligned} \tag{11}$$

in which we rewrite public goods expenditures as $\phi(b(\rho), z(\rho), \rho) = G_1(\rho)$ and $\psi(b(\rho), z(\rho), \rho) = \mathcal{G}_2(\rho)$, the first equality constraint gives the value function of regions of type-ρ when they are telling the truth, the second one is the fiscal budget constraint under pure intergovernmental grants, the third one is the first-order necessary condition for incentive compatibility, and the last one is the second-order sufficient condition for incentive compatibility.[13]

Similarly, for the case of observable output, the center takes into account truth-telling constraints and solves the following program:

$$\begin{aligned} &\max \int_{\underline{\xi}}^{\bar{\xi}} v(\xi) f(\xi) d\xi \\ s.t. \quad v(\xi) &= V(b(\xi), z(\xi), \xi); \\ \int_{\underline{\xi}}^{\bar{\xi}} z(\xi) f(\xi) d\xi &\leq 0; \\ \dot{v}(\xi) &= -u_2' \left(y_2 - b(\xi)(1+r) - \xi \psi(b(\xi), z(\xi), \xi)\right) \psi(b(\xi), z(\xi), \xi); \\ \dot{b}(\xi) &\leq 0 \end{aligned} \tag{12}$$

in which $\psi(b(\xi), z(\xi), \xi) = G_2(\xi)$ and the constraints can be similarly interpreted as those in program (11).

Now, solving problems (11) and (12) gives the following proposition.

Proposition 4.1 (Asymmetric Information Optimum under DTP Shocks) *In the asymmetric-information case without bunching, the welfare optimum under asymmetric information verifies:*

- *If expenditure of the IPGs is observable, the constrained optimum policy mix, $\{b^*(\rho), z^*(\rho)\}_{\rho \in \Upsilon}$, satisfies:*

  (i) Suppose Assumption 4.1 holds. Concerning the relationship between the intertemporal rate of substitution between current and future public goods consumption and the intertemporal rate of transformation, we have:

$$\frac{g_1'(G_1^*(\rho))}{g_2'(\theta G_1^*(\rho) + \rho \mathcal{G}_2^*(\rho))} \begin{cases} = \ \rho(1+r) - \theta & \text{for } \rho \in \{\underline{\rho}, \bar{\rho}\}; \\ > \ \rho(1+r) - \theta & \text{for } \rho \in (\underline{\rho}, \bar{\rho}). \end{cases}$$

[13]The derivation of this monotonicity constraint is given in the proof of Proposition 4.1.

(ii) Let $\mu_1(\rho) > 0$ be the Lagrange multiplier on the value constraint $v(\rho) \equiv V(b(\rho), z(\rho), \rho)$ of any type-ρ region who is reporting truthfully, then we have:

$$V_z(b^*(\rho), z^*(\rho), \rho) \begin{cases} = \gamma/\mu_1(\rho) & \text{for } \rho \in \{\underline{\rho}, \bar{\rho}\}; \\ > \gamma/\mu_1(\rho) & \text{for } \rho \in (\underline{\rho}, \bar{\rho}). \end{cases}$$

- *If output of the IPGs is observable, the constrained optimum policy mix, $\{b^*(\xi), z^*(\xi)\}_{\xi\in\Xi}$, satisfies:*

  (i) Suppose Assumption 4.2 holds. Concerning the relationship between the intertemporal rate of substitution between current and future private goods consumption and the intertemporal rate of transformation, we have:

$$\frac{u_1'(c_1^*(\xi))}{u_2'(c_2^*(\xi))} \begin{cases} = 1 + r & \text{for } \xi \in \{\underline{\xi}, \bar{\xi}\}; \\ < 1 + r & \text{for } \xi \in (\underline{\xi}, \bar{\xi}). \end{cases}$$

  (ii) Let $\mu_1(\xi) > 0$ be the Lagrange multiplier on the value constraint $v(\xi) \equiv V(b(\xi), z(\xi), \xi)$ of any type-ξ region who is reporting truthfully, then we have:

$$u_1'(c_1^*(\xi)) \begin{cases} = \gamma/\mu_1(\xi) & \text{for } \xi \in \{\underline{\xi}, \bar{\xi}\}; \\ < \gamma/\mu_1(\xi) & \text{for } \xi \in (\underline{\xi}, \bar{\xi}). \end{cases}$$

Proof. See the Appendix. ■

As shown in Proposition 3.1, the intertemporal allocation under asymmetric information is not distorted only at the endpoints of type distribution, and also there is incomplete insurance.

Proposition 4.2 (Policy Effect of Asymmetric Information on DTP) *For the current economic environment, the following statements are true.*

- *If expenditure is observable, then we have:*

  (i) If $\mu_1(\rho)$ is decreasing in ρ, then (i-a) there exists some $\breve{\rho} \in (\underline{\rho}, \bar{\rho})$ such that $z^*(\rho) < z^{FB}(\rho)$ for all $\rho \in [\breve{\rho}, \bar{\rho}]$; (i-b) $z^*(\underline{\rho}) > z^{FB}(\underline{\rho})$; and (i-c) $b^*(\underline{\rho}) < b^{FB}(\underline{\rho})$ and $b^*(\bar{\rho}) > b^{FB}(\bar{\rho})$.

  (ii) If $\mu_1(\rho)$ is increasing in ρ, then (ii-a) there exists some $\tilde{\rho} \in (\underline{\rho}, \bar{\rho})$ such that $z^*(\rho) < z^{FB}(\rho)$ for all $\rho \in [\underline{\rho}, \tilde{\rho}]$; (ii-b) $z^*(\bar{\rho}) > z^{FB}(\bar{\rho})$; and (ii-c) $b^*(\underline{\rho}) > b^{FB}(\underline{\rho})$ and $b^*(\bar{\rho}) < b^{FB}(\bar{\rho})$.

- *If output is observable, then we have:*

  (i) If $\mu_1(\xi)$ is decreasing in ξ, then (i-a) there exists some $\tilde{\xi} \in (\underline{\xi}, \bar{\xi})$ such that $z^*(\xi) > z^{FB}(\xi)$ for all $\xi \in [\underline{\xi}, \tilde{\xi}]$; (i-b) $z^*(\bar{\xi}) < z^{FB}(\bar{\xi})$; and (i-c) $b^*(\underline{\xi}) < b^{FB}(\underline{\xi})$ and $b^*(\bar{\xi}) > b^{FB}(\bar{\xi})$ whenever $g_1''/g_2'' \leq \rho\theta(1 + r)$.

  (ii) If $\mu_1(\xi)$ is increasing in ξ, then (ii-a) there exists some $\breve{\xi} \in (\underline{\xi}, \bar{\xi})$ such that $z^*(\xi) > z^{FB}(\xi)$ for all $\xi \in [\breve{\xi}, \bar{\xi}]$; (ii-b) $z^*(\underline{\xi}) < z^{FB}(\underline{\xi})$; and (ii-c) $b^*(\underline{\xi}) > b^{FB}(\underline{\xi})$ and $b^*(\bar{\xi}) < b^{FB}(\bar{\xi})$ whenever $g_1''/g_2'' \leq \rho\theta(1 + r)$.

Proof. See the Appendix. ■

Here, we define H-regions (L-regions) as regions of high (low) degrees of technological progress for producing the IPGs, namely, of large (small) ρ or small (large) ξ.

For the case of observable expenditure on the IPGs, if $\mu_1(\rho)$ is decreasing in ρ, then the shadow price of the value constraint under truth-telling is larger for L-regions than for H-regions. As a result, H-regions *incur* the information rent and receive less transfers under asymmetric information than they receive under complete information. Claim (ii-a) with $\mu_1(\rho)$ increasing in ρ can be interpreted in a similar way. Regardless of whether $\mu_1(\rho)$ is decreasing or increasing in ρ, the optimal allocation under asymmetric information is distorted for both top and bottom types, and importantly, the distortion is qualitatively reversed between the two extreme types. For example, if the shadow price of the value constraint under truth-telling is larger for L-regions than for H-regions, then the top type receives less transfers while issues more debt, and the bottom type receives more transfers while issues less debt than they would do, respectively, in the full-information optimum.

For the case of observable physical output of the IPGs, if $\mu_1(\xi)$ is decreasing in ξ, then the shadow price of the value constraint under truth-telling is larger for H-regions than for L-regions. As a result, H-regions *extract* the information rent and receive larger grants under asymmetric information than they receive under complete information. Claim (ii-a) with $\mu_1(\xi)$ increasing in ξ can be interpreted in a similar way. Regardless of whether $\mu_1(\xi)$ is decreasing or increasing in ξ, the optimal allocation under asymmetric information is distorted for both top ($\underline{\xi}$) and bottom ($\bar{\xi}$) types, and importantly, the distortion is qualitatively reversed between the two extreme types. For example, if $\mu_1(\xi)$ is decreasing in ξ, the bottom type receives smaller grants and issues more debt, whereas the top type receives larger grants and issues less debt than they would do, respectively, in the full-information optimum.[14]

4.3 Implementation

We now proceed to implement the welfare optimum through regionally decentralized debt decisions. For the case of observable expenditure on the IPGs, the maximization problem of regions of type-ρ is given by:

$$\max_{b(\rho)} V(b(\rho), z(\rho), \rho)$$

for any given $z(\rho)$. Rewriting private consumptions as $c_1 = \tilde{\phi}(b(\rho), z(\rho), \rho)$ and $c_2 = \tilde{\psi}(b(\rho), z(\rho), \rho)$ and applying the Envelope Theorem, the first-order condition is thus written as

$$\begin{aligned} &g_1'\left(y_1 + b(\rho) + z(\rho) - \tilde{\phi}(b(\rho), z(\rho), \rho)\right) = [\rho(1+r) - \theta]\times \\ &g_2'\left(\theta\left(y_1 + b(\rho) + z(\rho) - \tilde{\phi}(b(\rho), z(\rho), \rho)\right) + \rho\left(y_2 - (1+r)b(\rho) - \tilde{\psi}(b(\rho), z(\rho), \rho)\right)\right). \end{aligned} \tag{13}$$

[14]In particular, if $g_1(\cdot) = \ln G_1$ and $g_2(\cdot) = \ln(\theta G_1 + \xi G_2)$, then $g_1''/g_2'' \leq \rho\theta(1+r)$ implies that $G_2/G_1 \leq [\sqrt{\rho\theta(1+r)} - \theta]\rho$ with $\sqrt{\rho\theta(1+r)} > \theta$; if $g_1(\cdot) = G_1^\alpha$ and $g_2(\cdot) = (\theta G_1 + \xi G_2)^\alpha$ for some parameter $\alpha \in (0,1)$, then $g_1''/g_2'' \leq \rho\theta(1+r)$ implies that $G_2/G_1 \leq \{[\rho\theta(1+r)]^{1/(2-\alpha)} - \theta\}\rho$ with $[\rho\theta(1+r)]^{1/(2-\alpha)} > \theta$. That is, under log or power utility functions of public goods consumption, the technical conditions required for claims (i-c) and (ii-c) to hold in the case of observable physical output feature that the growth rate of local public goods provision must be bounded above.

Making use of (13) and Lemma 4.1, we immediately have that the full-information optimum is attained by simply setting $z(\rho) = z^{FB}(\rho)$ for all $\rho \in \Upsilon$.

Similarly, for the case of observable output, the maximization problem of regions of type-ξ is given by:

$$\max_{b(\xi)} V(b(\xi), z(\xi), \xi)$$

for any given $z(\xi)$. Rewriting public goods consumptions as $G_1 = \phi(b(\xi), z(\xi), \xi)$ and $G_2 = \psi(b(\xi), z(\xi), \xi)$ and applying the Envelope Theorem, the first-order condition is thus written as

$$\begin{aligned} & u_1' \left(y_1 + b(\xi) + z(\xi) - \phi(b(\xi), z(\xi), \xi) \right) \\ = & (1+r) u_2' \left(y_2 - (1+r) b(\xi) - \xi \psi(b(\xi), z(\xi), \xi) \right). \end{aligned} \tag{14}$$

Making use of (14) and Lemma 4.1, we immediately have that the full-information optimum is attained by simply setting $z(\xi) = z^{FB}(\xi)$ for all $\xi \in \Xi$.

Under asymmetric information, we obtain the following implementation scheme.

Proposition 4.3 (Optimal Decentralization under DTP Shocks) *The implementation scheme that decentralizes the asymmetric information optimum under DTP shocks is characterized as follows.*

(i) For the case of observable expenditure, suppose Assumption 4.1 holds and the second-order sufficient condition for incentive compatibility is not binding, namely $\dot{b}(\rho) < 0$ is fulfilled. Then, the grant scheme $z^*(b)$ that decentralizes $\{b^*(\rho), z^*(\rho)\}_{\rho \in \Upsilon}$ is a nonlinear non-increasing function of b, almost everywhere differentiable, with the slope

$$\frac{dz^*}{db} \begin{cases} = 0 & \text{for } b \in \{b^*(\underline{\rho}), b^*(\bar{\rho})\}; \\ < 0 & \text{for } b \in (b^*(\bar{\rho}), b^*(\underline{\rho})). \end{cases}$$

(ii) For the case of observable output, suppose Assumption 4.2 holds and the second-order sufficient condition for incentive compatibility is not binding, namely $\dot{b}(\xi) < 0$ is fulfilled. Then, the grant scheme $z^*(b)$ that decentralizes $\{b^*(\xi), z^*(\xi)\}_{\xi \in \Xi}$ is a nonlinear nondecreasing function of b, almost everywhere differentiable, with the slope

$$\frac{dz^*}{db} \begin{cases} = 0 & \text{for } b \in \{b^*(\underline{\xi}), b^*(\bar{\xi})\}; \\ > 0 & \text{for } b \in (b^*(\bar{\xi}), b^*(\underline{\xi})). \end{cases}$$

Proof. See the Appendix. ■

For the case of observable expenditure on the IPGs (i.e., the physical output is unobservable), the monotonicity constraint of $\dot{b}(\rho) < 0$ means that H-regions are allowed to issue less debt than L-regions. Given that the intertemporal rate of transformation is $\rho(1+r) - \theta$, as given by Proposition 4.1, H-regions with large ρ face higher opportunity costs of borrowing than the opportunity costs of borrowing facing L-regions with small ρ. As such, incentive compatibility is guaranteed by allowing regions with lower opportunity costs of borrowing to issue more government debt.

For the case of observable physical output of the IPGs (i.e., the expenditure is unobservable), however, the monotonicity constraint of $\dot{b}(\xi) < 0$ means that H-regions are allowed to issue more debt than L-regions. For residents of the second generation living in H-regions with small ξ, relative to their counterparts in L-regions with large ξ, they pay less taxes, denoted ξG_2, to finance a given amount of local public goods provision. As such, under the common individual income y_2 and interest rate r, the second generation of H-regions is able to repay a higher level of public debt plus interest than the counterpart of L-regions. This explains why the requirement of $\dot{b}(\xi) < 0$ guarantees incentive compatibility.

The intuition for a zero slope of the grant with respect to debt at the endpoints of type distribution is the same as that for Proposition 3.3. For regions of any other types, we always have nonzero slopes. In particular, for the case of observable expenditure, the slope of the grant with respect to debt is negative, whereas there is a positive slope for the case of observable output. The intuition of this difference is given as follows.

First, for the case of observable expenditure, we see from Proposition 4.1 that the asymmetric information optimum features that intertemporal rate of substitution is *greater* than intertemporal rate of transformation. This implies that these regions *borrow too little* relative to the first-best benchmark and, hence, they may face welfare losses in the presence of asymmetric information. For regions who are allowed to issue less debt under asymmetric information, they may face greater welfare losses. Given that incentive compatibility requires that H-regions are allowed to issue less debt than L-regions, then the grant scheme that decentralizes the asymmetric information optimum should be designed such that H-regions receive more grants than L-regions; otherwise, H-regions will choose to misreport their types by mimicking L-regions. This explains why the implementable grant scheme features that grants increase as debt decreases.

In contrast, for the case of observable output, we see from Proposition 4.1 that the asymmetric information optimum features that intertemporal rate of substitution is *smaller* than intertemporal rate of transformation. This implies that these regions *borrow too much* relative to the first-best benchmark. For regions who are allowed to issue more debt under asymmetric information, they may face greater welfare losses. Given that incentive compatibility requires that H-regions are allowed to issue more debt than L-regions, then the grant scheme that decentralizes the asymmetric information optimum should be designed such that H-regions receive more grants than L-regions; otherwise, H-regions will choose to misreport their types by mimicking L-regions. This explains why the implementable grant scheme features that grants increase as debt increases.

If the physical output of some IPGs, such as environmental protection, basic science and R&D, is unobservable by the center who is generally not involved in the production process, then we have the following optimal funding structure of such IPGs. More federal transfers plus less local borrowing for regions of high degrees of technological progress for producing the IPGs, while less federal transfers plus more local borrowing for regions of low degrees of technological progress. In consequence, when facing this kind of shocks and each region has the autonomy in choosing the level of government debt, federal transfers and local debt exhibit a sort of *substitutability* in terms of socially optimal regional insurance provision.

Meanwhile, for IPGs with observable physical output, such as parks, public schools and highways, if the total spending is unobservable by the center, then the optimal funding structure is given as follows. More (respectively less) federal transfers plus more (respectively less) local borrowing for regions of high (respectively low) degrees of technological progress for producing such IPGs. Therefore, when facing this kind of shocks and each region has the autonomy in choosing the level of government debt, federal transfers and local debt exhibit a sort of *complementarity* in terms of socially optimal regional insurance provision.

Table 1: Alternative asymmetric information optima and implementation schemes

regional heterogeneity \ mechanism design	intertemporal distortion	truthful implementation
DIE shocks	IRS = IRT for $\theta \in \{\underline{\theta}, \overline{\theta}\}$ IRS < IRT for $\theta \in (\underline{\theta}, \overline{\theta})$	$\frac{dz^*}{db} = 0$ for $\theta \in \{\underline{\theta}, \overline{\theta}\}$ $\frac{dz^*}{db} > 0$ for $\theta \in (\underline{\theta}, \overline{\theta})$
DTP shocks: observable expenditure	IRS = IRT for $\rho \in \{\underline{\rho}, \overline{\rho}\}$ IRS > IRT for $\rho \in (\underline{\rho}, \overline{\rho})$	$\frac{dz^*}{db} = 0$ for $\rho \in \{\underline{\rho}, \overline{\rho}\}$ $\frac{dz^*}{db} < 0$ for $\rho \in (\underline{\rho}, \overline{\rho})$
DTP shocks: observable output	IRS = IRT for $\xi \in \{\underline{\xi}, \overline{\xi}\}$ IRS < IRT for $\xi \in (\underline{\xi}, \overline{\xi})$	$\frac{dz^*}{db} = 0$ for $\xi \in \{\underline{\xi}, \overline{\xi}\}$ $\frac{dz^*}{db} > 0$ for $\xi \in (\underline{\xi}, \overline{\xi})$

5 Conclusion

This paper aims to study theoretically the design and implementation of optimal insurance provision to sub-national regions against privately observable shocks. We consider two types of shocks to regional economies, one of which is to the degree of intergenerational spillovers induced by IPGs and the other is to the degree of technological progress for producing the IPGs. We focus on the joint design of two widely-adopted public risk-sharing schemes — i.e., intergovernmental grants that provide *cross-region insurance* along the space dimension and public debt that provides *cross-generation insurance* along the time dimension. To the best of our knowledge, this paper is the first attempt in the literature on regional insurance provision within federations towards the joint design of these two risk-sharing schemes and the formal analysis of their interaction in the course of implementing welfare optimum.

The asymmetric information welfare optima under alternative regional economic shocks have three main features. Firstly, the informational asymmetries considered here preclude the completeness of public insurance under risk-averse individual preferences. Secondly, intertemporal allocation is distorted for all but the bottom and top types — namely, intertemporal rate of substitution (IRS) between current and future public goods consumption equals intertemporal rate of transformation (IRT) only at the endpoints of regional type distribution, as summarized in Table 1. Thirdly, if the intergovernmental grant received by the bottom type is distorted upward (or is large), then its public debt issuance must be distorted downward (or be low), and vice versa; meanwhile, the direction of distortion is qualitatively reversed between the bottom and top types.

To decentralize truthfully the welfare optima under regional debt issuance decisions, we have the following three predictions (see Table 1). First, for the top and bottom types of regions, the intergovernmental grant scheme that decentralizes the asymmetric-information optimum turns out to be independent of regional public debt, regardless of the source of shocks and whether the expenditure on, or the output of, public goods is observable when regions differ in the degree of technological progress. Second, for all other types of regions when they differ in the degree of intergenerational externality, regional debt complements the grant scheme that decentralizes the welfare optimum. Third, for all other types of regions when they differ in the degree of technological progress, regional debt complements the grant scheme that decentralizes the welfare optimum only when the physical output of public goods is observable; otherwise, regional debt and grant scheme act as substitutes in terms of decentralizing the asymmetric information welfare optimum. Therefore, it is worthwhile distinguishing between the case of observable input and the case of observable output for optimal regional insurance provision.

The additional insights of accounting for this distinction may be gained from combining the asymmetric information optimum with the corresponding implementation scheme. We observe that regions with a higher degree of technological progress should receive more federal transfers, ceteris paribus, regardless of whether expenditure or output is observable by the central government. This observation is intuitive provided that the center is a benevolent social planner. The transmission mechanism that leads to this claim under observable expenditure, however, departs from that under observable output. Firstly, concerning the optimal allocation of local debt issuance, the centralized optimum under asymmetric information shows different features in these two cases. Under observable expenditure, the sufficient condition for incentive compatibility requires that regions with a lower degree of technological progress should be provided with a higher level of debt issuance. That is, the incentive compatibility constraints of low-types (with low degrees of technological progress) rather than high-types are more likely to be binding in the optimum. In contrast, under observable output, the sufficient condition of truth-telling requires that regions with a higher degree of technological progress should be provided with a higher level of debt issuance. Now, the incentive compatibility constraints of high-types rather than low-types are more likely to be binding in the optimum. Therefore, this distinction is indeed relevant in terms of solving the self-selection problem facing the center in the presence of asymmetric information regarding the exogenous degree of technological progress. Secondly, to implement these asymmetric information optima when borrowing and spending decisions are decentralized at the regional level, we prove that federal transfers and local debt are substitutes under observable expenditure but are complements under observable output.

As such, the observation mentioned above can be obtained by exploiting the following logic: under observable expenditure, regions with a *lower* degree of technological progress should be provided with a *higher* level of debt issuance and hence *less* federal transfers under *policy substitutability*, ceteris paribus; whereas, under observable output, regions with a *lower* degree of technological progress should be provided with a *lower* level of debt issuance and hence *less* federal transfers under *policy complementarity*. Although in both cases regions with lower degrees of technological progress should receive less transfers due to *efficiency* considerations, in the course of achieving this goal through fiscal decentralization and intergovernmental transfer schemes the two insurance policies act as substitutes under observable expenditure while complements under observable output because the *self-selection problems* facing the center are essentially different in these two cases.

In addition to the previous analysis based on the assumption of unidimensional regional heterogeneity, we also discuss in Appendix B a more general case where there is heterogeneity in both the degree of intergenerational externality and the degree of technological progress that are unobservable by the center. We assume that there is a certain correlation between these two parameters, thereby reducing the multidimensional screening problem to a one-dimensional screening problem for the sake of technical tractability. Two main results are obtained. First, it remains true that the informational asymmetry between the center and regions prevents complete public insurance from happening. Second, federal transfers and local debt exhibit policy complementarity in terms of truthfully implementing asymmetric information optima, regardless of whether it is the physical output of, or the expenditure on, the IPGs that is observable by the center.

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Appendix: Proofs

Proof of Proposition 3.1. We shall complete the proof in 4 steps.

Step 1: We define the value function to a type-θ region when it is truth-telling as

$$v(\theta) \equiv V(b(\theta), z(\theta), \theta). \tag{15}$$

Applying the Envelope Theorem to (2), we get the following first-order necessary condition for the truth-telling constraints (5) to be satisfied:

$$\dot{v}(\theta) = g_2'(\theta\phi(b(\theta), z(\theta), \theta) + \psi(b(\theta), z(\theta), \theta))\phi(b(\theta), z(\theta), \theta), \tag{16}$$

in which $G_1(\theta) \equiv \phi(b(\theta), z(\theta), \theta)$ and $G_2(\theta) \equiv \psi(b(\theta), z(\theta), \theta)$.

We now derive the second-order sufficient condition for incentive compatibility. After some algebra, the local second-order condition of (5) can be written as

$$\dot{b}(\theta) \cdot V_z(b(\theta), z(\theta), \theta) \cdot \frac{\partial}{\partial \tilde{\theta}} \left(\frac{V_b(b(\theta), z(\theta), \tilde{\theta})}{V_z(b(\theta), z(\theta), \tilde{\theta})} \right) \Bigg|_{\tilde{\theta}=\theta} \geq 0.$$

Noting that $V_z(\cdot) = g_1' + \theta g_2' > 0$ and the Spence-Mirrlees property reads as

$$\frac{\partial}{\partial \theta}\left(\frac{V_b}{V_z}\right) = \frac{(1+r)[(g_2')^2 - G_1 g_1' g_2'']}{(g_1' + \theta g_2')^2} > 0,$$

we thus must have

$$\dot{b}(\theta) \geq 0, \tag{17}$$

which gives the desired monotonicity constraint. It is easy to verify that the local second-order condition also implies global optimality of the truth-telling strategy with the help of the above Spence-Mirrlees property.

We can equivalently rewrite (17) as

$$\dot{b}(\theta) \;=\; \beta(\theta), \quad \beta(\theta) \;\geq\; 0. \tag{18}$$

The problem of the center is therefore to choose piecewise continuous control variables $b(\theta)$ and $z(\theta)$ to maximize

$$\int_{\underline{\theta}}^{\bar{\theta}} v(\theta) f(\theta) d\theta$$

subject to constraints (6), (15), (16) and (18).

Step 2: To solve the optimal control problem with integral and inequality constraints, we write the generalized Hamiltonian as:

$$\begin{aligned}\mathcal{H} \;&= v(\theta)f(\theta) + \mu_1(\theta)[V(b(\theta), z(\theta), \theta) - v(\theta)]f(\theta) + \mu_2(\theta)\beta(\theta) - \gamma z(\theta) f(\theta) \\ &\quad + \eta_1(\theta) g_2'(\theta\phi(b(\theta), z(\theta), \theta) + \psi(b(\theta), z(\theta), \theta))\phi(b(\theta), z(\theta), \theta) + \eta_2(\theta)\beta(\theta),\end{aligned}$$

where $\mu_1(\theta)$, $\mu_2(\theta)$ and γ are non-negative Lagrange multipliers, and $\eta_1(\theta)$ and $\eta_2(\theta)$ are co-state variables. The first-order necessary conditions for a solution to the optimal control problem can now be stated as the state equations (16) and (18), plus

$$\mathcal{H}_z \;=\; \mu_1(\theta) V_z(b(\theta), z(\theta), \theta) f(\theta) - \gamma f(\theta) + \eta_1(\theta)[g_2''(\theta\phi_z + \psi_z)\phi + g_2'\phi_z] = 0, \tag{19}$$

$$\mathcal{H}_\beta \;=\; \mu_2(\theta) + \eta_2(\theta) = 0, \tag{20}$$

and

$$\dot{\eta}_1(\theta) \;=\; -\mathcal{H}_v \;=\; [\mu_1(\theta) - 1] f(\theta), \tag{21}$$

$$\dot{\eta}_2(\theta) \;=\; -\mathcal{H}_b \;=\; -\mu_1(\theta)[g_1' - (1 + r - \theta) g_2'] f(\theta) - \eta_1(\theta)[g_2''(\theta\phi_b + \psi_b)\phi + g_2'\phi_b]. \tag{22}$$

In addition, we have the following transversality conditions:

$$\eta_1(\theta) \;=\; \eta_2(\theta) \;=\; 0 \;\text{ for } \forall \theta \in \{\underline{\theta}, \bar{\theta}\}. \tag{23}$$

Step 3: Using (3) and the assumption that $\rho = 1$, we can write private consumptions as functions of debt, transfers and the degree of intergenerational spillovers: $c_1 \equiv \tilde{\phi}(b, z, \theta)$ and $c_2 \equiv \tilde{\psi}(b, z, \theta)$. Applying the Implicit Function Theorem to (3), we have these partial derivatives:

$$\begin{aligned}\tilde{\phi}_b(b, z, \theta) &= \frac{g_1''(u_2'' + g_2'') - (1 + r - \theta)\theta u_2'' g_2''}{\Sigma}, \\ \tilde{\psi}_b(b, z, \theta) &= \frac{\theta u_1'' g_2'' - (1 + r)(u_1'' + g_1'') g_2''}{\Sigma};\end{aligned} \tag{24}$$

and

$$\tilde{\phi}_z(b,z,\theta) = \frac{g_1''(u_2''+g_2'') + \theta^2 u_2'' g_2''}{\Sigma}, \ \tilde{\psi}_z(b,z,\theta) = \frac{\theta u_1'' g_2''}{\Sigma}; \tag{25}$$

with $\Sigma \equiv (u_1''+g_1'')(u_2''+g_2'') + \theta^2 u_2'' g_2'' > 0$. Using $\phi(b,z,\theta) = y_1 + b + z - \tilde{\phi}(b,z,\theta)$, $\psi(b,z,\theta) = y_2 - b(1+r) - \tilde{\psi}(b,z,\theta)$, (24) and (25), we obtain

$$\begin{aligned} \phi_b &= \frac{u_1''(u_2''+g_2'') + \theta^2 u_2'' g_2'' + (1+r-\theta)\theta u_2'' g_2''}{\Sigma} > 0, \\ \psi_b &= -\frac{\theta u_1'' g_2'' + (1+r)[(u_1''+g_1'')u_2'' + \theta^2 u_2'' g_2'']}{\Sigma} < 0; \end{aligned} \tag{26}$$

and

$$\phi_z = \frac{u_1''(u_2''+g_2'')}{\Sigma} > 0, \ \psi_z = -\frac{\theta u_1'' g_2''}{\Sigma} < 0. \tag{27}$$

Using (26) and (27), we get

$$\theta\phi_z + \psi_z = \frac{\theta u_1'' u_2''}{\Sigma} > 0, \ \theta\phi_b + \psi_b = -\frac{(1+r-\theta)u_1'' u_2'' + (1+r)g_1'' u_2''}{\Sigma} < 0. \tag{28}$$

Using (28) and (27) gives

$$g_2''(\theta\phi_z + \psi_z)\phi + g_2'\phi_z = \frac{(\theta G_1 g_2'' + g_2')u_1'' u_2'' + u_1'' g_2'' g_2'}{\Sigma} > 0 \tag{29}$$

under Assumption 3.1. Also, it is immediate from (28) and (26) that

$$g_2''(\theta\phi_b + \psi_b)\phi + g_2'\phi_b > 0. \tag{30}$$

Step 4: Since we are interested in the case without bunching, the monotonicity constraint (17) must be $\dot{b}(\theta) > 0$, and hence $\mu_2(\theta) = 0$ for all $\theta \in \Theta$ based on the complementary slackness conditions. By (20), we must have $\eta_2(\theta) = 0$ everywhere, yielding $\dot{\eta}_2 \equiv 0$. Consequently, we get from (22) and (30) that

$$\mu_1(\theta)[g_1' - (1+r-\theta)g_2']f(\theta) \ = \ \underbrace{-\eta_1(\theta)[g_2''(\theta\phi_b + \psi_b)\phi + g_2'\phi_b]}_{\leq 0},$$

by which combined with (23) and $\mu_1(\theta)f(\theta) > 0$ for all $\theta \in \Theta$ we have established the result in part (i).

Moreover, using (19) and (29) gives rise to

$$\mu_1(\theta)V_z(b(\theta), z(\theta), \theta)f(\theta) - \gamma f(\theta) \ = \ \underbrace{-\eta_1(\theta)[g_2''(\theta\phi_z + \psi_z)\phi + g_2'\phi_z]}_{\leq 0}$$

under Assumption 3.1. This combined with (23) and $\mu_1(\theta)f(\theta) > 0$ for all $\theta \in \Theta$ completes the proof of part (ii). ■

Proof of Lemma 3.2. It follows from (21) and (23) that

$$\int_{\underline{\theta}}^{\bar{\theta}} [\mu_1(\theta) - 1]f(\theta)d\theta \ = \ \eta_1(\bar{\theta}) - \eta_1(\underline{\theta}) \ = \ 0. \tag{31}$$

By (19), $\mu_1(\theta)$ must be everywhere continuous. Therefore, if $\mu_1(\theta)$ is decreasing in θ, then (31) implies that $\mu_1(\theta)-1$ is first positive and then negative as θ increases, and that the application of the Intermediate Value Theorem yields that there must be some $\tilde{\theta} \in (\underline{\theta}, \bar{\theta})$ such that $\mu_1(\tilde{\theta}) = 1$, as desired in part (i). The proof of part (ii) can be done analogously. ■

Proof of Proposition 3.2. We shall complete the proof in 2 steps.

Step 1: Here we just need to show the proof of part (i) because that of part (ii) is similar. We have by applying $\rho = 1$ and the Envelope Theorem to (2) that $V_z = g_1'(\phi(b,z,\theta)) + \theta g_2'(\theta\phi(b,z,\theta) + \psi(b,z,\theta))$. Using this, (27) and (28) gives

$$V_{zz}(b,z,\theta) \;=\; g_1''\phi_z + \theta g_2''(\theta\phi_z + \psi_z) < 0,$$

which combined with Lemma 3.2 produces the the desired results (i-a) and (i-b).

Step 2: We now proceed to prove result (i-c). It follows from Lemma 3.1 and Proposition 3.1 that the optimal debt policy is a solution to the equation

$$g_1'(\phi(b,z,\theta)) \;=\; (1+r-\theta)g_2'(\theta\phi(b,z,\theta) + \psi(b,z,\theta)) \tag{32}$$

for any $\theta \in \{\underline{\theta}, \bar{\theta}\}$. Differentiating both sides of equation (32) with respect to z and rearranging the algebra reveal that

$$[g_1''\phi_b - (1+r-\theta)g_2''(\theta\phi_b + \psi_b)]\frac{db}{dz} = [(1+r-\theta)\theta g_2'' - g_1'']\phi_z + (1+r-\theta)g_2''\psi_z.$$

Using (26) and (28) shows that $g_1''\phi_b - (1+r-\theta)g_2''(\theta\phi_b + \psi_b) < 0$. Differentiating both sides of equation (32) with respect to G_1 reveals that $(1+r-\theta)\theta g_2'' = g_1''$. Moreover, using (27) leads us to that $(1+r-\theta)g_2''\psi_z > 0$. In consequence, we must have $db/dz < 0$ for any $\theta \in \{\underline{\theta}, \bar{\theta}\}$. This combined with results (i-a) and (i-b) completes the proof. ■

Proof of Proposition 3.3. By (5) and applying the Envelope Theorem to (2), the first-order condition for incentive compatibility can be written as:

$$(g_1' + \theta g_2')\frac{dz}{d\theta} \;=\; [(1+r-\theta)g_2' - g_1']\frac{db}{d\theta},$$

by which we arrive at

$$\frac{dz}{db} \;=\; \frac{dz}{d\theta}\frac{d\theta}{db} \;=\; \frac{(1+r-\theta)g_2' - g_1'}{g_1' + \theta g_2'}. \tag{33}$$

It follows from (19) and (22) that

$$g_1' + \theta g_2' \;=\; \frac{\gamma}{\mu_1(\theta)} - \frac{\eta_1(\theta)}{\mu_1(\theta)f(\theta)}[g_2''(\theta\phi_z + \psi_z)\phi + g_2'\phi_z] \tag{34}$$

and

$$(1+r-\theta)g_2' - g_1' \;=\; \frac{\eta_1(\theta)}{\mu_1(\theta)f(\theta)}[g_2''(\theta\phi_b + \psi_b)\phi + g_2'\phi_b] \tag{35}$$

whenever there is no bunching. Plugging (34) and (35) in (33) results in

$$\frac{dz}{db} \;=\; \frac{\eta_1(\theta(b))[g_2''(\theta(b)\phi_b + \psi_b)\phi + g_2'\phi_b]}{\gamma f(\theta(b)) - \eta_1(\theta(b))[g_2''(\theta(b)\phi_z + \psi_z)\phi + g_2'\phi_z]}$$

in which $\theta(b)$ is the inverse of $b(\theta)$, which exists given that $\dot{b}(\theta) > 0$. As is obvious, dz/db satisfies the property required. ■

Proof of Proposition 4.1. We shall complete the proof in 5 steps.

Step 1: Applying the Envelope Theorem to the value function $V(b, z, \rho)$ and simplifying the algebra, we obtain the Spence-Mirrlees property:

$$\frac{\partial}{\partial\rho}\left[\frac{V_b(b,z,\rho)}{V_z(b,z,\rho)}\right] = -(1+r)\frac{\theta(g_2')^2 + g_1'[g_2' + \rho\mathcal{G}_2 g_2'']}{(g_1' + \theta g_2')^2} \;<\; 0$$

under Assumption 4.1. Noting that $V_z(\cdot) = g_1' + \theta g_2' > 0$, the second-order condition for incentive compatibility can be written as

$$\dot{b}(\rho) \cdot V_z(b(\rho), z(\rho), \rho) \cdot \frac{\partial}{\partial\tilde{\rho}}\left(\frac{V_b(b(\rho), z(\rho), \tilde{\rho})}{V_z(b(\rho), z(\rho), \tilde{\rho})}\right)\Bigg|_{\tilde{\rho}=\rho} \geq 0,$$

which leads to $\dot{b}(\rho) \leq 0$ under Assumption 4.1, as desired in (11). Let us equivalently rewrite this monotonicity constraint as $\dot{b}(\rho) = \beta(\rho)$ and $\beta(\rho) \leq 0$, then the Hamiltonian of the optimal control problem (11) is given by

$$\begin{aligned}\mathcal{H} \;&= v(\rho)f(\rho) + \mu_1(\rho)[V(b(\rho), z(\rho), \rho) - v(\rho)]f(\rho) - \mu_2(\rho)\beta(\rho) - \gamma z(\rho)f(\rho)\\ &\quad + \eta_1(\rho)g_2'(\theta\phi(b(\rho), z(\rho), \rho) + \rho\psi(b(\rho), z(\rho), \rho))\psi(b(\rho), z(\rho), \rho) + \eta_2(\rho)\beta(\rho),\end{aligned}$$

where $\phi(b(\rho), z(\rho), \rho) = G_1(\rho)$, $\psi(b(\rho), z(\rho), \rho) = \mathcal{G}_2(\rho)$, $\mu_1(\rho)$, $\mu_2(\rho)$ and γ are non-negative Lagrange multipliers, and $\eta_1(\rho)$ and $\eta_2(\rho)$ are co-state variables. The first-order necessary conditions are given by

$$\mathcal{H}_z \;=\; \mu_1(\rho)V_z(b(\rho), z(\rho), \rho)f(\rho) - \gamma f(\rho) + \eta_1(\rho)[g_2''(\theta\phi_z + \rho\psi_z)\psi + g_2'\psi_z] = 0, \tag{36}$$

$$\mathcal{H}_\beta \;=\; -\mu_2(\rho) + \eta_2(\rho) = 0, \tag{37}$$

and

$$\dot{\eta}_1(\rho) \;=\; -\mathcal{H}_v \;=\; [\mu_1(\rho) - 1]f(\rho), \tag{38}$$

$$\dot{\eta}_2(\rho) \;=\; -\mathcal{H}_b \;=\; -\mu_1(\rho)\{g_1' - [\rho(1+r) - \theta]g_2'\}f(\rho) - \eta_1(\rho)[g_2''(\theta\phi_b + \rho\psi_b)\psi + g_2'\psi_b]. \tag{39}$$

In addition, we have the following transversality conditions:

$$\eta_1(\rho) \;=\; \eta_2(\rho) \;=\; 0 \;\text{ for } \forall\rho \in \{\underline{\rho}, \bar{\rho}\}. \tag{40}$$

Step 2: Applying the Implicit Function Theorem to (8) gives rise to:

$$\phi_b \;=\; \frac{u_1''(u_2'' + \rho^2 g_2'') + \rho\theta(1+r)u_2''g_2''}{M} > 0,\;\; \phi_z \;=\; \frac{u_1''(u_1'' + \rho^2 g_2'')}{M} > 0; \tag{41}$$

and

$$\psi_b \;=\; -\frac{(1+r)[(u_1'' + g_1'')u_2'' + \theta^2 u_2''g_2''] + \rho\theta u_1''g_2''}{M} < 0,\quad \psi_z \;=\; -\frac{\rho\theta u_1''g_2''}{M} < 0 \tag{42}$$

in which $M \equiv (u_1'' + g_1'')(u_2'' + \rho^2 g_2'') + \theta^2 u_2''g_2'' > 0$. Making use of (41) and (42), we have

$$g_2''(\theta\phi_z + \rho\psi_z)\psi + g_2'\psi_z \;<\; 0 \tag{43}$$

given that

$$\theta\phi_z + \rho\psi_z \;=\; \frac{\theta u_1'' u_2''}{M} \;>\; 0. \tag{44}$$

In addition, we get by (41), (42) and

$$\theta\phi_b + \rho\psi_b \;=\; -\frac{[\rho(1+r)-\theta]u_1''u_2'' + \rho(1+r)u_2''g_1''}{M} \;<\; 0 \tag{45}$$

that

$$\begin{aligned} & g_2''(\theta\phi_b + \rho\psi_b)\psi + g_2'\psi_b \\ = \;\; & -\frac{[\rho\mathcal{G}_2 g_2'' + g_2'](1+r)(u_1''+g_1'')u_2'' - \theta\mathcal{G}_2 u_1''u_2''g_2'' + [(1+r)\theta u_2'' + \rho u_1'']\theta g_2' g_2''}{M} < 0 \end{aligned} \tag{46}$$

under Assumption 4.1.

Step 3: Since we focus on the case without bunching, we must have $\mu_2(\rho) = 0$ for all $\rho \in \Upsilon$. By (37), we have $\eta_2(\rho) = 0$ everywhere, implying that $\dot{\eta}_2 \equiv 0$. Applying this, (40) and (46) to (39) yields the desired assertion in part (i). Finally, applying (43), (40) and $\mu_1(\rho)f(\rho) > 0$ to (36) produces the desired assertion in part (ii) for the case of observable expenditure on the IPGs.

Step 4: As before, for the case of observable physical output of the IPGs, the Hamiltonian of the optimal control problem (12) is given by

$$\begin{aligned} \mathcal{H} \; = & \; v(\xi)f(\xi) + \mu_1(\xi)[V(b(\xi), z(\xi), \xi) - v(\xi)]f(\xi) - \mu_2(\xi)\beta(\xi) - \gamma z(\xi)f(\xi) \\ & - \eta_1(\xi)u_2'\left(y_2 - (1+r)b(\xi) - \xi\psi(b(\xi), z(\xi), \xi)\right)\psi(b(\xi), z(\xi), \xi) + \eta_2(\xi)\beta(\xi). \end{aligned}$$

The first-order necessary conditions are given by

$$\mathcal{H}_z \;=\; \mu_1(\xi)V_z(b(\xi), z(\xi), \xi)f(\xi) - \gamma f(\xi) - \eta_1(\xi)(-\xi u_2''\psi_z\psi + u_2'\psi_z) = 0, \tag{47}$$

$$\mathcal{H}_\beta \;=\; -\mu_2(\xi) + \eta_2(\xi) = 0, \tag{48}$$

and

$$\dot{\eta}_1(\xi) \;=\; -\mathcal{H}_v \;=\; [\mu_1(\xi) - 1]f(\xi), \tag{49}$$

$$\dot{\eta}_2(\xi) \;=\; -\mathcal{H}_b \;=\; -\mu_1(\xi)[u_1' - (1+r)u_2']f(\xi) + \eta_1(\xi)[-(1+r)u_2''\psi - \xi u_2''\psi_b\psi + u_2'\psi_b]. \tag{50}$$

In addition, we have the following transversality conditions:

$$\eta_1(\xi) \;=\; \eta_2(\xi) \;=\; 0 \;\; \text{for } \forall\xi \in \{\underline{\xi}, \bar{\xi}\}. \tag{51}$$

Step 5: Applying the Implicit Function Theorem to (10) gives rise to:

$$\phi_b \;=\; \frac{u_1''(\xi^2 u_2'' + g_2'') + \xi\theta(1+r)u_2''g_2''}{Q} > 0, \;\; \phi_z \;=\; \frac{u_1''(\xi^2 u_2'' + g_2'')}{Q} > 0; \tag{52}$$

and

$$\psi_b \;=\; -\frac{\xi(1+r)(u_1'' + g_1'' + \theta^2 g_2'')u_2'' + \theta u_1'' g_2''}{Q} < 0, \;\;\; \psi_z \;=\; -\frac{\theta u_1'' g_2''}{Q} < 0 \tag{53}$$

in which $Q \equiv (u_1'' + g_1'')(\xi^2 u_2'' + g_2'') + \xi^2\theta^2 u_2'' g_2'' > 0$. Now, applying (51) and (53) to (47) gives the desired assertion in part (ii). Moreover, using (53) again reveals that

$$\begin{aligned} & -(1+r)u_2''\psi - \xi u_2''\psi_b\psi + u_2'\psi_b \;= \\ & -\frac{(1+r)(u_1''+g_1'')u_2''(g_2''\psi + g_2') + \theta u_1''g_2''(u_2' - \xi\psi u_2'') + \theta^2(1+r)u_1''u_2''g_2'}{Q} < 0 \end{aligned} \tag{54}$$

under Assumption 4.2. In the case of no bunching, applying (48), (51) and (54) to (50) produces part (i) for the case of observable output of the IPGs. ■

Proof of Proposition 4.2. We shall complete the proof in 2 steps.

Step 1: Using (38), the proof is quite similar to that of Proposition 3.2. Here we just need to show the following for the case of observable expenditure on the IPGs. Firstly, using (41) and (44) reveals that $V_{zz} = g_1''\phi_z + \theta g_2''(\theta\phi_z + \rho\psi_z) < 0$ for all $\rho \in \Upsilon$. Secondly, by differentiating both sides of equation $g_1' = [\rho(1+r) - \theta]g_2'$ with respect to z, we obtain

$$\underbrace{[g_1''\phi_b - [\rho(1+r) - \theta]g_2''(\theta\phi_b + \rho\psi_b)]}_{<0}\frac{db}{dz} \;=\; [\rho(1+r) - \theta]g_2''(\theta\phi_z + \rho\psi_z) - g_1''\phi_z$$

under (41) and (45). As we get from (41) and (44) that

$$[\rho(1+r) - \theta]g_2''(\theta\phi_z + \rho\psi_z) - g_1''\phi_z \;=\; -\frac{\rho^2 u_1'' g_1'' g_2''}{M} > 0,$$

we thus have $db/dz < 0$ at the welfare optimum.

Step 2: For the case of observable output of the IPGs, the proof follows from using (49), (51), Lemma 4.1 and Proposition 4.1. Here, we just give by using (52), (53) and $u_1' = (1+r)u_2'$ evaluated at the welfare optimum that

$$V_{zz} = u_1''(1-\phi_z) = \frac{u_1'' g_1''(\xi^2 u_2'' + g_2'') + \theta^2\xi^2 u_1'' u_2'' g_2''}{Q} < 0$$

and

$$[(1-\phi_b)u_1'' + (1+r)(1+r+\xi\psi_b)u_2'']\frac{db}{dz} \;=\; -\xi(1+r)u_2''\psi_z - (1-\phi_z)u_1''$$

in which

$$\begin{aligned} &(1-\phi_b)u_1'' + (1+r)(1+r+\xi\psi_b)u_2'' \\ =\;& \frac{u_1'' g_1''(\xi^2 u_2'' + g_2'') + (1+r-\theta\xi)^2 u_1'' u_2'' g_2'' + (1+r)^2 u_2'' g_1'' g_2''}{Q} < 0 \end{aligned}$$

and

$$\begin{aligned} &-\xi(1+r)u_2''\psi_z - (1-\phi_z)u_1'' \\ =\;& -\frac{\xi u_1'' u_2''[\xi g_1'' - (1+r)\theta g_2''] + \theta^2\xi^2 u_1'' u_2'' g_2'' + u_1'' g_1'' g_2''}{Q} > 0 \end{aligned}$$

whenever $g_1'' \leq \rho\theta(1+r)g_2''$ holds. ■

Proof of Proposition 4.3. We shall complete the proof in 2 steps.

Step 1: The key for a grant scheme to decentralize the asymmetric-information optimum is that it takes into account the incentive-compatibility constraint. First, making use of the first-order necessary condition for incentive compatibility, we get

$$\frac{dz}{db} = \frac{dz}{d\rho}\frac{d\rho}{db} = -\frac{V_b}{V_z}.$$

As the monotonicity constraint is assumed to be not binding, we get from (36) and (39) that

$$\frac{dz}{db} = \frac{\eta_1(\rho(b))\left[g_2''(\theta\phi_b + \rho(b)\psi_b)\psi + g_2'\psi_b\right]}{\gamma f(\rho(b)) - \eta_1(\rho(b))\left[g_2''(\theta\phi_z + \rho(b)\psi_z)\psi + g_2'\psi_z\right]},$$

in which $\rho(b)$ denotes the inverse of $b(\rho)$. Secondly, making use of (40), (43) and (46), the proof of part (i) is immediately complete.

Step 2: As we focus on the case of no bunching, we have by using the first-order necessary condition for incentive compatibility, (47) and (50) that

$$\frac{dz^*}{db} = \frac{-\eta_1(\xi(b))[-(1+r)u_2''\psi - \xi(b)u_2''\psi_b\psi + u_2'\psi_b]}{\gamma f(\xi(b)) + \eta_1(\xi(b))[-\xi(b)u_2''\psi_z\psi + u_2'\psi_z]},$$

in which $\xi(b)$ denotes the inverse of $b(\xi)$ under the assumption of $\dot{b}(\xi) < 0$. By using (54) and (51), we see that dz^*/db satisfies the property required in part (ii). ■

Appendix B: Discussion on Multidimensional Heterogeneity

Following Lockwood (1999), in the main text we focus on the analysis of one-dimensional unobserved heterogeneity, which is either the degree of intergenerational externality induced by IPGs, denoted by parameter θ, or the degree of technological progress for producing the IPGs, denoted by parameter ρ (or equivalently ξ). Here we attempt to analyze the case with multidimensional heterogeneity, i.e., regions differ in both the degree of intergenerational externality and the degree of technological progress. Nevertheless, for the sake of obtaining some interesting theoretical results, we, as in Dai and Tian (2020), need to impose the following restriction:

Assumption 5.1 *Let $\xi \equiv \Psi(\theta)$ and $\rho = 1/\xi = 1/\Psi(\theta) \equiv \Phi(\theta)$, in which $\Psi(\cdot)$ is a continuously differentiable function satisfying $\Psi'(\cdot) > 0$.*

Since both the degree of intergenerational externality and the degree of technological progress are closely related to the IPGs, by Assumption 5.1 we mean that there is a publicly observable functional relationship that governs these two parameters. In particular, $\Psi'(\theta) > 0$ means that a higher degree of intergenerational externality induced by IPGs leads to a higher per unit cost of producing the IPGs. Intuitively, we assume that if a public good is of higher quality, durability or intergenerational spillovers, then the per unit cost of producing it tends to be higher. For example, a study of the construction costs of high-speed railways in China by Ollivier, Bullock, Ying, and Zhou (2014) shows that the weighted average unit cost for a passenger-dedicated line was RMB 129 million per km for a 350 km/h project and RMB 87 million per km for a 250 km/h project. As we could reasonably expect that more and more passengers in the future are willing to take trains of higher speeds, we might roughly interpret that a 350 km/h project generates higher intergenerational spillovers than a 250 km/h project, somehow justifying Assumption 5.1. Also, Assumption 5.1 helps us to consider the case with multidimensional heterogeneity but with one-dimensional asymmetric information.

As in Section 4, whenever regions face shocks to the degree of technological progress for producing public goods, we need to distinguish the case with observable expenditure on public goods to the case with observable physical output of public goods.

I. The Case with Observable Expenditure on Public Goods

Applying Assumption 5.1, the FOCs given by equation (8) can be rewritten as:

$$u_1'(c_1) = g_1'(G_1) + \theta g_2'(\theta G_1 + \Phi(\theta)\mathcal{G}_2) \quad \text{and} \quad u_2'(c_2) = \Phi(\theta) g_2'(\theta G_1 + \Phi(\theta)\mathcal{G}_2). \tag{55}$$

As in the main text, let the value function be written as $v(\theta) \equiv V(b(\theta), z(\theta), \theta)$. Applying Assumption 5.1 and the Envelope Theorem to (2) produces the following first-order necessary condition for incentive compatibility:

$$\begin{aligned} \dot{v}(\theta) &= g_2' \left(\theta \phi(b(\theta), z(\theta), \theta) + \Phi(\theta)\psi(b(\theta), z(\theta), \theta)\right) \\ &\quad \times \left[\phi(b(\theta), z(\theta), \theta) + \Phi'(\theta)\psi(b(\theta), z(\theta), \theta)\right], \end{aligned} \tag{56}$$

in which $G_1(\theta) \equiv \phi(b(\theta), z(\theta), \theta)$ and $\mathcal{G}_2(\theta) \equiv \psi(b(\theta), z(\theta), \theta)$. Applying the Envelope Theorem to the value function $V(b, z, \theta)$ again gives us $V_b(b, z, \theta) = g_1' + g_2' \cdot [\theta - \Phi(\theta)(1+r)]$ and $V_z(b, z, \theta) = g_1' + \theta g_2' > 0$ for all $\theta \in \Theta$, by which we can obtain:

Lemma 5.1 *Under Assumption 5.1, if $G_1(\theta)/\mathcal{G}_2(\theta) \geq -\Phi'(\theta)$, then the global optimality of truth-telling strategy is guaranteed by the second-order condition $\dot{b}(\theta) \geq 0$ for all $\theta \in \Theta$.*

Proof. Since by equation (2) and Assumption 5.1 we get

$$\begin{aligned} \frac{\partial}{\partial \theta}\left[\frac{V_b(b, z, \theta)}{V_z(b, z, \theta)}\right] &= \frac{1+r}{(g_1' + \theta g_2')^2} \\ &\times \left\{ \underbrace{\left[\Phi(\theta) - \theta\Phi'(\theta)\right](g_2')^2 - \Phi'(\theta) g_1' g_2'}_{>0} - \underbrace{\Phi(\theta) g_1' g_2''}_{<0} \left[G_1 + \Phi'(\theta)\mathcal{G}_2\right] \right\}, \end{aligned}$$

we thus have

$$\frac{\partial}{\partial \theta}\left[\frac{V_b(b, z, \theta)}{V_z(b, z, \theta)}\right] > 0 \tag{57}$$

whenever $G_1 + \Phi'(\theta)\mathcal{G}_2 \geq 0$. Condition (57) thus guarantees the Spence-Mirrlees property. The second-order condition for incentive compatibility can be expressed as:

$$\dot{b}(\theta) \cdot V_z(b(\theta), z(\theta), \theta) \cdot \frac{\partial}{\partial \tilde{\theta}} \left(\frac{V_b(b(\theta), z(\theta), \tilde{\theta})}{V_z(b(\theta), z(\theta), \tilde{\theta})} \right) \Bigg|_{\tilde{\theta} = \theta} \geq 0,$$

which combined with the fact $V_z > 0$ and Spence-Mirrlees property (57) reveals that $\dot{b}(\theta) \geq 0$ must hold. Applying the standard argument given on page 143 of Laffont and Martimort (2002), the proof is then complete. ■

Lemma 5.1 states that truth-telling calls for a regional debt allocation which is non-decreasing in the degree of intergenerational externality. Condition $G_1(\theta)/\mathcal{G}_2(\theta) \geq -\Phi'(\theta)$ means that the ratio of period-1 public goods expenditure to period-2 public goods expenditure is greater than some lower bound. For later use, we give

Assumption 5.2 $G_1(\theta)/\mathcal{G}_2(\theta) \geq -\Phi'(\theta)$ *for all* $\theta \in \Theta$.

Now, by safely replacing the global incentive-compatibility condition (5) by (56) and $\dot{b}(\theta) \geq 0$ established in Lemma 5.1, the optimization problem facing the center is formalized as:

$$\max \int_{\underline{\theta}}^{\bar{\theta}} v(\theta) f(\theta) d\theta$$

$$\begin{aligned} s.t. \quad v(\theta) &= V(b(\theta), z(\theta), \theta); \\ \int_{\underline{\theta}}^{\bar{\theta}} z(\theta) f(\theta) d\theta &\leq 0; \\ \dot{v}(\theta) &= g_2' \left(\theta \phi(b(\theta), z(\theta), \theta) + \Phi(\theta) \psi(b(\theta), z(\theta), \theta)\right) \\ &\quad \times \left[\phi(b(\theta), z(\theta), \theta) + \Phi'(\theta) \psi(b(\theta), z(\theta), \theta)\right]; \\ \dot{b}(\theta) &\geq 0. \end{aligned}$$

By solving this problem we arrive at the following proposition:

Proposition 5.1 *Suppose Assumptions 5.1 and 5.2 hold. In the asymmetric-information case without bunching, the welfare optimum $\{b^*(\theta), z^*(\theta)\}_{\theta \in \Theta}$ satisfies:*

(i) Concerning the relationship between the intertemporal rate of substitution between current and future public goods consumption and the intertemporal rate of transformation, we have:

$$\frac{g_1'(G_1^*(\theta))}{g_2'(\theta G_1^*(\theta) + \Phi(\theta) \mathcal{G}_2^*(\theta))} \begin{cases} = \ \Phi(\theta)(1+r) - \theta & \text{for } \theta \in \{\underline{\theta}, \bar{\theta}\}; \\ < \ \Phi(\theta)(1+r) - \theta & \text{for } \theta \in (\underline{\theta}, \bar{\theta}). \end{cases}$$

(ii) Let $\mu_1(\theta) > 0$ be the Lagrange multiplier on the value constraint $v(\theta) \equiv V(b(\theta), z(\theta), \theta)$ of any type-θ region who is reporting truthfully, we have:

- *$V_z(b^*(\theta), z^*(\theta), \theta) = \gamma/\mu_1(\theta)$ for $\theta \in \{\underline{\theta}, \bar{\theta}\}$;*
- *If the ratio $G_1(\theta)/\mathcal{G}_2(\theta)$ is sufficiently close to $-\Phi'(\theta)$, then*

  $$V_z(b^*(\theta), z^*(\theta), \theta) < \gamma/\mu_1(\theta)$$

  for $\theta \in (\underline{\theta}, \bar{\theta})$;
- *If the ratio $G_1(\theta)/\mathcal{G}_2(\theta)$ is sufficiently larger than $-\Phi'(\theta)$, then we have for any $\theta \in (\underline{\theta}, \bar{\theta})$ that:*

  $$V_z(b^*(\theta), z^*(\theta), \theta) \begin{cases} < \ \gamma/\mu_1(\theta) & \text{for } |\varepsilon_{g_2', \theta\phi + \Phi(\theta)\psi}| \cdot \varepsilon_{\theta\phi + \Phi(\theta)\psi, z} < \varepsilon_{\phi + \Phi'(\theta)\psi, z}, \\ = \ \gamma/\mu_1(\theta) & \text{for } |\varepsilon_{g_2', \theta\phi + \Phi(\theta)\psi}| \cdot \varepsilon_{\theta\phi + \Phi(\theta)\psi, z} = \varepsilon_{\phi + \Phi'(\theta)\psi, z}, \\ > \ \gamma/\mu_1(\theta) & \text{for } |\varepsilon_{g_2', \theta\phi + \Phi(\theta)\psi}| \cdot \varepsilon_{\theta\phi + \Phi(\theta)\psi, z} > \varepsilon_{\phi + \Phi'(\theta)\psi, z}, \end{cases}$$

  in which $|\varepsilon_{g_2', \theta\phi + \Phi(\theta)\psi}|$ represents the absolute value of the elasticity of g_2' with respect to the amount of period-2 public goods consumption $\theta\phi + \Phi(\theta)\psi$, $\varepsilon_{\theta\phi + \Phi(\theta)\psi, z} > 0$ represents the elasticity of the amount of period-2 public goods consumption with respect to the federal transfers z, and $\varepsilon_{\phi + \Phi'(\theta)\psi, z} > 0$ represents the elasticity of $\phi + \Phi'(\theta)\psi$ with respect to z.

Proof. We shall complete the proof in 4 steps.

Step 1: First, we let $\dot{b}(\theta) \equiv \beta(\theta)$ as before, and write the generalized Hamiltonian as:

$$\begin{aligned}\mathcal{H} &= v(\theta)f(\theta) + \mu_1(\theta)[V(b(\theta), z(\theta), \theta) - v(\theta)]f(\theta) + \mu_2(\theta)\beta(\theta) - \gamma z(\theta)f(\theta) \\ &\quad + \eta_1(\theta)g_2'(\theta\phi(b(\theta), z(\theta), \theta) + \Phi(\theta)\psi(b(\theta), z(\theta), \theta)) \\ &\quad \times [\phi(b(\theta), z(\theta), \theta) + \Phi'(\theta)\psi(b(\theta), z(\theta), \theta)] + \eta_2(\theta)\beta(\theta),\end{aligned}$$

where $\mu_1(\theta)$, $\mu_2(\theta)$ and γ are non-negative Lagrange multipliers, and $\eta_1(\theta)$ and $\eta_2(\theta)$ are co-state variables. The first-order necessary conditions are

$$\begin{aligned}\mathcal{H}_z &= \mu_1(\theta)V_z(b(\theta), z(\theta), \theta)f(\theta) - \gamma f(\theta) \\ &\quad + \eta_1(\theta)\{g_2'' \cdot [\theta\phi_z + \Phi(\theta)\psi_z][\phi + \Phi'(\theta)\psi] + g_2' \cdot [\phi_z + \Phi'(\theta)\psi_z]\} = 0,\end{aligned} \tag{58}$$

$$\mathcal{H}_\beta = \mu_2(\theta) + \eta_2(\theta) = 0, \tag{59}$$

$$\dot{\eta}_1(\theta) = -\mathcal{H}_v = [\mu_1(\theta) - 1]f(\theta), \tag{60}$$

and

$$\begin{aligned}\dot{\eta}_2(\theta) &= -\mathcal{H}_b \\ &= -\mu_1(\theta)\{g_1' - [\Phi(\theta)(1+r) - \theta]g_2'\}f(\theta) \\ &\quad - \eta_1(\theta)\{g_2'' \cdot [\theta\phi_b + \Phi(\theta)\psi_b][\phi + \Phi'(\theta)\psi] + g_2' \cdot [\phi_b + \Phi'(\theta)\psi_b]\}.\end{aligned} \tag{61}$$

In addition, we have the following transversality conditions:

$$\eta_1(\theta) = \eta_2(\theta) = 0 \text{ for } \forall\theta \in \{\underline{\theta}, \bar{\theta}\}. \tag{62}$$

Step 2: Following the first-order approach, we must have $\mu_2(\theta) = 0$ for all $\theta \in \Theta$. By (59) we thus have $\eta_2(\theta) = 0$ for all $\theta \in \Theta$, and hence we must have $\dot{\eta}_2 \equiv 0$. Applying $\dot{\eta}_2 \equiv 0$, $\mu_1(\theta)f(\theta) > 0$ and (62) to (61) reveals that $g_1' = [\Phi(\theta)(1+r) - \theta]g_2'$ for $\theta \in \{\underline{\theta}, \bar{\theta}\}$, as desired in part (i). Also, applying (62) to (58) reveals that $\mu_1(\theta)V_z(b(\theta), z(\theta), \theta) = \gamma$ for $\theta \in \{\underline{\theta}, \bar{\theta}\}$, as desired in part (ii).

Step 3: By applying the Implicit Function Theorem to the FOCs given by (55), we can still have those partial derivatives given by equations (41) and (42). In consequence, we get by (41), (42), (45) and Assumption 5.2 that

$$\underbrace{g_2''}_{<0} \cdot \underbrace{[\theta\phi_b + \Phi(\theta)\psi_b]}_{<0} \cdot \underbrace{[\phi + \Phi'(\theta)\psi]}_{\geq 0} + \underbrace{g_2'}_{>0} \cdot \underbrace{[\phi_b + \Phi'(\theta)\psi_b]}_{>0} > 0, \tag{63}$$

which combined with $\dot{\eta}_2 \equiv 0$, $\mu_1(\theta)f(\theta) > 0$ and (61) concludes the proof of part (i).

Step 4: Finally, using (41), (42), (44) and Assumption 5.2 shows that

$$\underbrace{g_2'' \cdot [\theta\phi_z + \Phi(\theta)\psi_z]}_{<0} \cdot \underbrace{[\phi + \Phi'(\theta)\psi]}_{\geq 0} + \underbrace{g_2' \cdot [\phi_z + \Phi'(\theta)\psi_z]}_{>0}.$$

Thus, the sign of this formula is positive whenever $\phi + \Phi'(\theta)\psi$ is sufficiently close to zero from above, applying which to equation (58) gives the second result in part (ii). If, however, $\phi + \Phi'(\theta)\psi$ is sufficiently larger than zero, then we have by using (41), (42), (44) and Assumption 5.2 again that:

$$\begin{aligned}&\underbrace{g_2'' \cdot [\theta\phi_z + \Phi(\theta)\psi_z]}_{-} \cdot \underbrace{[\phi + \Phi'(\theta)\psi]}_{+} + \underbrace{g_2' \cdot [\phi_z + \Phi'(\theta)\psi_z]}_{+} < 0 \\ \Leftrightarrow\ &\underbrace{-\frac{[\theta\phi + \Phi(\theta)\psi]g_2''}{g_2'}}_{|\varepsilon_{g_2', \theta\phi+\Phi(\theta)\psi}|} \cdot \underbrace{\frac{z[\theta\phi_z + \Phi(\theta)\psi_z]}{\theta\phi + \Phi(\theta)\psi}}_{\varepsilon_{\theta\phi+\Phi(\theta)\psi, z}} > \underbrace{\frac{z[\phi_z + \Phi'(\theta)\psi_z]}{\phi + \Phi'(\theta)\psi}}_{\varepsilon_{\phi+\Phi'(\theta)\psi, z}}\end{aligned}$$

applying which to equation (58), therefore, concludes the proof of part (ii). ∎

The key message conveyed by Proposition 5.1 can be summarized as follows. First, in the asymmetric-information optimum only the intertemporal allocation at the endpoints of type distribution is not distorted with respect to the first-best. Second, the presence of the asymmetric information between center and regions prevents full insurance from happening.

We now proceed to the implementation of the asymmetric-information optimum established in Proposition 5.1 through decentralized regional debt decisions, which lead to the intertemporal allocation features that intertemporal rate of substitution between current and future public goods consumption equals intertemporal rate of transformation:

$$\frac{g_1'\left(\phi(b(\theta), z(\theta), \theta)\right)}{g_2'\left(\theta\phi(b(\theta), z(\theta), \theta) + \Phi(\theta)\psi(b(\theta), z(\theta), \theta)\right)} = \Phi(\theta)(1+r) - \theta,$$

which is desired by each region for any given amount of federal transfers. The task facing the center is to design intergovernmental grants scheme that guarantees incentive compatibility for all regions. Indeed, we can obtain the following result:

Proposition 5.2 *The grant scheme $z^*(b)$ that decentralizes the asymmetric-information optimum $\{b^*(\theta), z^*(\theta)\}_{\theta\in\Theta}$ is a nonlinear nondecreasing function of b, almost everywhere differentiable, with the slope*

$$\frac{dz^*}{db} \begin{cases} = 0 & \text{for } b \in \{b^*(\underline{\theta}), b^*(\bar{\theta})\}; \\ > 0 & \text{for } b \in (b^*(\underline{\theta}), b^*(\bar{\theta})). \end{cases}$$

Proof. Making use of the first-order necessary condition for incentive compatibility, we get

$$\frac{dz}{db} = \frac{dz}{d\theta}\frac{d\theta}{db} = -\frac{V_b}{V_z},$$

in which we have shown above that $V_z > 0$ always holds true. As we focus on the case without bunching, we get from (61) and (63) that

$$\begin{aligned} & - V_b\left(b(\theta), z(\theta), \theta\right) \\ & = \frac{\eta_1(\theta)}{\mu_1(\theta)f(\theta)}\{g_2'' \cdot [\theta\phi_b + \Phi(\theta)\psi_b][\phi + \Phi'(\theta)\psi] + g_2' \cdot [\phi_b + \Phi'(\theta)\psi_b]\} > 0 \end{aligned}$$

for all $\theta \in (\underline{\theta}, \bar{\theta})$. We thus have for all $b \in (b^*(\underline{\theta}), b^*(\bar{\theta}))$ that:

$$\frac{dz}{db} = \frac{\eta_1(\theta(b))\{g_2'' \cdot [\theta(b)\phi_b + \Phi(\theta(b))\psi_b][\phi + \Phi'(\theta(b))\psi] + g_2' \cdot [\phi_b + \Phi'(\theta(b))\psi_b]\}}{\mu_1(\theta(b))f(\theta(b))V_z} > 0$$

in which by a little abuse of notation $\theta(b)$ denotes the inverse of $b(\theta)$, which does exist by Lemma 5.1. Finally, applying (62) immediately completes the proof. ∎

The main message of Proposition 5.2 is that federal transfers and local debt in general play a complementary role for regional insurance provision. Except for the bottom and top types whose intertemporal allocations are not distorted in the asymmetric-information optimum, the optimal funding structure in the case with observable expenditure on IPGs exhibits the following feature: regions of a higher degree of intergenerational externality should issue more debt and receive more federal transfers than regions of a lower degree of intergenerational externality.

II. The Case with Observable Physical Output of Public Goods

Using Assumption 5.1, the value function given by (9) can be rewritten as:

$$V(b,z,\theta) \equiv \max_{G_1,G_2} \; u_1(y_1+b+z-G_1)+g_1(G_1) \\ \qquad + u_2(y_2-b(1+r)-\Psi(\theta)G_2)+g_2(\theta G_1+G_2). \tag{64}$$

The first-order conditions are thus given by:

$$\begin{aligned} u_1'(y_1+b+z-G_1) &= g_1'(G_1)+\theta g_2'(\theta G_1+G_2) \;\text{ and} \\ \Psi(\theta)u_2'(y_2-b(1+r)-\Psi(\theta)G_2) &= g_2'(\theta G_1+G_2). \end{aligned} \tag{65}$$

Let $v(\theta) \equiv V(b(\theta),z(\theta),\theta)$. Applying the Envelope Theorem to (64) produces the following first-order necessary condition for incentive compatibility:

$$\begin{aligned} \dot{v}(\theta) &= -u_2'\left(y_2-b(\theta)(1+r)-\Psi(\theta)\psi(b(\theta),z(\theta),\theta)\right)\Psi'(\theta)\psi(b(\theta),z(\theta),\theta) \\ &\quad + g_2'\left(\theta\phi(b(\theta),z(\theta),\theta)+\psi(b(\theta),z(\theta),\theta)\right)\phi(b(\theta),z(\theta),\theta), \end{aligned} \tag{66}$$

in which $G_1(\theta) \equiv \phi(b(\theta),z(\theta),\theta)$ and $G_2(\theta) \equiv \psi(b(\theta),z(\theta),\theta)$.

Similar to Lemma 5.1, we now arrive at:

Lemma 5.2 *Under Assumption 5.1, then the global optimality of truth-telling strategy is guaranteed by the second-order condition $\dot{b}(\theta) \le 0$ for all $\theta \in \Theta$.*

Proof. Applying again the Envelope Theorem to the value function (64) gives us $V_b(b,z,\theta) = u_1' - (1+r)u_2'$ and $V_z(b,z,\theta) = u_1' > 0$ for all $\theta \in \Theta$. Under Assumption 5.1, we get

$$\frac{\partial}{\partial\theta}\left[\frac{V_b(b,z,\theta)}{V_z(b,z,\theta)}\right] = (1+r)\frac{u_2''\Psi'(\theta)G_2}{u_1'} < 0, \tag{67}$$

which thus guarantees the Spence-Mirrlees property. The second-order condition for incentive compatibility can be expressed as:

$$\dot{b}(\theta)\cdot V_z(b(\theta),z(\theta),\theta)\cdot\frac{\partial}{\partial\tilde{\theta}}\left(\frac{V_b(b(\theta),z(\theta),\tilde{\theta})}{V_z(b(\theta),z(\theta),\tilde{\theta})}\right)\Bigg|_{\tilde{\theta}=\theta} \ge 0,$$

which combined with (67) reveals that $\dot{b}(\theta) \le 0$ must hold. ■

Now, exploiting (66) and Lemma 5.2, the optimization problem facing the center is formalized as follows:

$$\max \int_{\underline{\theta}}^{\bar{\theta}} v(\theta)f(\theta)d\theta$$

$$\begin{aligned} s.t. \quad v(\theta) &= V(b(\theta),z(\theta),\theta); \\ \int_{\underline{\theta}}^{\bar{\theta}} z(\theta)f(\theta)d\theta &\le 0; \\ \dot{v}(\theta) &= -u_2'\left(y_2-b(\theta)(1+r)-\Psi(\theta)\psi(b(\theta),z(\theta),\theta)\right)\Psi'(\theta)\psi(b(\theta),z(\theta),\theta) \\ &\quad + g_2'\left(\theta\phi(b(\theta),z(\theta),\theta)+\psi(b(\theta),z(\theta),\theta)\right)\phi(b(\theta),z(\theta),\theta); \\ \dot{b}(\theta) &\le 0. \end{aligned}$$

By solving this problem we arrive at the following proposition:

Proposition 5.3 *Suppose Assumption 5.1 holds. In the asymmetric-information case without bunching, the welfare optimum* $\{b^*(\theta), z^*(\theta)\}_{\theta\in\Theta}$ *satisfies:*

(i) Suppose Assumption 4.2 holds. Concerning the relationship between the intertemporal rate of substitution between current and future public goods consumption and the intertemporal rate of transformation, we have:

$$\frac{u_1'(c_1^*(\theta))}{u_2'(c_2^*(\theta))} \begin{cases} = & 1+r \quad \text{for } \theta \in \{\underline{\theta}, \bar{\theta}\}; \\ < & 1+r \quad \text{for } \theta \in (\underline{\theta}, \bar{\theta}). \end{cases}$$

(ii) Let $\mu_1(\theta) > 0$ *be the Lagrange multiplier on the value constraint* $v(\theta) \equiv V(b(\theta), z(\theta), \theta)$ *of any type-*θ *region who is reporting truthfully, we have:*

- $V_z(b^*(\theta), z^*(\theta), \theta) = \gamma/\mu_1(\theta)$ *for* $\theta \in \{\underline{\theta}, \bar{\theta}\}$;
- *If* $|\varepsilon_{g_2', \theta G_1}| \leq 1$, *in which the elasticity is given by* $\varepsilon_{g_2', \theta G_1} \equiv g_2'' \cdot \theta\phi/g_2'$, *then*
$$V_z(b^*(\theta), z^*(\theta), \theta) < \gamma/\mu_1(\theta)$$
for $\theta \in (\underline{\theta}, \bar{\theta})$.

Proof. We shall complete the proof in 4 steps.

Step 1: First, we let $\dot{b}(\theta) \equiv \beta(\theta)$ as before, and write the generalized Hamiltonian as:

$$\begin{aligned} \mathcal{H} &= v(\theta)f(\theta) + \mu_1(\theta)[V(b(\theta), z(\theta), \theta) - v(\theta)]f(\theta) - \mu_2(\theta)\beta(\theta) - \gamma z(\theta)f(\theta) \\ &\quad - \eta_1(\theta)u_2'\left(y_2 - b(\theta)(1+r) - \Psi(\theta)\psi(b(\theta), z(\theta), \theta)\right)\Psi'(\theta)\psi(b(\theta), z(\theta), \theta) \\ &\quad + \eta_1(\theta)g_2'\left(\theta\phi(b(\theta), z(\theta), \theta) + \psi(b(\theta), z(\theta), \theta)\right)\phi(b(\theta), z(\theta), \theta) + \eta_2(\theta)\beta(\theta), \end{aligned}$$

where $\mu_1(\theta)$, $\mu_2(\theta)$ and γ are non-negative Lagrange multipliers, and $\eta_1(\theta)$ and $\eta_2(\theta)$ are co-state variables. The first-order necessary conditions are

$$\begin{aligned} \mathcal{H}_z &= \mu_1(\theta)V_z(b(\theta), z(\theta), \theta)f(\theta) - \gamma f(\theta) \\ &\quad + \eta_1(\theta)\left[u_2'' \cdot \Psi(\theta)\Psi'(\theta)\psi\psi_z - u_2' \cdot \Psi'(\theta)\psi_z\right] \\ &\quad + \eta_1(\theta)\left[g_2'' \cdot (\theta\phi_z + \psi_z)\phi + g_2' \cdot \phi_z\right] = 0, \end{aligned} \tag{68}$$

$$\mathcal{H}_\beta = -\mu_2(\theta) + \eta_2(\theta) = 0, \tag{69}$$

$$\dot{\eta}_1(\theta) = -\mathcal{H}_v = [\mu_1(\theta) - 1]f(\theta), \tag{70}$$

and

$$\begin{aligned} \dot{\eta}_2(\theta) &= -\mathcal{H}_b \\ &= -\mu_1(\theta)[u_1' - (1+r)u_2']f(\theta) \\ &\quad + \eta_1(\theta)\{-u_2'' \cdot [1 + r + \Psi(\theta)\psi_b]\Psi'(\theta)\psi + u_2' \cdot \Psi'(\theta)\psi_b\} \\ &\quad - \eta_1(\theta)[g_2'' \cdot (\theta\phi_b + \psi_b)\phi + g_2' \cdot \phi_b]. \end{aligned} \tag{71}$$

In addition, we have the following transversality conditions:

$$\eta_1(\theta) = \eta_2(\theta) = 0 \quad \text{for } \forall\theta \in \{\underline{\theta}, \bar{\theta}\}. \tag{72}$$

Step 2: Following the first-order approach, we must have $\mu_2(\theta) = 0$ for all $\theta \in \Theta$. By (69) we thus have $\eta_2(\theta) = 0$ for all $\theta \in \Theta$, and hence we must have $\dot{\eta}_2 \equiv 0$. Applying

$\dot{\eta}_2 \equiv 0$, $\mu_1(\theta)f(\theta) > 0$ and (72) to (71) reveals that $u_1' = (1+r)u_2'$ for $\theta \in \{\underline{\theta}, \bar{\theta}\}$, as desired in part (i). Also, applying (72) to (68) reveals that $\mu_1(\theta)V_z(b(\theta), z(\theta), \theta) = \gamma$ for $\theta \in \{\underline{\theta}, \bar{\theta}\}$, as desired in part (ii).

Step 3: By applying the Implicit Function Theorem to the FOCs given by (65), we can still have those partial derivatives given by equations (52) and (53). In consequence, we get by (52) and (53) that

$$\theta\phi_b + \psi_b = \frac{\{[\theta\Psi(\theta) - (1+r)]u_1'' - (1+r)g_1''\}\Psi(\theta)u_2''}{Q} < 0 \tag{73}$$

and

$$1 + r + \Psi(\theta)\psi_b = \frac{[1 + r - \theta\Psi(\theta)]u_1''g_1'' + (1+r)g_1''g_2''}{Q} > 0. \tag{74}$$

Applying $\dot{\eta}_2 \equiv 0$, Assumption 5.1, (52), (53), (73) and (74) to (71) yields that

$$\begin{aligned} & \mu_1(\theta)[u_1' - (1+r)u_2']f(\theta) \\ = \ & \eta_1(\theta)\{\underbrace{-u_2'' \cdot [1 + r + \Psi(\theta)\psi_b]\Psi'(\theta)\psi}_{+} + \underbrace{u_2' \cdot \Psi'(\theta)\psi_b}_{-}\} \\ & - \eta_1(\theta)\underbrace{[g_2'' \cdot (\theta\phi_b + \psi_b)\phi + g_2' \cdot \phi_b]}_{+}. \end{aligned} \tag{75}$$

As such, applying $\mu_1(\theta)f(\theta) > 0$, (54) (which uses Assumption 4.2), and Assumption 5.1 to (75) concludes the proof of part (i).

Step 4: In addition, applying (52), (53) and Assumption 5.1 to (68) gives rise to

$$\begin{aligned} & [\mu_1(\theta)V_z(b(\theta), z(\theta), \theta) - \gamma]\, f(\theta) \\ = \ & -\eta_1(\theta)\underbrace{[u_2'' \cdot \Psi(\theta)\Psi'(\theta)\psi\psi_z - u_2' \cdot \Psi'(\theta)\psi_z]}_{+} \\ & -\eta_1(\theta)\left\{[g_2' + g_2'' \cdot \theta\phi]\underbrace{\phi_z}_{+} + \underbrace{g_2'' \cdot \psi_z\phi}_{+}\right\}, \end{aligned}$$

which combined with

$$g_2' + g_2'' \cdot \theta\phi \geq 0 \;\Leftrightarrow\; \underbrace{-\frac{\theta\phi g_2''}{g_2'}}_{\varepsilon_{g_2', \theta G_1}} \leq 1$$

completes the proof of part (ii). ■

The key message conveyed by Proposition 5.3 can be summarized as follows. First, in the asymmetric-information optimum only the intertemporal allocation at the endpoints of type distribution is not distorted with respect to the first-best. Second, the presence of the asymmetric information between center and regions prevents full insurance from happening.

Next we characterize the scheme of federal transfers than decentralizes the asymmetric-information optimum established in Proposition 5.3.

Proposition 5.4 *Suppose Assumptions 4.2 and 5.1 hold. The grant scheme $z^*(b)$ that decentralizes the asymmetric-information optimum $\{b^*(\theta), z^*(\theta)\}_{\theta\in\Theta}$ is a nonlinear non-decreasing function of b, almost everywhere differentiable, with the slope*

$$\frac{dz^*}{db}\begin{cases} = \ 0 & \text{for } b \in \{b^*(\underline{\theta}), b^*(\bar{\theta})\}; \\ > \ 0 & \text{for } b \in (b^*(\bar{\theta}), b^*(\underline{\theta})). \end{cases}$$

Proof. Using the first-order necessary condition for incentive compatibility, we get

$$\frac{dz}{db} = \frac{dz}{d\theta}\frac{d\theta}{db} = -\frac{V_b}{V_z},$$

in which we have shown above that $V_z > 0$ always holds true. As we focus on the case without bunching, we get from (75) (which uses Assumption 5.1) and (54) (which uses Assumption 4.2) that

$$\begin{aligned} & - V_b\left(b(\theta), z(\theta), \theta\right) \\ = & - \frac{\eta_1(\theta)}{\mu_1(\theta)f(\theta)}\underbrace{\{-u_2'' \cdot [1 + r + \Psi(\theta)\psi_b]\Psi'(\theta)\psi + u_2' \cdot \Psi'(\theta)\psi_b\}}_{-} \\ & + \frac{\eta_1(\theta)}{\mu_1(\theta)f(\theta)}\underbrace{[g_2'' \cdot (\theta\phi_b + \psi_b)\phi + g_2' \cdot \phi_b]}_{+} \end{aligned}$$

for all $\theta \in (\underline{\theta}, \bar{\theta})$. We thus have for all $b \in (b^*(\bar{\theta}), b^*(\underline{\theta}))$ that:

$$\begin{aligned} \frac{dz}{db} = & - \frac{\eta_1(\theta(b))\{-u_2'' \cdot [1 + r + \Psi(\theta(b))\psi_b]\Psi'(\theta(b))\psi + u_2' \cdot \Psi'(\theta(b))\psi_b\}}{\mu_1(\theta(b))f(\theta(b))V_z} \\ & + \frac{\eta_1(\theta(b))[g_2'' \cdot (\theta(b)\phi_b + \psi_b)\phi + g_2' \cdot \phi_b]}{\mu_1(\theta(b))f(\theta(b))V_z} > 0, \end{aligned}$$

in which, by a little abuse of notation, $\theta(b)$ denotes the inverse of $b(\theta)$, which does exist in light of Lemma 5.2. Finally, applying (72) immediately completes the proof. ■

The main message of Proposition 5.4 is that federal transfers and local debt in general play a complementary role for regional insurance provision. Except for the bottom and top types whose intertemporal allocations are not distorted in the asymmetric-information optimum, the optimal funding structure in the case with observable physical output of IPGs exhibits the following feature: regions of a higher degree of intergenerational externality should issue less debt and receive less federal transfers than regions of a lower degree of intergenerational externality.