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Optimal Regional Insurance Provision: Do Federal Transfers Complement Local Debt?

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Abstract
We study optimal regional insurance provision in federations with regionally and privately observable shocks to the degree of intergenerational externality (DIE) induced by local intergenerational public goods (IPGs), or to the degree of technological progress (DTP) for producing the IPGs. Federal transfers provide interregional insurance, and local debt provides intergenerational insurance. If optimal federal transfers increase (decrease) with a region's debt level, we say the two insurance policies are complements (substitutes). We address such questions as whether it is efficiency-enhancing to adopt both schemes for providing regional insurance and how the answer varies with these two different economic shocks. The paper's main results are twofold: first, under the DIE shocks, federal transfers and local debt act as complements in implementing the asymmetric information optimum when borrowing and spending decisions are decentralized at the regional level; second, under the DTP shocks, they act as complements with observable output of IPGs, but act as substitutes with observable expenditure on the IPGs.

Keywords Intergovernmental transfer · Local government debt · Intergenerational spillover · Regional economic shocks · Asymmetric information · Mechanism design

JEL Classifications H41 · H74 · H77 · D82

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1 Introduction

Intergovernmental grants implemented by the central government of a federal fiscal system are justified on the grounds that they internalize interregional spillovers generated by the provision of local public goods (Oates 1972) or by inter-jurisdictional migrations,[1] redistribute income between regions,[2] and, in particular, serve as a risk-sharing device against region-specific shocks.[3] As shown by Sala-i-Martin and Sachs (1992), even policies aimed at income redistribution may have an effect on the degree of interregional risk-sharing. Indeed, there is empirical evidence showing that fiscal transfers from the federal government provide substantial insurance against regional economic fluctuations in the United States, Canada, Japan, and Norway.[4]

On the other hand, local government debt serves as a public contract for sharing risks between generations or over life-cycles in a given region. For instance, in an infinite-horizon economy where individuals face uninsurable risks to their human capital accumulation, Gottardi et al. (2015) show that the benefits of government debt increase with the magnitude of risks and the degree of risk aversion. Since present generations are imperfectly altruistic (e.g., Altonji et al. 1992, 1997), however, the design of optimal public debt to account for intergenerational conflicts turns out to be a nontrivial mechanism design task (e.g., Dai et al. 2019a; Huber and Runkel 2008; Rangel 2003, 2005).

Given the insurance role played by both federal transfers and local debt, the following questions arise. Is it socially optimal or welfare-enhancing to adopt both schemes for the provision of regional insurance? Under decentralized borrowing decisions made by local governments, how would interregional insurance provided by the central government interact with intergenerational insurance provided locally? In addition, how might the answers to these questions vary with alternative sources of regional economic shocks? The goal of this paper is to address these questions via tackling the optimal design and implementation of risk-sharing contracts consisting of both intergovernmental grants and regional public debt along space and time dimensions, respectively.

Indeed, whether federal grants and local debt act as complements or substitutes matters greatly in regional insurance design. Specifically, if the two insurance schemes complement each other, then a joint implementation for regional insurance provision is justified on efficiency grounds. If they act as substitutes, on the other hand, then efficiency considerations require using either federal grants or local debt

[1] See, e.g., Breuillé and Gary-Bobo (2007), Cremer et al. (1997), Dai et al. (2019b), Figuieres and Hindriks (2002), and Hercowitz and Pines (1991).

[2] See, e.g., Bordignon et al. (2001), Cornes and Silva (2000), Cremer and Pestieau (1997), and Raff and Wilson (1997).

[3] See, e.g., Bucovetsky (1998), Cornes and Silva (2000), Jüßen (2006) , Lockwood (1999), and Persson and Tabellini (1996a, 1996b).

[4] See, e.g., Asdrubali et al. (1996), Athanasoulis and van Wincoop (2001), Atkeson and Bayoumi (1993), Borge and Matsen (2004), Evers (2015), Kalemli-Ozcan et al. (2003), and Mélitz and Zumer (1999). Even in the presence of complete markets, Farhi and Werning (2017) provide a rationale for government intervention in terms of public risk sharing.

but not both of them simultaneously in providing regional insurance. In particular, identifying the case with substitutability creates a sort of policy flexibility; that is, they can be adopted simultaneously while targeting alternative policy goals facing the national government. For example, federal transfers are used for interregional income redistribution or for horizontal externality correction, whereas local debt is used for regional insurance provision. Or, federal transfers are used for regional insurance provision, whereas local debt can be strictly constrained to defuse local government debt bomb (e.g., The Economist 2015), which is practically relevant for countries with a high risk of local government debt default such as China (e.g., Huang et al. 2021, 2020).

We consider a country that consists of a central government and many sub-national governments located in geographically decentralized regions. The center is in charge of revenue transfers across regions whereas local governments are responsible for collecting taxes used to provide local public goods. Each region is populated by a continuum of identical residents who live for one period only. The economy lasts for two periods, thus enabling us to incorporate intergenerational concerns into the model. The current generation chooses how much debt to pass on to the future generation and how much to invest in intergenerational public goods (IPGs), namely durable public goods that entail positive intergenerational spillovers, such as basic science, environmental protection, and public capital. Initially, as the centralization benchmark to which we refer, we let the center jointly determine the amount of public debt a region can issue and the transfers it can receive. We then move to the more realistic situation with decentralized leadership in which local governments have the autonomy to choose the level of regional public debt. Therefore, we characterize the federal grants scheme that implements the asymmetric information welfare optimum through decentralized regional borrowing decisions.

Regions are assumed to be ex ante identical but are subject to stochastic shocks to either the degree of intergenerational externality induced by, or the degree of technological progress for producing, the IPGs. In this context, while regional heterogeneity in shock realizations creates a natural role for interregional insurance represented by transfers from the center to the regions, a potential role of intergenerational insurance played by government debt is also easy to understand because both types of shocks primarily affect the future generations. Firstly, the present generation incurs the cost of an IPG investment that generates a positive externality on future generations. Secondly, it is well recognized that the progress made in fields like basic science, space exploration, and environmental protection benefits from standing on the shoulders of giants, and, hence, a high degree of technological progress to be realized in the future appeals to knowledge accumulation and R&D investments in the present.

As is customarily assumed in the fiscal federalism literature, regional governments are better informed about the shocks than is the federal government.[5] We are thus interested in incentive-compatible schemes for regional insurance provision.

[5] See, for example, Bucovetsky (1998), Cornes and Silva (2000, 2002), Dai et al. (2019a, 2019b), Huber and Runkel (2008), Lockwood (1999), Oates (1972).

That is, intergovernmental grants and regional public debt form the risk-sharing contracts designed by the center, subject to the federal fiscal budget balance and truth-telling constraints. From solving the mechanism design problem facing the center, conducting the comparison with the full-information optimum (or the first-best allocation), and implementing the optimal allocation through regional debt issuance decisions, we obtain the following four results, regardless of the source of shocks. While the first two results characterize the asymmetric information welfare optimum, the last two results show the key features of the implementation scheme over the entire shock distribution.

First, the intertemporal allocation is undistorted only for the bottom and top types, i.e., the intertemporal rate of substitution between current and future consumption equals the intertemporal rate of transformation only at the endpoints of shock distribution. As such, relative to the full-information optimum in which the intertemporal rate of substitution equals the intertemporal rate of transformation for all types, regions of types between the bottom and top may borrow too much or too little in the asymmetric information optimum. So, regions of some types are likely to face welfare losses under asymmetric information. Also, full insurance is not achievable for any type. That is, the amount of public/private goods consumption in the future is not the same independent of the realizations of economic shocks. We thus conclude that the informational asymmetry between the center and regions restricts the availability of complete public insurance in this setting.

Second, relative to the full-information optimal allocation, if the intergovernmental grant received by the bottom type is distorted upward (or is large), then the amount of public debt it can issue must be distorted downward (or be small), and vice versa; meanwhile, the direction of distortion is qualitatively reversed between the bottom and top types. Let us explain this result for the case of shocks to intergenerational externality. Recall that the intertemporal rate of transformation is the rate at which savings in the first period can be transformed into consumption in the second period, and an increase in which implies an increase in the opportunity cost of borrowing. Note that the positive intergenerational spillovers of IPGs partly offset the negative intergenerational externality of debt issuance; the bottom-type regions face the largest opportunity cost of borrowing. In consequence, the incentive-compatible insurance contracts must feature an upward distortion on transfers while a downward distortion on debt for the bottom-type regions.

Third, for all but the bottom and top types, to implement the welfare optimum truthfully through decentralized regional debt decisions, the intergovernmental grant scheme enforced by the center must be a nonlinear, almost everywhere differentiable and monotonic function of local debt. Since the intertemporal allocation is distorted for these types in the asymmetric information optimum, the amounts of public debt allocated to these regions are different from the ones determined by maximizing their respective regional goals. As a result, if borrowing decisions are decentralized to the regional governments, the grant scheme enforced by the center must depend on regional debt such that regions have incentives to reveal their types truthfully.

And fourth, for the bottom and top types, the grant scheme that decentralizes the welfare optimum is independent of local government debt. The reason is that the intertemporal allocations desired by regions of the extreme types are not distorted in the asymmetric information welfare optimum, namely, the relationship of intertemporal rate of substitution and intertemporal rate of transformation characterized by regional welfare maximization coincides with that in the asymmetric information welfare optimum determined by the central government. Consequently, incentive compatibility can be guaranteed for such types by directly setting the grants established in the asymmetric information optimum.

Moreover, concerning the potentially different effects of different sources of regional shocks on the center's optimal allocation of federal transfers across heterogeneous regions, it would be interesting to analyze the interactive relationship of the two insurance instruments for decentralizing the asymmetric information welfare optimum. We just need to focus on regional types between the top and bottom of shock distribution because only for these types do nontrivial interactions between federal transfers and local debt arise from the task of optimal decentralization.

When regions differ in the degree of intergenerational externality induced by IPGs, federal transfers and local debt act as complements for regional insurance provision. The immediate implication is that it is socially optimal to use the two insurance schemes simultaneously when facing these sorts of shocks. The intuition for this result is the following. Given that the asymmetric information optimum under such shocks features intertemporal rates of substitution that are smaller than intertemporal rates of transformation for these regions, they thus borrow too much relative to the first-best allocation and, hence, welfare losses may emerge. Indeed, the higher the level of debt a regional government issues, ceteris paribus, the greater the welfare loss it may face. As such, to implement the asymmetric information optimum with the feature given above, this region should receive increased federal transfers as a sort of compensation; otherwise, it may misreport its type by mimicking those regions issuing less debt than this region issues. That is, local debt and federal transfers act as complements to guarantee self-selection through regional debt decisions in the course of decentralizing the asymmetric information optimum.

When regions differ in the degree of technological progress for producing the IPGs, federal transfers and local debt act as complements if it is the physical output of public goods that is observable, whereas they act as substitutes if it is the regional expenditure on public goods that is observable. The intuition for this latter result is as follows: The asymmetric information optimum under observable expenditure features an intertemporal rate of substitution that is greater than the intertemporal rate of transformation, implying that these regions borrow too little relative to the first-best allocation. The lower the level of debt that a regional government issues, ceteris paribus, the greater the welfare loss it may face. As such, this region should receive more federal transfers as a sort of compensation; otherwise, this region may mimic those regions issuing more debt than it issues. This explains why local debt and federal transfers act as substitutes for implementing asymmetric information optimum. The result under observable physical output can be explained analogously. Note that the corresponding asymmetric information optimum features an intertemporal rate of substitution that is smaller than the intertemporal rate of transformation, implying that these regions borrow too much relative to the first-best allocation. Due to the opposing features of the asymmetric information optima, therefore, the endogenous interaction between local debt and federal transfers reverses from the case of observable expenditure to the case of observable output to guarantee desirable self-selection under asymmetric information, even though under the same source of economic shocks.

This distinction between the case of observable output and the case of observable expenditure is practically relevant as well. For example, the physical output of some IPGs, such as parks, public schools, and highways, is observable, whereas the output of some IPGs such as environmental protection, basic science, and R&D is unobservable, at least in the short run, by the center, which is generally not involved in the process of producing these public goods. Consequently, whether it is the input or output of IPGs that is observable makes a nontrivial difference in determining whether the insurance provided by federal grants and the insurance provided by local debt should be used jointly or in isolation. In terms of identifying the effect of alternative observability on the implementation of information-constrained optima, this finding contributes to the public finance and regional science literature. In sum, these results characterize the connections between public risk-sharing schemes and the underlying economic environment, thereby helping us understand how they *should* be adopted in a federation.

The present study is related to the literature that examines theoretically the design for the optimal provision of regional insurance in a federation, such as Bucovetsky (1998), Cornes and Silva (2000), Lockwood (1999), and Persson and Tabellini (1996a, 1996b). A comparison with these studies reveals three distinctive features of our study. Firstly, rather than adopting a one-period static setting, we consider a two-period setting that accounts for intergenerational concerns and gives a natural insurance provision role to public debt. Secondly, the sources of regional economic shocks considered in the paper are novel for analyzing optimal regional insurance provision. In particular, these types of shocks have nontrivial effects on intergenerational resource allocation. Thirdly, instead of treating local debt and federal grants as unrelated policy variables, we study the *joint design* of the two risk-sharing schemes along space and time dimensions, namely intergovernmental grants along the interregional dimension and public debt along the intertemporal/intergenerational dimension, and further analyze their interaction in the course of optimal decentralization. Indeed, this paper represents the first attempt on the joint design of two public insurance schemes widely used by governments in both developed and underdeveloped economies. For these features, our paper extends and complements existing literature.

The remainder of the paper is organized as follows. Section 2 describes the economic environment. Section 3 derives the welfare optimum and discusses its implementation when regions differ in the degree of intergenerational externality. Section 4 conducts a similar analysis when regions differ in the degree of technological progress for producing local IPGs. Section 5 concludes. Proofs are relegated to the "Appendix". "Appendix B" further examines the case in which regions differ in both DIE and DTP, and details the conditions under which the two insurance policies act as complements, regardless of whether output or expenditure is observable by the central government.

2 Environment

We consider a two-period economy of a federation consisting of a federal government (also referred to as the "center") and n regions, inhabited by a representative immobile resident in each period.[6] In other words, each resident only lives for one period. The social welfare of region i, for $i = 1, 2, ..., n$, is given as

$$\underbrace{u_1(c_1^i) + g_1(G_1^i)}_{\text{utility of Generation 1}} + \underbrace{u_2(c_2^i) + g_2(\theta^i G_1^i + G_2^i)}_{\text{utility of Generation 2}}, \tag{1}$$

where c_1^i and c_2^i are private consumptions, G_1^i and G_2^i are public goods, and $\theta^i \in (0, 1]$ is a parameter measuring the degree of intergenerational externality of the IPG, G_1^i.[7] All four functions in (1) are strictly increasing and concave, twice continuously differentiable, and satisfy the usual Inada conditions.

The representative resident of Generation t, for $t = 1, 2$, in region i has private budget constraint $c_t^i + \tau_t^i = y_t$, where y_t is the commonly given income across all regions.[8] Lump sum tax τ_t^i is collected by the local government to finance the provision of local public goods. In Period 1, the local government receives a transfer z^i from the center ($z^i < 0$ means that the local government pays a tax to the center) and issues debt b^i. Debt and interest payments must be repayed in Period 2, taking as given the common interest rate $r > 0$.[9] The fiscal budget constraints of region i in Periods 1 and 2 can be written as $G_1^i = \tau_1^i + b^i + z^i$, and $\mathcal{G}_2^i = \tau_2^i - (1 + r)b^i$, respectively. We let $\mathcal{G}_2^i = \xi^i G_2^i$, in which the parameter $\xi^i \in (0, 1]$ measures the per unit cost of Period-2 public goods provision. The case of $\xi^i < 1$ captures the effect of technological progress, which Rangel (2005) argues is important for the provision of IPGs, such as infrastructure, space exploration, and environmental capital. In addition, whether the expenditure $\mathcal{G}_2^i$ or the physical output G_2^i is observable to the mechanism designer generally influences the implementation (Maskin and Riley 1985). If the expenditure is observable, we need to express the output as a function of expenditure, namely, $G_2^i = \mathcal{G}_2^i / \xi^i \equiv \rho^i \mathcal{G}_2^i$, where $\rho^i \equiv 1/\xi^i$.

6 We have not covered the matter of horizontal fiscal externalities induced by cross-region labor mobility; that can be investigated in future research.

7 IPG is a kind of public good produced in Generation 1, and is still (partially) usable in Generation 2 (Conley et al. 2019; Dai et al. 2021; Dai and Tian 2022; Rangel 2005). To guarantee tractability, we focus on the effect of intergenerational externalities and assume away the external effect a region's local public goods may exert on neighboring regions.

8 Instead of considering income heterogeneity across regions, which has been well studied in terms of designing optimal intergovernmental grants, we consider some novel dimensions of cross-region heterogeneity. Also, one can interpret our model as restricting attention to regions of similar personal incomes, such as California and Texas in the United States, or Jiangsu and Zhejiang provinces in China.

9 Assuming that there is a common capital market within a federation, there is a unique rental price level of capital such that arbitrage opportunities are eliminated.

For expositional convenience, the region index i is suppressed in the remainder of the set-up. After combining the private budget constraints with the public budget constraints, and applying them to Eq. (1), a region's social welfare maximization problem is given by

$$\begin{aligned}\max_{c_1,c_2} \quad & u_1(c_1) + g_1(y_1 + b + z - c_1) \\ & \qquad + u_2(c_2) + g_2(\theta(y_1 + b + z - c_1) + \rho(y_2 - b(1+r) - c_2)),\end{aligned} \tag{2}$$

where $\rho \geq 1$. Note that in Problem (2), choosing c_1 and c_2 is equivalent to choosing τ_1 and τ_2. Therefore, we write the first-order conditions as

$$u_1'(c_1) = g_1'(G_1) + \theta g_2'(\theta G_1 + G_2) \quad \text{and} \quad u_2'(c_2) = \rho g_2'(\theta G_1 + G_2), \tag{3}$$

which represent the Samuelson conditions for the optimal provision of public goods.

We allow regions to differ in two dimensions in terms of privately observable shocks: the degree of intergenerational externality measured by θ, and the degree of technological progress for producing IPGs measured by ξ (or, equivalently, by ρ). Both θ and ξ (or ρ) are considered as private information of local governments. The per unit cost of public goods provision has previously been treated as the private information of local governments by Boadway et al. (1999), Cornes and Silva (2002), and Lockwood (1999). Interpreted as a measure of the quality or durability of local IPGs, there are two plausible reasons to assume that θ is only privately observable by local governments.

Firstly, the quality of the physical output of some IPGs, such as basic science, local environmental protection, and R&D, is in the short-term, objectively unobservable by the center, which is little involved in the process of producing these public goods. Secondly, local politicians have subjective incentives to hide/misreport such information to obtain more transfers, personal promotions, or avoidance of punishments. For example, local politicians in China may achieve higher level promotions due to previous experience in public infrastructure investment, having established a business friendly environment. Alternatively, they may get punished for being responsible for *tofu-dreg projects*[10] in the provision of local IPGs, such as public schools, bridges and dams, which may end up being of either low quality or even tragic failures.

Note that to obtain meaningful results in the presence of multidimensional private information is analytically intractable (Armstrong and Rochet 1999; Rochet and Choné 1998). Therefore, we consider two separate cases: (1) privately observable shocks to the degree of intergenerational externality (DIE); (2) privately observable shocks to the degree of technological progress (DTP). The random variables are assumed to be continuously distributed in intervals $[\underline{\theta}, \bar{\theta}] \equiv \Theta$ and $[\underline{\xi}, \bar{\xi}] \equiv \Xi$ (or $[\underline{\rho}, \bar{\rho}] \equiv \Upsilon$), and are also identically and independently distributed across regions. The publicly observable density and distribution functions are denoted by $f = F' > 0$ and F, respectively.

[10] This is a well-known phrase coined by Zhu Rongji, the former premier of the People's Republic of China, on a visit to Jiujiang City, Jiangxi Province to describe a jerry-built dam.

3 Welfare optimum and implementation when regions differ in intergenerational externality

This section concentrates on the optimal provision of regional insurance against shocks to the degree of intergenerational externality, θ^i. We assume that all regions have the same degree of technological progress, and let ρ^i (or ξ^i) $= 1$ for all i without loss of generality. We firstly introduce the problem of the center. We then proceed to derive welfare optimum in cases of complete and asymmetric information between the center and regions, followed by a discussion about implementation.

3.1 The problem pf the center

The center is responsible for determining time-consistent (or credible) policies of regional debt and cross-region transfers as non-market insurances against shocks to the intergenerational externality.[11] Assuming it treats all regions equally and the realization of shocks can be privately observed by each region, we can conclude that it maximizes the expectation of the value function (2) of any region, subject to federal fiscal budget balance and incentive-compatibility constraints.

We follow the mechanism design approach, and apply the direct revelation principle. The center offers each region i a contract stipulating the federal transfer and the region's debt, which is only conditional on its report of type θ^i, with the report belonging to set Θ. Since all regions are ex-ante identical, the insurance contract can be considered signed between the principal (the center) and the agent (a region) whose type θ belongs to set Θ following the distribution F.[12]

The timing of the underlying game unfolds as follows:

- Shock occurs, i.e., a natural event makes the first move.
- Local governments privately observe shock realizations.
- The federal government offers the contract menu, $\{b(\theta), z(\theta)\}$, for all $\theta \in \Theta$.
- The local governments simultaneously pick a contract (or equivalently report their types), and the game ends.

We write the value function generated by the maximization problem (2) as $V(b, z, \theta)$. As all regions are ex-ante identical, the objective function of the center can be written as:

[11] This study will not consider the issue of soft budget constraints which may emerge when the central government cannot commit to the transfers it sets ex-ante.

[12] In fact, the center offers each region i a contract stipulating the federal transfer and the region's debt conditional only on its report of type θ^i, which is denoted by $\hat{\theta}^i$, i.e., $b^i = b(\hat{\theta}^i)$ and $z^i = z(\hat{\theta}^i)$. To formulate the constraints facing the mechanism designer, we consider the limiting case with the number of regions being large, i.e., $n \to \infty$. Making use of the weak law of large numbers, the empirical distributions of b^i and z^i across regions approximate the theoretical distributions generated by $b^i = b(\hat{\theta}^i)$, $z^i = z(\hat{\theta}^i)$ and F.

$$EU = \int_{\underline{\theta}}^{\bar{\theta}} V(b(\theta), z(\theta), \theta) f(\theta) \mathrm{d}\theta. \tag{4}$$

The truth-telling constraints require that any region with shock realization θ prefers to report θ rather than some θ', which is formally written as

$$V(b(\theta), z(\theta), \theta) \geq V(b(\theta'), z(\theta'), \theta) \;\; \forall \theta' \neq \theta, \; \theta', \theta \in \Theta. \tag{5}$$

The federal budget balance constraint for large n is written as

$$\int_{\underline{\theta}}^{\bar{\theta}} z(\theta) f(\theta) \mathrm{d}\theta \leq 0, \tag{6}$$

which implies that in aggregate transfers must sum to at most 0.

The problem facing the center is the need to choose $\{b(\theta), z(\theta)\}_{\theta \in \Theta}$ to maximize (4), subject to constraints (5) and (6). Following common practice in existing mechanism design literature, we allow b and z to be piecewise continuously differentiable functions, and let $b(\theta)$ be everywhere continuous.

3.2 Welfare optimum

As a standard benchmark result, we start our analysis by deriving the full-information (first-best) allocation, which maximizes (4) subject to (6) only. We index the first-best optimum by the superscript, ^{FB}.

Lemma 3.1 *In the full-information case, the welfare optimum* $\{b^{FB}(\theta), z^{FB}(\theta)\}_{\theta \in \Theta}$ *satisfies*:

(i) The intertemporal rate of substitution between current and future public goods consumption equals the intertemporal rate of transformation, as shown by

$$\frac{g_1'(G_1^{FB}(\theta))}{g_2'(\theta G_1^{FB}(\theta) + G_2^{FB}(\theta))} = 1 + r - \theta \;\; \text{for any } \theta \in \Theta.$$

(ii) Full insurance is achievable, as shown by

$$V_z\big(b^{FB}(\theta), z^{FB}(\theta), \theta\big) = \gamma \;\; \text{for any } \theta \in \Theta,$$

in which $\gamma > 0$ denotes the Lagrange multiplier of the budget constraint (6).

Proof Straightforward and omitted. □

Part (i) yields that the intertemporal allocation of any type of region in the first-best optimum features that the intertemporal rate of substitution is equal to the intertemporal rate of transformation. Part (ii) provides the standard insurance condition which states that the consumption of Period-2 public goods remains the same, irrespective of the shock realization on the degree of intergenerational spillovers.

We now turn to the more relevant case: there is asymmetric information between the center and regions. In this case, the realization of the random variable measuring the degree of intergenerational externality is private information so that regions of a certain type may mimic regions of another type in order to obtain (more) insurance transfers. We now index the second-best allocation by the superscript *.

We shall need the following assumption:

Assumption 3.1 $-\theta G_1 g_2'' \leq g_2'$ for all $\theta \in (\underline{\theta}, \bar{\theta})$, namely the absolute value of the elasticity of Generation 2's marginal utility from G_1 is no greater than 1 for all but the endpoints of the type distribution.

This is a technical restriction imposed on Generation 2's preference of public goods. It is easy to verify that this assumption is satisfied for log and power utility functions. We can interpret Assumption 3.1 by applying the Envelope Theorem to (2), as shown below:

$$V_z(b, z, \theta) = g_1'(G_1) + \theta g_2'(\theta G_1 + G_2) > 0,$$
$$V_{z\theta}(b, z, \theta) = g_2'(\theta G_1 + G_2) + \theta G_1 g_2''(\theta G_1 + G_2).$$

As such, Assumption 3.1 guarantees that $V_{z\theta}(b, z, \theta) \geq 0$ for all $\theta \in (\underline{\theta}, \bar{\theta})$. Federal transfers and intergenerational spillovers are in other words complementary in terms of enhancing the welfare of these regions; the higher the degree of intergenerational spillovers realized within a region, the greater the effect of federal transfers with regards to enhancing the region's welfare. Alternatively, the larger the number of federal transfers received by a region, the greater the effect a positive shock to intergenerational spillovers has on enhancing a region's welfare.

Proposition 3.1 *(Asymmetric Information Optimum under DIE Shocks) In the asymmetric-information case without bunching, namely* $\dot{b}(\theta) > 0$*, the welfare optimum* $\{b^*(\theta), z^*(\theta)\}_{\theta \in \Theta}$ *satisfies:*

(i) With regards to the relationship between the intertemporal rate of substitution between current and future public goods consumption and the intertemporal rate of transformation, we arrive at:

$$\frac{g_1'(G_1^*(\theta))}{g_2'(\theta G_1^*(\theta) + G_2^*(\theta))} \begin{cases} = \ 1 + r - \theta & \text{for } \theta \in \{\underline{\theta}, \bar{\theta}\}; \\ < \ 1 + r - \theta & \text{for } \theta \in (\underline{\theta}, \bar{\theta}). \end{cases}$$

(ii) Suppose Assumption 3.1 holds. If we let $\mu_1(\theta) > 0$ be the Lagrange multiplier on the value constraint $v(\theta) \equiv V(b(\theta), z(\theta), \theta)$ of any type-θ region who is reporting truthfully, we then arrive at:

$$V_z(b^*(\theta), z^*(\theta), \theta) \begin{cases} = \ \gamma/\mu_1(\theta) & \text{for } \theta \in \{\underline{\theta}, \bar{\theta}\}; \\ < \ \gamma/\mu_1(\theta) & \text{for } \theta \in (\underline{\theta}, \bar{\theta}). \end{cases}$$

Proof See the "Appendix". □

This proposition provides two main findings: (1) only at the endpoints of shock distribution the intertemporal allocation under asymmetric information is not distorted, relative to the first-best allocation given by Lemma 3.1. The intertemporal rate substitution still equals the intertemporal rate of transformation, but only for regions of the highest and lowest degrees of intergenerational externality. However, the intertemporal allocations of all other regions that fall in between, are distorted relative to their respective first-best allocations, such that the intertemporal rate substitution becomes smaller than the intertemporal rate of transformation. In fact, given that $V_{bb}(b, z, \theta) = g_1''(G_1) + [\theta - \rho(1+r)]^2 g_2''(\theta G_1 + G_2) < 0$, we must have $b^*(\theta) > b^{FB}(\theta)$ for any $\theta \in (\underline{\theta}, \bar{\theta})$, namely, in the asymmetric information optimum, regions of these types should borrow more than they would borrow in the full-information optimum; (2) given that the multiplier $\mu(\theta)$ is type-dependent, there is incomplete insurance under asymmetric information.

As informational friction is the focus of this study, it is important to identify the effect of asymmetric information between the center and regions on optimal debt and intergovernmental grants policies. Hence, it is worthy to provide a detailed characterization of the Lagrange multiplier, $\mu_1(\theta)$.

Lemma 3.2 *For the current economic environment, the following statements are true.*

(i) If $\mu_1(\theta)$ is decreasing in θ, then there exists some $\tilde{\theta} \in (\underline{\theta}, \bar{\theta})$, such that $\mu_1(\theta) > 1$ for $\theta \in [\underline{\theta}, \tilde{\theta})$, $\mu_1(\theta) = 1$ for $\theta = \tilde{\theta}$, and $\mu_1(\theta) < 1$ for $\theta \in (\tilde{\theta}, \bar{\theta}]$.

(ii) If $\mu_1(\theta)$ is increasing in θ, then there exists some $\breve{\theta} \in (\underline{\theta}, \bar{\theta})$, such that $\mu_1(\theta) < 1$ for $\theta \in [\underline{\theta}, \breve{\theta})$, $\mu_1(\theta) = 1$ for $\theta = \breve{\theta}$, and $\mu_1(\theta) > 1$ for $\theta \in (\breve{\theta}, \bar{\theta}]$.

Proof See the "Appendix". □

Although $\mu(\theta)$ could be a complex nonlinear function of θ, we focus on the case of monotonicity to obtain clear-cut results. In light of Lemma 3.2, the following proposition is established by comparing Lemma 3.1 with Proposition 3.1.

Proposition 3.2 *(Policy Effect of Asymmetric Information on DIE) Under Assumption 3.1, the following statements are true.*

(i) If $\mu_1(\theta)$ is decreasing in θ, then (i-a) $z^*(\theta) > z^{FB}(\theta)$ for all $\theta \in [\underline{\theta}, \tilde{\theta}]$; (i-b) $z^*(\bar{\theta}) < z^{FB}(\bar{\theta})$; and (i-c) $b^*(\underline{\theta}) < b^{FB}(\underline{\theta})$ and $b^*(\bar{\theta}) > b^{FB}(\bar{\theta})$.

(ii) If $\mu_1(\theta)$ is increasing in θ, then (ii-a) $z^*(\theta) > z^{FB}(\theta)$ for all $\theta \in [\breve{\theta}, \bar{\theta}]$; (ii-b) $z^*(\underline{\theta}) < z^{FB}(\underline{\theta})$; and (ii-c) $b^*(\underline{\theta}) > b^{FB}(\underline{\theta})$ and $b^*(\bar{\theta}) < b^{FB}(\bar{\theta})$.

Proof See the "Appendix". □

If $\mu_1(\theta)$ is decreasing in θ, then the shadow price of the value constraint under truth-telling is larger for low types (i.e., regions with low degrees of intergenerational externality) than for high types. In other words, the incentive compatibility constraints of low types are binding. Subsequently, as claim (i-a) shows, low types *extract* the information rent and receive more transfers under asymmetric information than they would receive under complete information. Claim (ii-a) can be interpreted in a similar way.

Regardless of whether $\mu_1(\theta)$ is decreasing or increasing in θ, the optimal policy mix of federal transfers and local debt under asymmetric information is distorted for both top and bottom types. Importantly, the distortion is qualitatively reversed between the two extreme types. For instance, if $\mu_1(\theta)$ is decreasing in θ, then, relative to their respective federal transfers received and local debt issued in the full-information optimum, the bottom type receives more transfers and issues less debt, whilst the top type receives less transfers and issues more debt in the asymmetric information optimum. The intuition for this result is the following.

Firstly, we recall that the intertemporal rate of transformation is the rate at which savings in the first period can be transformed into consumption in the second period, and an increase in which implies an increase in the opportunity cost of borrowing. Secondly, we must note that the *positive intergenerational spillovers* of the IPGs partially offset the *negative intergenerational externality* caused by local government borrowing. The bottom-type regions have the largest opportunity cost of borrowing because $1 + r - \underline{\theta} > 1 + r - \theta$ for any $\theta > \underline{\theta}$. Consequently, a decrease of $\mu_1(\theta)$ in θ imposes an upward distortion on transfers. Yet, a downward distortion on debt provides appropriate incentives, such that the bottom-type regions reveal their type truthfully under asymmetric information.

3.3 Implementation

We established the welfare optimum under both complete and asymmetric information in the previous subsection, and are now proceeding to consider its implementation via regionally decentralized debt decisions. In other words, both regions choose a level of public debt to maximize their regional welfare, taking as given the intergovernmental grants scheme provided by the center. The maximization problem of regions of type-θ is

$$\max_{b(\theta)} \; V(b(\theta), z(\theta), \theta)$$

assuming the federal transfers received $z(\theta)$. We then rewrite private consumptions as $c_1 = \tilde{\phi}(b(\theta), z(\theta), \theta)$ and $c_2 = \tilde{\psi}(b(\theta), z(\theta), \theta)$, and apply the Envelope Theorem, enabling the first-order condition to be written as

$$\frac{g_1'\Big(y_1 + b(\theta) + z(\theta) - \tilde{\phi}(b(\theta), z(\theta), \theta)\Big)}{g_2'\Big(\theta\Big(y_1 + b(\theta) + z(\theta) - \tilde{\phi}(b(\theta), z(\theta), \theta)\Big) + y_2 - (1 + r)b(\theta) - \tilde{\psi}(b(\theta), z(\theta), \theta)\Big)} = 1 + r - \theta, \tag{7}$$

showing that the intertemporal rate of substitution must be equal to the intertemporal rate of transformation at the regional welfare optimum.

Application of (7) and Lemma 3.1 leads to the full-information optimum being attained by simply setting $z(\theta) = z^{FB}(\theta)$ for all $\theta \in \Theta$. The reason for this implementation scheme to be valid is that firstly, the center can observe the type of each region, and secondly, the full-information optimum does not distort the intertemporal allocation desired by each region.

Under asymmetric information, the center must design an intergovernmental grants scheme that guarantees incentive compatibility for all regions. Proposition 3.1 shows that the intertemporal allocation of regions of all but top and bottom types is distorted. Thus, the asymmetric-information optimum can no longer be implemented through decentralized debt decisions characterized by (7), whereby the center simply sets $z(\theta) = z^*(\theta)$. Therefore, we establish the following proposition.

Proposition 3.3 *(Optimal Decentralization under DIE Shocks) Suppose the second-order sufficient condition for incentive compatibility is not binding, namely, $\dot{b}(\theta) > 0$ is fulfilled. Then, the grant scheme $z^*(b)$ that decentralizes $\{b^*(\theta), z^*(\theta)\}_{\theta\in\Theta}$ is a nonlinear nondecreasing function of b, and is almost everywhere differentiable, with the slope satisfying*

$$\frac{dz^*}{db}\begin{cases} = 0 & \text{for } b \in \{b^*(\underline{\theta}), b^*(\bar{\theta})\}; \\ > 0 & \text{for } b \in (b^*(\underline{\theta}), b^*(\bar{\theta})). \end{cases}$$

Proof See the "Appendix". □

The monotonicity constraint derived from guaranteeing incentive compatibility — i.e., $\dot{b}(\theta) > 0$ — means that regions with high degrees of intergenerational spillovers (H-regions) are allowed to issue more debt than regions with low degrees (L-regions). This condition is logical in terms of efficiency, as H-regions face smaller opportunity costs of borrowing than those faced by L-regions. We now elaborate further, and identify that when the intertemporal rate of transformation is given by $1 + r - \theta$, relative to L-regions with small θ, H-regions with large θ seem to have a stronger ability to mitigate the negative intergenerational externality induced by borrowing. In other words, they reduce the amount of debt repayment plus interest placed on future generations.

In view of Proposition 3.1 we observe that the intertemporal allocation under asymmetric information is not distorted at the endpoints of type distribution, thus the socially optimal levels of local debt for regions of bottom and top types, namely $b^*(\underline{\theta})$ and $b^*(\bar{\theta})$, can be realized by simply setting $z(\underline{\theta}) = z^*(\underline{\theta})$ and $z(\bar{\theta}) = z^*(\bar{\theta})$, respectively. This explains why we have $dz^*/db = 0$ at the endpoints of type distribution in Proposition 3.3.

The intertemporal allocations of regions of types between $\underline{\theta}$ and $\bar{\theta}$ are distorted relative to the first-best. Subsequently, if H-regions are allowed to issue more debt than L-regions, then the grant scheme that decentralizes the asymmetric information optimum should be designed such that H-regions are allocated more grants than L-regions. We can intuitively assume the following: Proposition 3.1 demonstrates that the asymmetric information optimum for types between $\underline{\theta}$ and $\bar{\theta}$ is such that the intertemporal rate of substitution is smaller than the intertemporal rate of transformation. This implies that relative to the first-best, these types borrow too much. Therefore, welfare loss emerges in the presence of asymmetric information. In particular, the more debt a region issues, the greater the welfare loss it will face. Given that the monotonicity constraint shows that H-regions issue more debt than do L-regions, H-regions thus face greater welfare losses than L-regions, ceteris paribus. Consequently, the center must allocate more transfers to H-regions to prevent them from mimicking L-regions under asymmetric information.

When facing this kind of shock, the implication for the optimal funding structure of IPGs is as follows: Regions with higher (respectively, lower) degrees of intergenerational externality generated by the IPGs should obtain more (respectively, less) federal transfers and issue more (respectively, less) local government debt, ceteris paribus. Therefore, federal transfers and local debt exhibit *complementarity* in the case of shocks to the DIE in order to guarantee incentive compatibility when the leadership of local borrowing is decentralized to each region.

4 Welfare optimum and implementation when regions differ in technological progress

In order to analyze the optimal regional insurance provision when regions differ in the degree of technological progress, we assume that all regions have the same degree of intergenerational externality, as denoted θ. Now, ρ^i (or ξ^i) is a random variable, whose realization is region i's private information. As shown by Lockwood (1999), Maskin and Riley (1985), whether or not the expenditure, $\mathcal{G}_2^i$, or the physical output, G_2^i, is observable to the mechanism designer, generally impacts on the implementation of the asymmetric information optimum. Both possibilities are discussed further below.

4.1 The first-best benchmark

With regards to observable expenditure on the IPGs, the first-order conditions of Problem (2) are re-written as

$$u_1'(c_1) = g_1'(G_1) + \theta g_2'(\theta G_1 + \rho \mathcal{G}_2) \quad \text{and} \quad u_2'(c_2) = \rho g_2'(\theta G_1 + \rho \mathcal{G}_2), \tag{8}$$

and the corresponding regional value function is written as $V(b, z, \rho)$.

With regards to observable physical output of the IPGs, the value function of regions of type-ξ reads as follows:

$$\begin{aligned} V(b, z, \xi) \equiv & \max_{G_1, G_2} u_1(y_1 + b + z - G_1) + g_1(G_1) \\ & + u_2(y_2 - b(1 + r) - \xi G_2) + g_2(\theta G_1 + G_2). \end{aligned} \tag{9}$$

The first-order conditions are given by

$$\begin{aligned} u_1'(y_1+b+z-G_1) = &\ g_1'(G_1)+\theta g_2'(\theta G_1+G_2) \text{ and} \\ \xi u_2'(y_2-b(1+r)-\xi G_2) = &\ g_2'(\theta G_1+G_2). \end{aligned} \tag{10}$$

Applying the Envelope Theorem, the first-best allocation can be characterized as stated in Lemma 4.1. The proof is straightforward and omitted.

Lemma 4.1 *In the full-information case, the welfare optimum verifies*:

- If expenditure on the IPGs is observable, then the first-best policy mix, $\{b^{FB}(\rho), z^{FB}(\rho)\}_{\rho\in\Upsilon}$, satisfies:
  - (i) The intertemporal rate of substitution between current and future public goods consumption equals intertemporal rate of transformation, namely,
  $$\frac{g_1'(G_1^{FB}(\rho))}{g_2'(\theta G_1^{FB}(\rho)+\rho \mathcal{G}_2^{FB}(\rho))} = \rho(1+r)-\theta \text{ for any } \rho\in\Upsilon.$$
  - (ii) Full insurance is achievable, namely,
  $$V_z\big(b^{FB}(\rho), z^{FB}(\rho), \rho\big) = \gamma \text{ for any } \rho\in\Upsilon,$$
  in which $\gamma>0$ denotes the Lagrange multiplier on the budget constraint, $\int_{\underline{\rho}}^{\bar{\rho}} z(\rho)f(\rho)d\rho\leq 0$.

- If output of the IPGs is observable, then the first-best policy mix, $\{b^{FB}(\xi), z^{FB}(\xi)\}_{\xi\in\Xi}$, satisfies:
  - (i) The intertemporal rate of substitution between current and future private goods consumption equals intertemporal rate of transformation, namely,
  $$\frac{u_1'(c_1^{FB}(\xi))}{u_2'(c_2^{FB}(\xi))} = 1+r \text{ for any } \xi\in\Xi.$$
  - (ii) Full insurance is achievable, namely,
  $$u_1'\big(c_1^{FB}(\xi)\big) = \gamma \text{ for any } \xi\in\Xi,$$
  in which $\gamma>0$ denotes the Lagrange multiplier on the budget constraint, $\int_{\underline{\xi}}^{\bar{\xi}} z(\xi)f(\xi)d\xi\leq 0$.

In the presence of complete information, there is no difference in the first-best welfare optimum of the two cases stated in Lemma 4.1, as is verifiable by comparing the FOCs given by (8) and (10). Therefore, when comparing the full-information optimum with the corresponding asymmetric information optimum, the first-best optimum is characterized in terms of public goods consumption in the case of observable expenditure, while it is characterized in terms of private goods consumption in the case of observable output. There are two key features of the full-information optimum: Part (i) shows that the intertemporal allocation of any type of region is not distorted in the sense that the intertemporal rate of substitution equals the intertemporal rate of transformation; Part (ii) gives the standard full insurance condition.

4.2 Asymmetric information optimum

To derive the asymmetric information optimum, we make the following assumptions:

Assumption 4.1 For the case of observable expenditure, we have $-\rho\mathcal{G}_2 g_2'' \leq g_2'$ for all $\rho \in (\underline{\rho}, \bar{\rho})$. In other words, the absolute value of the elasticity of marginal utility from consuming public good $G_2 = \rho\mathcal{G}_2$ is no greater than 1, excluding the endpoints of the type distribution.

Assumption 4.2 For the case of observable output, we have $-G_2 g_2'' \leq g_2'$ for all $\xi \in (\underline{\xi}, \bar{\xi})$. In other words, the absolute value of the elasticity of marginal utility from G_2 for Generation 2 is no greater than 1, excluding the endpoints of the type distribution.

These are technical restrictions imposed on the preferences of public goods consumption and they can be interpreted analogously to Assumption 3.1.

For the case of observable expenditure, the center under Assumption 4.1 is thought to solve the following maximization problem:

$$\begin{aligned}
&\max \int_{\underline{\rho}}^{\bar{\rho}} v(\rho) f(\rho) d\rho \\
&s.t. v(\rho) = V(b(\rho), z(\rho), \rho); \\
&\int_{\underline{\rho}}^{\bar{\rho}} z(\rho) f(\rho) d\rho \leq 0; \\
&\dot{v}(\rho) = g_2'(\theta\phi(b(\rho), z(\rho), \rho) + \rho\psi(b(\rho), z(\rho), \rho))\psi(b(\rho), z(\rho), \rho); \\
&\dot{b}(\rho) \leq 0
\end{aligned} \tag{11}$$

in which public goods expenditures are rewritten as $\phi(b(\rho), z(\rho), \rho) = G_1(\rho)$ and $\psi(b(\rho), z(\rho), \rho) = \mathcal{G}_2(\rho)$. The first equality constraint gives the value function of regions of type-ρ when they are telling the truth, whilst the second is the fiscal budget constraint under pure intergovernmental grants, the third is the first-order necessary condition for incentive compatibility, and the last constraint is the second-order sufficient condition for incentive compatibility.[13]

[13] The derivation of this monotonicity constraint is given in the proof of Proposition 4.1.

In the case of observable output, the center similarly takes truth-telling constraints into account, and solves the following program:

$$\begin{aligned}
&\max \int_{\underline{\xi}}^{\bar{\xi}} v(\xi)f(\xi)d\xi \\
&s.t. v(\xi) = V(b(\xi), z(\xi), \xi); \\
&\int_{\underline{\xi}}^{\bar{\xi}} z(\xi)f(\xi)d\xi \leq 0; \\
&\dot{v}(\xi) = -u_2'(y_2 - b(\xi)(1+r) - \xi\psi(b(\xi), z(\xi), \xi))\psi(b(\xi), z(\xi), \xi); \\
&\dot{b}(\xi) \leq 0
\end{aligned} \tag{12}$$

in which $\psi(b(\xi), z(\xi), \xi) = G_2(\xi)$ and the constraints can be similarly interpreted as those in program (11).

We now solve Problems (11) and (12) and arrive at the following proposition:

Proposition 4.1 *(Asymmetric Information Optimum under DTP Shocks) In the asymmetric-information case without bunching, the welfare optimum under asymmetric information verifies:*

- If expenditure of the IPGs is observable, then the constrained optimum policy mix, $\{b^*(\rho), z^*(\rho)\}_{\rho\in\Upsilon}$, satisfies:

  (i) Suppose Assumption 4.1 holds. Concerning the relationship between the intertemporal rate of substitution between current and future public goods consumption, and the intertemporal rate of transformation, we have:

  $$\frac{g_1'(G_1^*(\rho))}{g_2'(\theta G_1^*(\rho) + \rho\mathcal{G}_2^*(\rho))} \begin{cases} = \rho(1+r) - \theta & \text{for } \rho \in \{\underline{\rho}, \bar{\rho}\}; \\ > \rho(1+r) - \theta & \text{for } \rho \in (\underline{\rho}, \bar{\rho}). \end{cases}$$

  (ii) If we let $\mu_1(\rho) > 0$ be the Lagrange multiplier on the value constraint $v(\rho) \equiv V(b(\rho), z(\rho), \rho)$ of any type-ρ region that is reporting truthfully, then we have:

  $$V_z(b^*(\rho), z^*(\rho), \rho) \begin{cases} = \gamma/\mu_1(\rho) & \text{for } \rho \in \{\underline{\rho}, \bar{\rho}\}; \\ > \gamma/\mu_1(\rho) & \text{for } \rho \in (\underline{\rho}, \bar{\rho}). \end{cases}$$

- If output of the IPGs is observable, then the constrained optimum policy mix, $\{b^*(\xi), z^*(\xi)\}_{\xi\in\Xi}$, satisfies:

(i) Suppose Assumption 4.2 holds. With regards to the relationship between the intertemporal rate of substitution between current and future private goods consumption, and the intertemporal rate of transformation, we have:

$$\frac{u_1'(c_1^*(\xi))}{u_2'(c_2^*(\xi))}\begin{cases} = \ 1+r & \text{for } \xi \in \{\underline{\xi}, \bar{\xi}\}; \\ < \ 1+r & \text{for } \xi \in (\underline{\xi}, \bar{\xi}). \end{cases}$$

(ii) Let $\mu_1(\xi) > 0$ be the Lagrange multiplier on the value constraint $v(\xi) \equiv V(b(\xi), z(\xi), \xi)$ of any type-ξ region that is reporting truthfully, then we have:

$$u_1'\left(c_1^*(\xi)\right)\begin{cases} = \ \gamma/\mu_1(\xi) & \text{for } \xi \in \{\underline{\xi}, \bar{\xi}\}; \\ < \ \gamma/\mu_1(\xi) & \text{for } \xi \in (\underline{\xi}, \bar{\xi}). \end{cases}$$

Proof See the "Appendix". □

As shown in Proposition 3.1, the intertemporal allocation under asymmetric information is only undistorted at the endpoints of type distribution, and there is incomplete insurance.

Proposition 4.2 *(Policy Effect of Asymmetric Information on DTP) For the current economic environment, the following statements are true.*

- If expenditure is observable, then we have:
  - (i) If $\mu_1(\rho)$ is decreasing in ρ, then we have: (i-a) there exists some $\breve{\rho} \in (\underline{\rho}, \bar{\rho})$, such that $z^*(\rho) < z^{FB}(\rho)$ for all $\rho \in [\breve{\rho}, \bar{\rho}]$; (i-b) $z^*(\underline{\rho}) > z^{FB}(\underline{\rho})$; (i-c) $b^*(\underline{\rho}) < b^{FB}(\underline{\rho})$ and $b^*(\bar{\rho}) > b^{FB}(\bar{\rho})$.
  - (ii) If $\mu_1(\rho)$ is increasing in ρ, then we have: (ii-a) there exists some $\tilde{\rho} \in (\underline{\rho}, \bar{\rho})$, such that $z^*(\rho) < z^{FB}(\rho)$ for all $\rho \in [\underline{\rho}, \tilde{\rho}]$; (ii-b) $z^*(\bar{\rho}) > z^{FB}(\bar{\rho})$; (ii-c) $b^*(\underline{\rho}) > b^{FB}(\underline{\rho})$ and $b^*(\bar{\rho}) < b^{FB}(\bar{\rho})$.

- If output is observable, then we have:
  - (i) If $\mu_1(\xi)$ is decreasing in ξ, then we have: (i-a) there exists some $\tilde{\xi} \in (\underline{\xi}, \bar{\xi})$, such that $z^*(\xi) > z^{FB}(\xi)$ for all $\xi \in [\underline{\xi}, \tilde{\xi}]$; (i-b) $z^*(\bar{\xi}) < z^{FB}(\bar{\xi})$; (i-c) $b^*(\underline{\xi}) < b^{FB}(\underline{\xi})$ and $b^*(\bar{\xi}) > b^{FB}(\bar{\xi})$ whenever $g_1''/g_2'' \leq \rho\theta(1+r)$.
  - (ii) If $\mu_1(\xi)$ is increasing in ξ, then we have: (ii-a) there exists some $\breve{\xi} \in (\underline{\xi}, \bar{\xi})$, such that $z^*(\xi) > z^{FB}(\xi)$ for all $\xi \in [\breve{\xi}, \bar{\xi}]$; (ii-b) $z^*(\underline{\xi}) < z^{FB}(\underline{\xi})$; (ii-c) $b^*(\underline{\xi}) > b^{FB}(\underline{\xi})$ and $b^*(\bar{\xi}) < b^{FB}(\bar{\xi})$ whenever $g_1''/g_2'' \leq \rho\theta(1+r)$.

Proof See the "Appendix". □

We now define H-regions (L-regions) as regions with high (low) degrees of technological progress for producing the IPGs, namely, with large (small) ρ or small (large) ξ.

For the case of observable expenditure on the IPGs, if $\mu_1(\rho)$ is decreasing in ρ, then the shadow price of the value constraint under truth-telling is larger for L-regions than for H-regions. Subsequently, H-regions *incur* the information rent and receive less transfers under asymmetric information than they would receive under complete information. The case, in which $\mu_1(\rho)$ is increasing in ρ, can be analyzed in a similar way. Regardless of whether $\mu_1(\rho)$ is decreasing or increasing in ρ, the optimal allocation under asymmetric information is distorted for both top and bottom types, and importantly, the distortion is qualitatively reversed between these two extreme types. For example, if the shadow price of the value constraint under truth-telling is larger for L-regions than for H-regions, then the top type receives less transfers, yet issues more debt, and the bottom type receives more transfers, yet issues less debt, respectively, in the full-information optimum.

For the case of observable physical output of the IPGs, if $\mu_1(\xi)$ is decreasing in ξ, then the shadow price of the value constraint under truth-telling is larger for H-regions than for L-regions. As a result, H-regions *extract* the information rent and receive larger grants under asymmetric information than they receive under complete information. The case, in which $\mu_1(\xi)$ is increasing in ξ, can be analyzed in a similar way. Regardless of whether $\mu_1(\xi)$ is decreasing or increasing in ξ, the optimal allocation under asymmetric information is distorted for both top ($\underline{\xi}$) and bottom ($\bar{\xi}$) types, and importantly, the distortion is qualitatively reversed between the bottom and top types. For example, if $\mu_1(\xi)$ is decreasing in ξ, then the bottom type receives smaller grants and issues more debt, whereas the top type receives larger grants and issues less debt, respectively, than they would do in the full-information optimum.[14]

4.3 Implementation

We now proceed to implement the welfare optimum through regionally decentralized debt decisions. For the case of observable expenditure on the IPGs, the maximization problem of regions of type-ρ is expressed as

[14] In particular, if $g_1(\cdot) = \ln G_1$ and $g_2(\cdot) = \ln(\theta G_1 + \xi G_2)$, then $g_1''/g_2'' \leq \rho\theta(1+r)$ implies that $G_2/G_1 \leq [\sqrt{\rho\theta(1+r)} - \theta]\rho$ with $\sqrt{\rho\theta(1+r)} > \theta$; if $g_1(\cdot) = G_1^\alpha$ and $g_2(\cdot) = (\theta G_1 + \xi G_2)^\alpha$ for some parameter $\alpha \in (0,1)$, then $g_1''/g_2'' \leq \rho\theta(1+r)$ implies that $G_2/G_1 \leq \{[\rho\theta(1+r)]^{1/(2-\alpha)} - \theta\}\rho$ with $[\rho\theta(1+r)]^{1/(2-\alpha)} > \theta$. In other words, under log or power utility functions of public goods consumption, the technical conditions required for claims (i-c) and (ii-c) to hold in the case of observable physical output feature that the growth rate of local public goods provision must be bounded above.

$$\max_{b(\rho)} \; V(b(\rho), z(\rho), \rho)$$

for any given $z(\rho)$. We rewrite private consumptions as $c_1 = \tilde{\phi}(b(\rho), z(\rho), \rho)$ and $c_2 = \tilde{\psi}(b(\rho), z(\rho), \rho)$, and then applying the Envelope Theorem, we write the first-order condition as

$$g_1'\Big(y_1 + b(\rho) + z(\rho) - \tilde{\phi}(b(\rho), z(\rho), \rho)\Big) = [\rho(1+r) - \theta] \times$$
$$g_2'\Big(\theta\Big(y_1 + b(\rho) + z(\rho) - \tilde{\phi}(b(\rho), z(\rho), \rho)\Big) + \rho\Big(y_2 - (1+r)b(\rho) - \tilde{\psi}(b(\rho), z(\rho), \rho)\Big)\Big). \tag{13}$$

Making use of (13) and Lemma 4.1, the full-information optimum is immediately attained by simply setting $z(\rho) = z^{FB}(\rho)$ for all $\rho \in \Upsilon$.

Similarly, for the case of observable output, the maximization problem of regions of type-ξ is expressed by:

$$\max_{b(\xi)} \; V(b(\xi), z(\xi), \xi)$$

for any given $z(\xi)$. Rewriting public goods consumptions as $G_1 = \phi(b(\xi), z(\xi), \xi)$ and $G_2 = \psi(b(\xi), z(\xi), \xi)$, and applying the Envelope Theorem, enable the first-order condition to be written as

$$\begin{aligned} &u_1'(y_1 + b(\xi) + z(\xi) - \phi(b(\xi), z(\xi), \xi)) \\ &= (1+r)u_2'(y_2 - (1+r)b(\xi) - \xi\psi(b(\xi), z(\xi), \xi)). \end{aligned} \tag{14}$$

Making use of (14) and Lemma 4.1, we immediately attain the full-information optimum by simply setting $z(\xi) = z^{FB}(\xi)$ for all $\xi \in \Xi$.

Under asymmetric information, we obtain the following implementation scheme.

Proposition 4.3 *(Optimal Decentralization under DTP Shocks) The implementation scheme that decentralizes the asymmetric information optimum under DTP shocks is characterized as follows.*

(i) For the case of observable expenditure, we suppose that Assumption 4.1 holds, and that the second-order sufficient condition for incentive compatibility is not binding, namely, $\dot{b}(\rho) < 0$ is fulfilled. The grant scheme $z^*(b)$ that decentralizes $\{b^*(\rho), z^*(\rho)\}_{\rho \in \Upsilon}$ is then a nonlinear non-increasing function of b, almost everywhere differentiable, with the slope

$$\frac{dz^*}{db} \begin{cases} = 0 & \text{for } b \in \{b^*(\underline{\rho}), b^*(\bar{\rho})\}; \\ < 0 & \text{for } b \in (b^*(\bar{\rho}), b^*(\underline{\rho})). \end{cases}$$

(ii) For the case of observable output, we suppose that Assumption 4.2 holds and the second-order sufficient condition for incentive compatibility is not binding, namely, $\dot{b}(\xi) < 0$ is fulfilled. Then, the grant scheme $z^*(b)$ that decentralizes $\{b^*(\xi), z^*(\xi)\}_{\xi\in\Xi}$ is a nonlinear nondecreasing function of b, almost everywhere differentiable, with the slope

$$\frac{dz^*}{db}\begin{cases} = 0 & \text{for } b \in \{b^*(\underline{\xi}), b^*(\bar{\xi})\}; \\ > 0 & \text{for } b \in (b^*(\bar{\xi}), b^*(\underline{\xi})). \end{cases}$$

Proof See the "Appendix". □

For the case of observable expenditure on the IPGs (i.e., the physical output is unobservable), the monotonicity constraint of $\dot{b}(\rho) < 0$ means that H-regions are allowed to issue less debt than L-regions. Given that the intertemporal rate of transformation is $\rho(1 + r) - \theta$, as given by Proposition 4.1, H-regions with large ρ face higher opportunity costs of borrowing than the opportunity costs of borrowing facing L-regions with small ρ. As such, incentive compatibility is guaranteed by allowing regions with lower opportunity costs of borrowing to issue more government debt.

However, regarding the observable physical output of the IPGs (i.e., the expenditure is unobservable), the monotonicity constraint of $\dot{b}(\xi) < 0$ means that H-regions are allowed to issue more debt than L-regions. Second-generation residents of H-regions with small ξ, relative to their counterparts in L-regions with large ξ, pay lower taxes, denoted by ξG_2, to finance a given amount of local public goods provisions. As such, under the common individual income y_2 and interest rate r, the second-generation residents of H-regions are able to repay a higher level of public debt plus interest than their counterparts in L-regions. This explains the requirement that $\dot{b}(\xi) < 0$ guarantees incentive compatibility.

The intuition for a zero slope of the grant with respect to debt at the endpoints of type distribution is the same as that for Proposition 3.3. Other types of regions always have nonzero slopes. With regards to observable expenditure, the slope of the grant with respect to debt is negative, whereas there is a positive slope for the case of observable output. We intuitively specify the reasons for this difference below.

Firstly, with regards to observable expenditure, we see from Proposition 4.1 that the asymmetric information optimum features that the intertemporal rate of substitution is *greater* than the intertemporal rate of transformation. This implies that these regions *borrow too little* in relation to the first-best benchmark, which means they may face welfare losses in the presence of asymmetric information. The regions that are allowed to issue less debt under asymmetric information, may face greater welfare losses. As the incentive compatibility requires that H-regions are allowed to issue less debt than L-regions, the grant scheme that decentralizes the asymmetric information optimum should be designed such that H-regions receive more grants than L-regions. Otherwise, H-regions will choose to misreport their types by mimicking L-regions. This explains why the implementable grant scheme features that grants increase as debt decreases.

Secondly, with regards to the observable output, Proposition 4.1 shows that the asymmetric information optimum is such that the intertemporal rate of substitution is *smaller* than the intertemporal rate of transformation. This implies that these regions *borrow too much* relative to the first-best benchmark. Those regions allowed to issue more debt under asymmetric information, may face greater welfare losses. Incentive compatibility requires that H-regions are allowed to issue more debt than L-regions. Therefore, the grants scheme that decentralizes the asymmetric information optimum should be designed such that H-regions receive more grants than L-regions. Otherwise, H-regions will choose to misreport their types by mimicking L-regions. This explains why the implementable grant scheme features that grants increase as debt increases.

If the physical output of some IPGs, such as environmental protection, basic science and R&D, is unobservable by the center who is generally not involved in the production process, we subsequently have the following optimal funding structure of such IPGs: greater federal transfers and less local borrowing for regions with high degrees of technological progress for producing the IPGs, whilst less federal transfers and more local borrowing for regions with low degrees of technological progress. Consequently, when each region faced with this kind of shock has the autonomy to choose the level of government debt, federal transfers and local debt exhibits a sort of *substitutability* in terms of socially optimal regional insurance provision.

Meanwhile, with regards to IPGs with observable physical output, such as parks, public schools and highways, if the total spending is unobservable by the center, then the subsequent optimal funding structure is as follows: more (respectively less) federal transfers plus more (respectively less) local borrowing for regions with high (respectively low) degrees of technological progress for producing such IPGs. Therefore, when each region faced with this kind of shock has the autonomy to choose the level of government debt, federal transfers and local debt exhibit a sort of *complementarity* in terms of socially optimal regional insurance provision.

5 Conclusion

This paper is a theoretical study of designing and implementing optimal insurance provisions to sub-national regions against privately observable shocks. We consider two types of shocks to regional economies, one of which is to the degree of intergenerational spillovers induced by IPGs, and the other is to the degree of technological progress for producing the IPGs. We focus on the joint design of two widely-adopted public risk-sharing schemes — intergovernmental grants that provide *cross-region insurance* along the space dimension and public debt that provides *cross-generation insurance* along the time dimension. To the best of our knowledge, this paper is the first attempt reported in the literature concerning the provision of regional insurance within federations towards the joint design of these two risk-sharing schemes and the formal analysis of their interaction in the course of implementing welfare optima.

The asymmetric information welfare optima under alternative regional economic shocks have three main features. Firstly, the informational asymmetries considered here preclude the completeness of public insurance under risk-averse individual preferences. Secondly, intertemporal allocation is distorted for all but the bottom and top types; namely, the intertemporal rate of substitution (IRS) between current and future public goods consumption equals the intertemporal rate of transformation (IRT) only at the endpoints of a regional type distribution, as summarized in Table 1. Thirdly, if the intergovernmental grant received by the bottom type is distorted upward (or is large), then its public debt issuance must be distorted downward (or be low), and vice versa; meanwhile, the direction of distortion is qualitatively reversed between the bottom and top types.

To decentralize the welfare optima truthfully under regional debt issuance decisions, we have the following three predictions (see Table 1). First, for the top and bottom types of regions, the intergovernmental grant scheme that decentralizes the asymmetric information optimum turns out to be independent of regional public debt, regardless of the source of shocks and whether the expenditure on, or the output of, public goods is observable when regions differ in the degree of technological progress. Second, for all other types of regions when they differ in the degree of intergenerational externality, regional debt complements the grant scheme that decentralizes the welfare optimum. Third, for all other types of regions when they differ in the degree of technological progress, regional debt complements the grant scheme that decentralizes the welfare optimum only when the physical output of public goods is observable; otherwise, regional debt and the grant scheme act as substitutes in terms of decentralizing the asymmetric information welfare optimum. Therefore, it is worthwhile distinguishing between the case of observable input and the case of observable output for optimal regional insurance provision.

The additional insights that account for this distinction may be gained from combining the asymmetric information optimum with the corresponding implementation scheme. We observe that regions with a higher degree of technological progress should receive more federal transfers, ceteris paribus, regardless of whether expenditure or output is observable by the central government. This observation is intuitive provided that the center is a benevolent social planner. The transmission mechanism that leads to this claim under observable expenditure, however, departs

Table 1 Alternative asymmetric information optima and implementation schemes

mechanism design		
regional heterogeneity	intertemporal distortion	truthful implementation
DIE shocks	IRS = IRT for $\theta \in \{\underline{\theta}, \overline{\theta}\}$	$\frac{dz^*}{db} = 0$ for $\theta \in \{\underline{\theta}, \overline{\theta}\}$
	IRS < IRT for $\theta \in (\underline{\theta}, \overline{\theta})$	$\frac{dz^*}{db} > 0$ for $\theta \in (\underline{\theta}, \overline{\theta})$
DTP shocks: observable expenditure	IRS = IRT for $\rho \in \{\underline{\rho}, \overline{\rho}\}$	$\frac{dz^*}{db} = 0$ for $\rho \in \{\underline{\rho}, \overline{\rho}\}$
	IRS > IRT for $\rho \in (\underline{\rho}, \overline{\rho})$	$\frac{dz^*}{db} < 0$ for $\rho \in (\underline{\rho}, \overline{\rho})$
DTP shocks: observable output	IRS = IRT for $\xi \in \{\underline{\xi}, \overline{\xi}\}$	$\frac{dz^*}{db} = 0$ for $\xi \in \{\underline{\xi}, \overline{\xi}\}$
	IRS < IRT for $\xi \in (\underline{\xi}, \overline{\xi})$	$\frac{dz^*}{db} > 0$ for $\xi \in (\underline{\xi}, \overline{\xi})$

from that under observable output. Firstly, concerning the optimal allocation of local debt issuance, the centralized optimum under asymmetric information shows different features in these two cases. Under observable expenditure, the sufficient condition for incentive compatibility requires that regions with a lower degree of technological progress should be provided with a higher level of debt issuance. That is, the incentive compatibility constraints of low-types (with low degrees of technological progress) rather than high-types are more likely to be binding in the optimum. In contrast, under observable output, the sufficient condition of truth-telling requires that regions with a higher degree of technological progress should be provided with a higher level of debt issuance. Now, the incentive compatibility constraints of high-types rather than low-types are more likely to be binding in the optimum. Therefore, this distinction is indeed relevant in terms of solving the self-selection problem facing the center in the presence of asymmetric information regarding the exogenous degree of technological progress. Secondly, to implement these asymmetric information optima when borrowing and spending decisions are decentralized at the regional level, we prove that federal transfers and local debt are substitutes under observable expenditures but are complements under observable output.

As such, the observation mentioned above can be obtained by exploiting the following logic: under observable expenditure, regions with a *lower* degree of technological progress should be provided with a *higher* level of debt issuance and hence *fewer* federal transfers under *policy substitutability*, ceteris paribus. However, under observable output, regions with a *lower* degree of technological progress should be provided with a *lower* level of debt issuance and hence *fewer* federal transfers under *policy complementarity*. Although in both cases, regions with lower degrees of technological progress should receive fewer transfers due to *efficiency* considerations, in the course of achieving this goal through fiscal decentralization and intergovernmental transfer schemes, the two insurance policies act as substitutes under observable expenditure but as complements under observable output because the *self-selection problems* facing the center are essentially different in these two cases.

In addition to the previous analysis based on the assumption of unidimensional regional heterogeneity, in "Appendix B", we also discuss a more general case where there is heterogeneity in both the degree of intergenerational externality (DIE) and the degree of technological progress (DTP), which are unobservable by the center. We assume that the parameter of DTP is a continuously differentiable and monotonic function of the parameter of DIE, thereby reducing the multidimensional screening problem to a one-dimensional screening problem for the sake of technical tractability. Two main results are obtained. First, it remains true that the informational asymmetry between the center and the regions prevents complete public insurance from occurring. Second, federal transfers and local debt exhibit policy complementarity in terms of truthful implementation of asymmetric information optima, regardless of whether the physical output of the IPGs, or the expenditure on the IPGs, is observable by the center.

Appendix: Proofs

Proof of Proposition 3.1 We shall complete the proof in four steps.
Step 1: When a type-θ region is truth-telling, we define the value function as

$$v(\theta) \;\equiv\; V(b(\theta), z(\theta), \theta). \tag{15}$$

Applying the Envelope Theorem to (2), we get the following first-order necessary condition for the truth-telling constraints (5) to be satisfied:

$$\dot{v}(\theta) \;=\; g_2'(\theta\phi(b(\theta), z(\theta), \theta) + \psi(b(\theta), z(\theta), \theta))\phi(b(\theta), z(\theta), \theta), \tag{16}$$

where $G_1(\theta) \equiv \phi(b(\theta), z(\theta), \theta)$ and $G_2(\theta) \equiv \psi(b(\theta), z(\theta), \theta)$.
We now derive the second-order sufficient condition for incentive compatibility. Following some algebra, the local second-order condition of (5) can be written as

$$\dot{b}(\theta) \cdot V_z(b(\theta), z(\theta), \theta) \cdot \frac{\partial}{\partial \tilde{\theta}} \left(\frac{V_b(b(\theta), z(\theta), \tilde{\theta})}{V_z(b(\theta), z(\theta), \tilde{\theta})} \right) \bigg|_{\tilde{\theta}=\theta} \geq 0.$$

As $V_z(\cdot) = g_1' + \theta g_2' > 0$, the Spence-Mirrlees property reads as

$$\frac{\partial}{\partial \theta} \left(\frac{V_b}{V_z} \right) = \frac{(1+r)[(g_2')^2 - G_1 g_1' g_2'']}{(g_1' + \theta g_2')^2} > 0,$$

we must therefore have

$$\dot{b}(\theta) \geq 0 \tag{17}$$

which gives the desired monotonicity constraint. It is easy to verify that the local second-order condition also implies global optimality of the truth-telling strategy with the help of the above Spence-Mirrlees property.
Equivalently (17) can be written as

$$\dot{b}(\theta) \;=\; \beta(\theta), \quad \beta(\theta) \;\geq\; 0. \tag{18}$$

The center's problem is therefore to choose piecewise continuous control variables $b(\theta)$ and $z(\theta)$ to maximize

$$\int_{\underline{\theta}}^{\bar{\theta}} v(\theta) f(\theta) d\theta$$

subject to constraints (6), (15), (16) and (18).
Step 2: To solve the optimal control problem with integral and inequality constraints, we write the generalized Hamiltonian as:

$$\begin{aligned} \mathcal{H} = {} & v(\theta)f(\theta) + \mu_1(\theta)[V(b(\theta), z(\theta), \theta) - v(\theta)]f(\theta) + \mu_2(\theta)\beta(\theta) - \gamma z(\theta)f(\theta) \\ & + \eta_1(\theta) g_2'(\theta\phi(b(\theta), z(\theta), \theta) + \psi(b(\theta), z(\theta), \theta))\phi(b(\theta), z(\theta), \theta) + \eta_2(\theta)\beta(\theta), \end{aligned}$$

where $\mu_1(\theta)$, $\mu_2(\theta)$ and γ are non-negative Lagrange multipliers, and $\eta_1(\theta)$ and $\eta_2(\theta)$ are co-state variables. The first-order necessary conditions for a solution to the optimal control problem can now be stated as the state Eqs. (16) and (18), plus

$$\mathcal{H}_z = \mu_1(\theta)V_z(b(\theta), z(\theta), \theta)f(\theta) - \gamma f(\theta) + \eta_1(\theta)[g_2''(\theta\phi_z + \psi_z)\phi + g_2'\phi_z] = 0, \tag{19}$$

$$\mathcal{H}_\beta = \mu_2(\theta) + \eta_2(\theta) = 0, \tag{20}$$

and

$$\dot{\eta}_1(\theta) = -\mathcal{H}_v = [\mu_1(\theta) - 1]f(\theta), \tag{21}$$

$$\dot{\eta}_2(\theta) = -\mathcal{H}_b = -\mu_1(\theta)[g_1' - (1 + r - \theta)g_2']f(\theta) - \eta_1(\theta)[g_2''(\theta\phi_b + \psi_b)\phi + g_2'\phi_b]. \tag{22}$$

In addition, we have the following transversality conditions:

$$\eta_1(\theta) = \eta_2(\theta) = 0 \text{ for } \forall\theta \in \{\underline{\theta}, \bar{\theta}\}. \tag{23}$$

Step 3: Using (3) and the assumption that $\rho = 1$, we can write private consumptions as functions of debt, transfers and the degree of intergenerational spillovers: $c_1 \equiv \tilde{\phi}(b, z, \theta)$ and $c_2 \equiv \tilde{\psi}(b, z, \theta)$. When applying the Implicit Function Theorem to (3), we have these partial derivatives:

$$\begin{aligned} \tilde{\phi}_b(b, z, \theta) &= \frac{g_1''(u_2'' + g_2'') - (1 + r - \theta)\theta u_2'' g_2''}{\Sigma}, \\ \tilde{\psi}_b(b, z, \theta) &= \frac{\theta u_1'' g_2'' - (1 + r)(u_1'' + g_1'')g_2''}{\Sigma}; \end{aligned} \tag{24}$$

and

$$\tilde{\phi}_z(b, z, \theta) = \frac{g_1''(u_2'' + g_2'') + \theta^2 u_2'' g_2''}{\Sigma}, \ \tilde{\psi}_z(b, z, \theta) = \frac{\theta u_1'' g_2''}{\Sigma}; \tag{25}$$

with $\Sigma \equiv (u_1'' + g_1'')(u_2'' + g_2'') + \theta^2 u_2'' g_2'' > 0$. Using $\phi(b, z, \theta) = y_1 + b + z - \tilde{\phi}(b, z, \theta)$, $\psi(b, z, \theta) = y_2 - b(1 + r) - \tilde{\psi}(b, z, \theta)$, (24) and (25), we obtain

$$\begin{aligned} \phi_b &= \frac{u_1''(u_2'' + g_2'') + \theta^2 u_2'' g_2'' + (1 + r - \theta)\theta u_2'' g_2''}{\Sigma} > 0, \\ \psi_b &= -\frac{\theta u_1'' g_2'' + (1 + r)[(u_1'' + g_1'')u_2'' + \theta^2 u_2'' g_2'']}{\Sigma} < 0; \end{aligned} \tag{26}$$

and

$$\phi_z = \frac{u_1''(u_2'' + g_2'')}{\Sigma} > 0, \ \psi_z = -\frac{\theta u_1'' g_2''}{\Sigma} < 0. \tag{27}$$

Using (26) and (27), we obtain

$$\theta\phi_z + \psi_z = \frac{\theta u_1'' u_2''}{\Sigma} > 0, \ \ \theta\phi_b + \psi_b = -\frac{(1+r-\theta)u_1''u_2'' + (1+r)g_1''u_2''}{\Sigma} < 0. \quad (28)$$

Then, using (28) and (27) enables us to arrive at:

$$g_2''(\theta\phi_z + \psi_z)\phi + g_2'\phi_z = \frac{(\theta G_1 g_2'' + g_2')u_1''u_2'' + u_1''g_2''g_2'}{\Sigma} > 0 \quad (29)$$

under Assumption 3.1. Additionally, it is immediate from (28) and (26) that

$$g_2''(\theta\phi_b + \psi_b)\phi + g_2'\phi_b > 0. \quad (30)$$

Step 4: Since we are interested in the case without bunching, the monotonicity constraint (17) must be $\dot{b}(\theta) > 0$, and hence $\mu_2(\theta) = 0$ for all $\theta \in \Theta$ based on the complementary slackness conditions. By (20), we must have $\eta_2(\theta) = 0$ everywhere, yielding $\dot{\eta}_2 \equiv 0$. We consequently obtain from (22) and (30) that

$$\mu_1(\theta)[g_1' - (1+r-\theta)g_2']f(\theta) \ = \ \underbrace{-\eta_1(\theta)[g_2''(\theta\phi_b + \psi_b)\phi + g_2'\phi_b]}_{\leq 0},$$

which in combination with (23) and $\mu_1(\theta)f(\theta) > 0$ for all $\theta \in \Theta$, enables us to establish the results in Part (i).
Moreover, using (19) and (29) gives rise to

$$\mu_1(\theta)V_z(b(\theta), z(\theta), \theta)f(\theta) - \gamma f(\theta) \ = \ \underbrace{-\eta_1(\theta)[g_2''(\theta\phi_z + \psi_z)\phi + g_2'\phi_z]}_{\leq 0}$$

under Assumption 3.1. This, combined with (23) and $\mu_1(\theta)f(\theta) > 0$ for all $\theta \in \Theta$ completes the proof of Part (ii). □

Proof of Lemma 3.2 It follows from (21) and (23) that

$$\int_{\underline{\theta}}^{\bar{\theta}} [\mu_1(\theta) - 1]f(\theta)d\theta \ = \ \eta_1(\bar{\theta}) - \eta_1(\underline{\theta}) \ = \ 0. \quad (31)$$

By (19), $\mu_1(\theta)$ must be everywhere continuous. Therefore, if $\mu_1(\theta)$ is decreasing in θ, then (31) implies that $\mu_1(\theta) - 1$ is first positive and then negative as θ increases, and that the application of the Intermediate Value Theorem yields that there must be some $\tilde{\theta} \in (\underline{\theta}, \bar{\theta})$, such that $\mu_1(\tilde{\theta}) = 1$, as desired in Part (i). The proof of Part (ii) can be done analogously. □

Proof of Proposition 3.2 We shall complete the proof in two steps.
Step 1: Here we only need to show the proof of Part (i) because that of Part (ii) is similar. Applying $\rho = 1$ and the Envelope Theorem to (2) leads us to $V_z = g_1'(\phi(b,z,\theta)) + \theta g_2'(\theta\phi(b,z,\theta) + \psi(b,z,\theta))$. Using $V_z = g_1'(\phi(b,z,\theta)) + \theta g_2'(\theta\phi(b,z,\theta) + \psi(b,z,\theta))$, (27) and (28), we arrive at:

$$V_{zz}(b,z,\theta) \;=\; g_1''\phi_z+\theta g_2''(\theta\phi_z+\psi_z)<0,$$

which when combined with Lemma 3.2, produces the desired results, (i-a) and (i-b).
Step 2: We now proceed to prove result (i-c). It follows from Lemma 3.1 and Proposition 3.1 that the optimal debt policy is a solution to the equation

$$g_1'(\phi(b,z,\theta)) \;=\; (1+r-\theta)g_2'(\theta\phi(b,z,\theta)+\psi(b,z,\theta)) \tag{32}$$

for any $\theta\in\{\underline{\theta},\bar{\theta}\}$. Differentiating both sides of Eq. (32) with respect to z, and rearranging the algebra, reveal that

$$[g_1''\phi_b-(1+r-\theta)g_2''(\theta\phi_b+\psi_b)]\frac{db}{dz}=[(1+r-\theta)\theta g_2''-g_1'']\phi_z+(1+r-\theta)g_2''\psi_z.$$

Using (26) and (28) shows that $g_1''\phi_b-(1+r-\theta)g_2''(\theta\phi_b+\psi_b)<0$. Differentiating both sides of Eq. (32) with respect to G_1 reveals that $(1+r-\theta)\theta g_2''=g_1''$. Moreover, using (27) leads to $(1+r-\theta)g_2''\psi_z>0$. Consequently, we must have $db/dz<0$ for any $\theta\in\{\underline{\theta},\bar{\theta}\}$. Combining results (i-a) and (i-b) completes the proof. □

Proof of Proposition 3.3 Using (5) and applying the Envelope Theorem to (2), the first-order condition for incentive compatibility can be written as:

$$\left(g_1'+\theta g_2'\right)\frac{dz}{d\theta} \;=\; \left[(1+r-\theta)g_2'-g_1'\right]\frac{db}{d\theta},$$

which enables us to arrive at:

$$\frac{dz}{db} \;=\; \frac{dz}{d\theta}\frac{d\theta}{db} \;=\; \frac{(1+r-\theta)g_2'-g_1'}{g_1'+\theta g_2'}. \tag{33}$$

It follows from (19) and (22) that

$$g_1'+\theta g_2' \;=\; \frac{\gamma}{\mu_1(\theta)}-\frac{\eta_1(\theta)}{\mu_1(\theta)f(\theta)}[g_2''(\theta\phi_z+\psi_z)\phi+g_2'\phi_z] \tag{34}$$

and

$$(1+r-\theta)g_2'-g_1' \;=\; \frac{\eta_1(\theta)}{\mu_1(\theta)f(\theta)}[g_2''(\theta\phi_b+\psi_b)\phi+g_2'\phi_b], \tag{35}$$

whenever there is no bunching. Plugging (34) and (35) in (33) results in

$$\frac{dz}{db} \;=\; \frac{\eta_1(\theta(b))[g_2''(\theta(b)\phi_b+\psi_b)\phi+g_2'\phi_b]}{\gamma f(\theta(b))-\eta_1(\theta(b))[g_2''(\theta(b)\phi_z+\psi_z)\phi+g_2'\phi_z]}$$

where $\theta(b)$ is the inverse of $b(\theta)$, which exists given that $\dot{b}(\theta)>0$. Evidently, *dz/db* satisfies the property required. □

Proof of Proposition 4.1 We shall complete the proof in five steps.
Step 1: Applying the Envelope Theorem to the value function $V(b, z, \rho)$, and simplifying the algebra, we obtain the Spence-Mirrlees property:

$$\frac{\partial}{\partial \rho}\left[\frac{V_b(b, z, \rho)}{V_z(b, z, \rho)}\right] = -(1+r)\frac{\theta(g_2')^2 + g_1'[g_2' + \rho\mathcal{G}_2 g_2'']}{(g_1' + \theta g_2')^2} < 0$$

under Assumption 4.1. Noting that $V_z(\cdot) = g_1' + \theta g_2' > 0$, the second-order condition for incentive compatibility can be written as

$$\dot{b}(\rho) \cdot V_z(b(\rho), z(\rho), \rho) \cdot \frac{\partial}{\partial \tilde{\rho}}\left(\frac{V_b(b(\rho), z(\rho), \tilde{\rho})}{V_z(b(\rho), z(\rho), \tilde{\rho})}\right)\bigg|_{\tilde{\rho}=\rho} \geq 0,$$

which leads to $\dot{b}(\rho) \leq 0$ under Assumption 4.1, as desired in (11). If we equivalently rewrite this monotonicity constraint as $\dot{b}(\rho) = \beta(\rho)$ and $\beta(\rho) \leq 0$, then the Hamiltonian of the optimal control problem (11) is given by

$$\begin{aligned}\mathcal{H} = {} & v(\rho)f(\rho) + \mu_1(\rho)[V(b(\rho), z(\rho), \rho) - v(\rho)]f(\rho) - \mu_2(\rho)\beta(\rho) - \gamma z(\rho)f(\rho) \\ & + \eta_1(\rho)g_2'(\theta\phi(b(\rho), z(\rho), \rho) + \rho\psi(b(\rho), z(\rho), \rho))\psi(b(\rho), z(\rho), \rho) + \eta_2(\rho)\beta(\rho),\end{aligned}$$

where $\phi(b(\rho), z(\rho), \rho) = G_1(\rho)$, $\psi(b(\rho), z(\rho), \rho) = \mathcal{G}_2(\rho)$, $\mu_1(\rho)$, $\mu_2(\rho)$ and γ are non-negative Lagrange multipliers, and $\eta_1(\rho)$ and $\eta_2(\rho)$ are co-state variables. The first-order necessary conditions are given by

$$\mathcal{H}_z = \mu_1(\rho)V_z(b(\rho), z(\rho), \rho)f(\rho) - \gamma f(\rho) + \eta_1(\rho)[g_2''(\theta\phi_z + \rho\psi_z)\psi + g_2'\psi_z] = 0, \tag{36}$$

$$\mathcal{H}_\beta = -\mu_2(\rho) + \eta_2(\rho) = 0, \tag{37}$$

and

$$\dot{\eta}_1(\rho) = -\mathcal{H}_v = [\mu_1(\rho) - 1]f(\rho), \tag{38}$$

$$\begin{aligned}\dot{\eta}_2(\rho) = -\mathcal{H}_b = {} & -\mu_1(\rho)\{g_1' - [\rho(1+r) - \theta]g_2'\}f(\rho) - \eta_1(\rho)[g_2''(\theta\phi_b + \rho\psi_b) \\ & \psi + g_2'\psi_b].\end{aligned} \tag{39}$$

In addition, we have the following transversality conditions:

$$\eta_1(\rho) = \eta_2(\rho) = 0 \ \text{ for } \forall\rho \in \{\underline{\rho}, \bar{\rho}\}. \tag{40}$$

Step 2: Applying the Implicit Function Theorem to (8) gives rise to:

$$\phi_b = \frac{u_1''(u_2'' + \rho^2 g_2'') + \rho\theta(1+r)u_2''g_2''}{M} > 0, \ \ \phi_z = \frac{u_1''(u_1'' + \rho^2 g_2'')}{M} > 0; \tag{41}$$

and

$$\psi_b = -\frac{(1+r)[(u_1''+g_1'')u_2''+\theta^2u_2''g_2'']+\rho\theta u_1''g_2''}{M}<0, \quad \psi_z = -\frac{\rho\theta u_1''g_2''}{M}<0 \tag{42}$$

in which $M \equiv (u_1''+g_1'')(u_2''+\rho^2g_2'')+\theta^2u_2''g_2''>0$. Making use of (41) and (42), we have

$$g_2''(\theta\phi_z+\rho\psi_z)\psi+g_2'\psi_z < 0 \tag{43}$$

given that

$$\theta\phi_z+\rho\psi_z = \frac{\theta u_1''u_2''}{M} > 0. \tag{44}$$

In addition, we get by (41), (42) and

$$\theta\phi_b+\rho\psi_b = -\frac{[\rho(1+r)-\theta]u_1''u_2''+\rho(1+r)u_2''g_1''}{M} < 0 \tag{45}$$

that

$$\begin{aligned} & g_2''(\theta\phi_b+\rho\psi_b)\psi+g_2'\psi_b \\ & =-\frac{[\rho\mathcal{G}_2g_2''+g_2'](1+r)(u_1''+g_1'')u_2''-\theta\mathcal{G}_2u_1''u_2''g_2''+[(1+r)\theta u_2''+\rho u_1'']\theta g_2'g_2''}{M}<0 \end{aligned} \tag{46}$$

under Assumption 4.1.

Step 3: Since we focus on the case without bunching, we must have $\mu_2(\rho)=0$ for all $\rho\in\Upsilon$. By (37), we have $\eta_2(\rho)=0$ everywhere, implying that $\dot\eta_2\equiv 0$. Applying $\dot\eta_2\equiv 0$, (40) and (46) to (39) yields the desired assertion in Part (i). Finally, applying (43), (40) and $\mu_1(\rho)f(\rho)>0$ to (36) produces the desired assertion in Part (ii) for the case of observable expenditure on the IPGs.

Step 4: As before, with regards to observable physical output of the IPGs, the Hamiltonian of the optimal control problem (12) is given by

$$\begin{aligned}\mathcal{H} &= v(\xi)f(\xi)+\mu_1(\xi)[V(b(\xi),z(\xi),\xi)-v(\xi)]f(\xi)-\mu_2(\xi)\beta(\xi)-\gamma z(\xi)f(\xi) \\ &\quad -\eta_1(\xi)u_2'(y_2-(1+r)b(\xi)-\xi\psi(b(\xi),z(\xi),\xi))\psi(b(\xi),z(\xi),\xi)+\eta_2(\xi)\beta(\xi).\end{aligned}$$

The first-order necessary conditions are given by

$$\mathcal{H}_z = \mu_1(\xi)V_z(b(\xi),z(\xi),\xi)f(\xi)-\gamma f(\xi)-\eta_1(\xi)(-\xi u_2''\psi_z\psi+u_2'\psi_z)=0, \tag{47}$$

$$\mathcal{H}_\beta = -\mu_2(\xi)+\eta_2(\xi)=0, \tag{48}$$

and

$$\dot\eta_1(\xi) = -\mathcal{H}_v = [\mu_1(\xi)-1]f(\xi), \tag{49}$$

$$\dot{\eta}_2(\xi) = -\mathcal{H}_b \ = \ -\mu_1(\xi)[u_1' - (1+r)u_2']f(\xi) + \eta_1 (\xi)[-(1+r)u_2''\psi - \xi u_2''\psi_b\psi + u_2'\psi_b]. \tag{50}$$

Additionally, we have the following transversality conditions:

$$\eta_1(\xi) \ = \ \eta_2(\xi) \ = \ 0 \ \text{ for } \forall\xi \in \{\underline{\xi}, \bar{\xi}\}. \tag{51}$$

Step 5: Applying the Implicit Function Theorem to (10) gives rise to:

$$\phi_b \ = \ \frac{u_1''(\xi^2 u_2'' + g_2'') + \xi\theta(1+r)u_2''g_2''}{Q} > 0, \ \ \phi_z \ = \ \frac{u_1''(\xi^2 u_2'' + g_2'')}{Q} > 0; \tag{52}$$

and

$$\psi_b \ = \ -\frac{\xi(1+r)(u_1'' + g_1'' + \theta^2 g_2'')u_2'' + \theta u_1''g_2''}{Q} < 0, \ \ \psi_z \ = \ -\frac{\theta u_1'' g_2''}{Q} < 0 \tag{53}$$

in which $Q \equiv (u_1'' + g_1'')(\xi^2 u_2'' + g_2'') + \xi^2\theta^2 u_2'' g_2'' > 0$. We now apply (51) and (53) to (47), which gives the desired assertion in Part (ii). Moreover, using (53) again reveals that

$$\begin{aligned} &-(1+r)u_2''\psi - \xi u_2''\psi_b\psi + u_2'\psi_b \ = \\ &-\frac{(1+r)(u_1'' + g_1'')u_2''(g_2''\psi + g_2') + \theta u_1''g_2''(u_2' - \xi\psi u_2'') + \theta^2(1+r)u_1''u_2''g_2'}{Q} < 0 \end{aligned} \tag{54}$$

under Assumption 4.2. In the case of no bunching, applying (48), (51) and (54) to (50) produces Part (i) for the case of observable output of the IPGs. □

Proof of Proposition 4.2 We shall complete the proof in two steps.
Step 1: Using (38), the proof is quite similar to that of Proposition 3.2. Therefore, we only need to show the following for the case of observable expenditure on the IPGs. Firstly, using (41) and (44) reveals that $V_{zz} = g_1''\phi_z + \theta g_2''(\theta\phi_z + \rho\psi_z) < 0$ for all $\rho \in \Upsilon$. Secondly, by differentiating both sides of equation $g_1' = [\rho(1+r) - \theta]g_2'$ with respect to z, we obtain

$$\underbrace{[g_1''\phi_b - [\rho(1+r) - \theta]g_2''(\theta\phi_b + \rho\psi_b)]}_{<0}\frac{db}{dz} \ = \ [\rho(1+r) - \theta]g_2''(\theta\phi_z + \rho\psi_z) - g_1''\phi_z$$

under (41) and (45). Using (41) and (44), we arrive at:

$$[\rho(1+r) - \theta]g_2''(\theta\phi_z + \rho\psi_z) - g_1''\phi_z \ = \ -\frac{\rho^2 u_1'' g_1'' g_2''}{M} > 0,$$

and thus, have $db/dz < 0$ at the welfare optimum.
Step 2: For the case of observable output of the IPGs, the proof follows from using (49), (51), Lemma 4.1 and Proposition 4.1. Here, using (52), (53), and the equation $u_1' = (1+r)u_2'$ evaluated at the welfare optimum, we only need to show that

$$V_{zz} = u_1''(1 - \phi_z) = \frac{u_1'' g_1''(\xi^2 u_2'' + g_2'') + \theta^2 \xi^2 u_1'' u_2'' g_2''}{Q} < 0$$

and

$$[(1 - \phi_b)u_1'' + (1 + r)(1 + r + \xi\psi_b)u_2''] \frac{db}{dz} = -\xi(1 + r)u_2''\psi_z - (1 - \phi_z)u_1''$$

in which

$$\begin{aligned}(1 - \phi_b)u_1'' &+ (1 + r)(1 + r + \xi\psi_b)u_2'' \\ &= \frac{u_1'' g_1''(\xi^2 u_2'' + g_2'') + (1 + r - \theta\xi)^2 u_1'' u_2'' g_2'' + (1 + r)^2 u_2'' g_1'' g_2''}{Q} < 0\end{aligned}$$

and

$$\begin{aligned}&- \xi(1 + r)u_2''\psi_z - (1 - \phi_z)u_1'' \\ &= - \frac{\xi u_1'' u_2''[\xi g_1'' - (1 + r)\theta g_2''] + \theta^2 \xi^2 u_1'' u_2'' g_2'' + u_1'' g_1'' g_2''}{Q} > 0\end{aligned}$$

whenever $g_1'' \leq \rho\theta(1 + r)g_2''$ holds. □

Proof of Proposition 4.3 We shall complete the proof in two steps.
Step 1: The key for a grant scheme to decentralize the asymmetric-information optimum is to take into account the incentive-compatibility constraint. Firstly, we make use of the first-order necessary condition for incentive compatibility and arrive at

$$\frac{dz}{db} = \frac{dz}{d\rho}\frac{d\rho}{db} = -\frac{V_b}{V_z}.$$

As the monotonicity constraint is assumed to be not binding, (36) and (39) enable us to arrive at

$$\frac{dz}{db} = \frac{\eta_1(\rho(b))\left[g_2''(\theta\phi_b + \rho(b)\psi_b)\psi + g_2'\psi_b\right]}{\gamma f(\rho(b)) - \eta_1(\rho(b))\left[g_2''(\theta\phi_z + \rho(b)\psi_z)\psi + g_2'\psi_z\right]},$$

in which $\rho(b)$ denotes the inverse of $b(\rho)$. Secondly, using (40), (43) and (46) enables the proof of Part (i) to be immediately complete.
Step 2: We next focus on the case of no bunching. We have by using the first-order necessary condition for incentive compatibility, (47) and (50) that

$$\frac{dz^*}{db} = \frac{-\eta_1(\xi(b))[-(1 + r)u_2''\psi - \xi(b)u_2''\psi_b\psi + u_2'\psi_b]}{\gamma f(\xi(b)) + \eta_1(\xi(b))[-\xi(b)u_2''\psi_z\psi + u_2'\psi_z]},$$

in which $\xi(b)$ denotes the inverse of $b(\xi)$, under the assumption that $\dot{b}(\xi) < 0$. Exploiting (54) and (51) yields that dz^*/db satisfies the property required in Part (ii). □

Appendix B: discussion on multidimensional heterogeneity

In the main text we focus on the analysis of one-dimensional, unobserved heterogeneity, which is either the degree of intergenerational externality induced by IPGs, as denoted by parameter θ, or the degree of technological progress for producing the IPGs, as denoted by parameter ρ (or equivalently ξ). In this appendix, we attempt to analyze the case with multidimensional heterogeneity, i.e., regions differ in both the degree of intergenerational externality and the degree of technological progress. Nevertheless, in order to obtain more meaningful theoretical results, we need to impose the following restriction (Dai and Tian 2022).

Assumption 5.1 Let $\xi \equiv \Psi(\theta)$ and $\rho = 1/\xi = 1/\Psi(\theta) \equiv \Phi(\theta)$, in which $\Psi(\cdot)$ is a continuously differentiable function satisfying $\Psi'(\cdot) > 0$.

Both the degree of intergenerational externality and the degree of technological progress are closely related to the IPGs, and so Assumption 5.1 infers there is a publicly observable functional relationship that governs these two parameters. In particular, $\Psi'(\theta) > 0$ indicates that a higher degree of intergenerational externality induced by IPGs leads to a higher per unit cost of producing the IPGs. Intuitively, we assume that if a public good is of higher quality, durability or intergenerational spillovers, then the per unit cost of production tends to be higher. For example, Ollivier et al. (2014) research the construction costs of high-speed railways in China and show that the weighted average unit cost for a passenger-dedicated line is RMB 129 million per km for a 350 km/h project, and RMB 87 million per km for a 250 km/h project. As we can reasonably assume that an increasing number of passengers will take high-speed trains in the future, we may roughly interpret that a 350 km/h project generates higher intergenerational spillovers than a 250 km/h project, thus justifying Assumption 5.1. Additionally, Assumption 5.1 helps us to consider the case with multidimensional heterogeneity, but with only one-dimensional asymmetric (or private) information.

As in Sect. 4, whenever regions face shocks to the degree of technological progress for producing public goods, we need to distinguish the case with observable expenditure on public goods to the case with observable physical output of public goods.

I. The case with observable expenditure on public goods

Applying Assumption 5.1, the FOCs given by Eq. (8) can be rewritten as:

$$u_1'(c_1) = g_1'(G_1) + \theta g_2'(\theta G_1 + \Phi(\theta)\mathcal{G}_2) \quad \text{and} \quad u_2'(c_2) = \Phi(\theta) g_2'(\theta G_1 + \Phi(\theta)\mathcal{G}_2). \tag{55}$$

As in the main text, we write the value function as $v(\theta) \equiv V(b(\theta), z(\theta), \theta)$. Applying Assumption 5.1 and the Envelope Theorem to (2) produces the following first-order necessary condition for incentive compatibility:

$$\begin{aligned} \dot{v}(\theta) = g_2'(\theta\phi(b(\theta), z(\theta), \theta) + \Phi(\theta)\psi(b(\theta), z(\theta), \theta)) \\ \times [\phi(b(\theta), z(\theta), \theta) + \Phi'(\theta)\psi(b(\theta), z(\theta), \theta)], \end{aligned} \tag{56}$$

in which $G_1(\theta) \equiv \phi(b(\theta), z(\theta), \theta)$ and $\mathcal{G}_2(\theta) \equiv \psi(b(\theta), z(\theta), \theta)$. If we apply the Envelope Theorem to value function $V(b, z, \theta)$, we obtain $V_b(b, z, \theta) = g_1' + g_2' \cdot [\theta - \Phi(\theta)(1 + r)]$, and $V_z(b, z, \theta) = g_1' + \theta g_2' > 0$ for all $\theta \in \Theta$, which enable us to establish:

Lemma 5.1 Under Assumption 5.1, if $G_1(\theta)/\mathcal{G}_2(\theta) \geq -\Phi'(\theta)$, then the global optimality of truth-telling strategy is guaranteed by the second-order condition, $\dot{b}(\theta) \geq 0$ for all $\theta \in \Theta$.

Proof In light of (2) and Assumption 5.1 we get

$$\begin{aligned} \frac{\partial}{\partial\theta}\left[\frac{V_b(b, z, \theta)}{V_z(b, z, \theta)}\right] &= \frac{1 + r}{(g_1' + \theta g_2')^2} \\ &\times \left\{\underbrace{[\Phi(\theta) - \theta\Phi'(\theta)](g_2')^2 - \Phi'(\theta)g_1'g_2'}_{>0} - \underbrace{\Phi(\theta)g_1'g_2''}_{<0}[G_1 + \Phi'(\theta)\mathcal{G}_2]\right\}, \end{aligned}$$

and so

$$\frac{\partial}{\partial\theta}\left[\frac{V_b(b, z, \theta)}{V_z(b, z, \theta)}\right] > 0 \tag{57}$$

whenever $G_1 + \Phi'(\theta)\mathcal{G}_2 \geq 0$. Condition (57) thus guarantees the Spence-Mirrlees property. The second-order condition for incentive compatibility can be expressed as:

$$\dot{b}(\theta) \cdot V_z(b(\theta), z(\theta), \theta) \cdot \frac{\partial}{\partial\tilde{\theta}}\left(\frac{V_b(b(\theta), z(\theta), \tilde{\theta})}{V_z(b(\theta), z(\theta), \tilde{\theta})}\right)\Bigg|_{\tilde{\theta}=\theta} \geq 0,$$

which when combined with $V_z > 0$ and Spence-Mirrlees property (57) reveals that $\dot{b}(\theta) \geq 0$ must hold. Applying the standard argument given by Laffont and Martimort (2002), the proof is then complete. □

Lemma 5.1 states that truth-telling requires a regional debt allocation, which is non-decreasing in the degree of intergenerational externality. Condition $G_1(\theta)/\mathcal{G}_2(\theta) \geq -\Phi'(\theta)$ means that the ratio of Period-1 public goods expenditure to Period-2 public goods expenditure is greater than some lower bound. For later use, we provide

Assumption 5.2 $G_1(\theta)/\mathcal{G}_2(\theta) \geq -\Phi'(\theta)$ for all $\theta \in \Theta$.

We now safely replace the global incentive-compatibility condition (5) by (56) and $\dot{b}(\theta) \geq 0$ established in Lemma 5.1, and so the optimization problem facing the center is formalized as:

$$\max \int_{\underline{\theta}}^{\bar{\theta}} v(\theta)f(\theta)d\theta$$
$$s.t. v(\theta) = V(b(\theta), z(\theta), \theta);$$
$$\int_{\underline{\theta}}^{\bar{\theta}} z(\theta)f(\theta)d\theta \ \leq 0;$$
$$\dot{v}(\theta) \ = g_2'(\theta\phi(b(\theta), z(\theta), \theta) + \Phi(\theta)\psi(b(\theta), z(\theta), \theta)) \times [\phi(b(\theta), z(\theta), \theta) + \Phi'(\theta)\psi(b(\theta), z(\theta), \theta)];$$
$$\dot{b}(\theta) \ \geq 0.$$

By solving this problem, we arrive at the following proposition:

Proposition 5.1 Let us suppose Assumptions 5.1 and 5.2 hold. In the asymmetric-information case without bunching, the welfare optimum $\{b^*(\theta), z^*(\theta)\}_{\theta\in\Theta}$ satisfies:

(i) With regards to the relationship between the intertemporal rate of substitution between current and future public goods consumption and the intertemporal rate of transformation, we reach:

$$\frac{g_1'(G_1^*(\theta))}{g_2'(\theta G_1^*(\theta) + \Phi(\theta)\mathcal{G}_2^*(\theta))} \begin{cases} = \ \Phi(\theta)(1+r) - \theta & \text{for } \theta \in \{\underline{\theta}, \bar{\theta}\}; \\ < \ \Phi(\theta)(1+r) - \theta & \text{for } \theta \in (\underline{\theta}, \bar{\theta}). \end{cases}$$

(ii) Let $\mu_1(\theta) > 0$ be the Lagrange multiplier on the value constraint $v(\theta) \equiv V(b(\theta), z(\theta), \theta)$ of any type-θ region that is reporting truthfully, and we obtain:

- $V_z(b^*(\theta), z^*(\theta), \theta) = \gamma/\mu_1(\theta)$ for $\theta \in \{\underline{\theta}, \bar{\theta}\}$;
- If the ratio $G_1(\theta)/\mathcal{G}_2(\theta)$ is sufficiently close to $-\Phi'(\theta)$, then

$$V_z(b^*(\theta), z^*(\theta), \theta) < \gamma/\mu_1(\theta)$$

  for $\theta \in (\underline{\theta}, \bar{\theta})$;
- If the ratio $G_1(\theta)/\mathcal{G}_2(\theta)$ is sufficiently larger than $-\Phi'(\theta)$, then for any $\theta \in (\underline{\theta}, \bar{\theta})$ we have that:

$$V_z(b^*(\theta), z^*(\theta), \theta) \begin{cases} < \ \gamma/\mu_1(\theta) & \text{for } |\varepsilon_{g_2', \theta\phi+\Phi(\theta)\psi}| \cdot \varepsilon_{\theta\phi+\Phi(\theta)\psi, z} < \varepsilon_{\phi+\Phi'(\theta)\psi, z}, \\ = \ \gamma/\mu_1(\theta) & \text{for } |\varepsilon_{g_2', \theta\phi+\Phi(\theta)\psi}| \cdot \varepsilon_{\theta\phi+\Phi(\theta)\psi, z} = \varepsilon_{\phi+\Phi'(\theta)\psi, z}, \\ > \ \gamma/\mu_1(\theta) & \text{for } |\varepsilon_{g_2', \theta\phi+\Phi(\theta)\psi}| \cdot \varepsilon_{\theta\phi+\Phi(\theta)\psi, z} > \varepsilon_{\phi+\Phi'(\theta)\psi, z}, \end{cases}$$

  in which $|\varepsilon_{g_2', \theta\phi+\Phi(\theta)\psi}|$ represents the absolute value of the elasticity of g_2' with respect to the amount of Period-2 public goods consumption $\theta\phi + \Phi(\theta)\psi$, $\varepsilon_{\theta\phi+\Phi(\theta)\psi, z} > 0$ represents the elasticity of the amount of

Period-2 public goods consumption with respect to the federal transfers z, and $\varepsilon_{\phi+\Phi'(\theta)\psi,z} > 0$ represents the elasticity of $\phi + \Phi'(\theta)\psi$ with respect to z.

Proof We shall complete the proof in four steps.
Step 1: Firstly, we let $\dot{b}(\theta) \equiv \beta(\theta)$ as before, and write the generalized Hamiltonian as:

$$\begin{aligned}\mathcal{H} &= v(\theta)f(\theta) + \mu_1(\theta)[V(b(\theta), z(\theta), \theta) - v(\theta)]f(\theta) + \mu_2(\theta)\beta(\theta) - \gamma z(\theta)f(\theta)\\ &+ \eta_1(\theta)g_2'(\theta\phi(b(\theta), z(\theta), \theta) + \Phi(\theta)\psi(b(\theta), z(\theta), \theta))\\ &\times [\phi(b(\theta), z(\theta), \theta) + \Phi'(\theta)\psi(b(\theta), z(\theta), \theta)] + \eta_2(\theta)\beta(\theta),\end{aligned}$$

where $\mu_1(\theta)$, $\mu_2(\theta)$ and γ are non-negative Lagrange multipliers, and $\eta_1(\theta)$ and $\eta_2(\theta)$ are co-state variables. The first-order necessary conditions are

$$\begin{aligned}\mathcal{H}_z &= \mu_1(\theta)V_z(b(\theta), z(\theta), \theta)f(\theta) - \gamma f(\theta)\\ &+ \eta_1(\theta)\{g_2'' \cdot [\theta\phi_z + \Phi(\theta)\psi_z][\phi + \Phi'(\theta)\psi] + g_2' \cdot [\phi_z + \Phi'(\theta)\psi_z]\} = 0,\end{aligned} \tag{58}$$

$$\mathcal{H}_\beta = \mu_2(\theta) + \eta_2(\theta) = 0, \tag{59}$$

$$\dot{\eta}_1(\theta) = -\mathcal{H}_v = [\mu_1(\theta) - 1]f(\theta), \tag{60}$$

and

$$\begin{aligned}\dot{\eta}_2(\theta) &= -\mathcal{H}_b\\ &= -\mu_1(\theta)\{g_1' - [\Phi(\theta)(1+r) - \theta]g_2'\}f(\theta)\\ &- \eta_1(\theta)\{g_2'' \cdot [\theta\phi_b + \Phi(\theta)\psi_b][\phi + \Phi'(\theta)\psi] + g_2' \cdot [\phi_b + \Phi'(\theta)\psi_b]\}.\end{aligned} \tag{61}$$

In addition, we have the following transversality conditions:

$$\eta_1(\theta) = \eta_2(\theta) = 0 \ \text{ for } \forall\theta \in \{\underline{\theta}, \bar{\theta}\}. \tag{62}$$

Step 2: Following the first-order approach, we must have $\mu_2(\theta) = 0$ for all $\theta \in \Theta$. Using (59) yields $\eta_2(\theta) = 0$ for all $\theta \in \Theta$, and hence we must have $\dot{\eta}_2 \equiv 0$. Applying $\dot{\eta}_2 \equiv 0$, $\mu_1(\theta)f(\theta) > 0$ and (62) to (61) reveals that $g_1' = [\Phi(\theta)(1+r) - \theta]g_2'$ for $\theta \in \{\underline{\theta}, \bar{\theta}\}$, as desired in Part (i). Additionally, applying (62) to (58) reveals that $\mu_1(\theta)V_z(b(\theta), z(\theta), \theta) = \gamma$ for $\theta \in \{\underline{\theta}, \bar{\theta}\}$, as desired in Part (ii).
Step 3: By applying the Implicit Function Theorem to the FOCs given by (55), we can still have those partial derivatives given by Eqs. (41) and (42). Consequently, (41), (42), (45) and Assumption 5.2 enable us to arrive at

$$\underbrace{g_2''}_{<0} \cdot \underbrace{[\theta\phi_b + \Phi(\theta)\psi_b]}_{<0} \cdot \underbrace{[\phi + \Phi'(\theta)\psi]}_{\geq 0} + \underbrace{g_2'}_{>0} \cdot \underbrace{[\phi_b + \Phi'(\theta)\psi_b]}_{>0} > 0, \tag{63}$$

which when combined with $\dot{\eta}_2 \equiv 0$, $\mu_1(\theta)f(\theta) > 0$ and (61) concludes the proof of Part (i).

Step 4: Finally, using (41), (42), (44) and Assumption 5.2 shows that

$$\underbrace{g_2'' \cdot [\theta\phi_z + \Phi(\theta)\psi_z]}_{<0} \cdot \underbrace{[\phi + \Phi'(\theta)\psi]}_{\geq 0} + \underbrace{g_2' \cdot [\phi_z + \Phi'(\theta)\psi_z]}_{>0}.$$

Thus, the sign of this formula is positive whenever $\phi + \Phi'(\theta)\psi$ is sufficiently close to 0 from above, which when applied to Eq. (58) gives the second result in Part (ii). However, if $\phi + \Phi'(\theta)\psi$ is sufficiently larger than 0, then again using (41), (42), (44) and Assumption 5.2 yields:

$$\underbrace{g_2'' \cdot [\theta\phi_z + \Phi(\theta)\psi_z]}_{-} \cdot \underbrace{[\phi + \Phi'(\theta)\psi]}_{+} + \underbrace{g_2' \cdot [\phi_z + \Phi'(\theta)\psi_z]}_{+} < 0$$

$$\Leftrightarrow \underbrace{-\frac{[\theta\phi + \Phi(\theta)\psi]g_2''}{g_2'}}_{|\varepsilon_{g_2',\theta\phi+\Phi(\theta)\psi}|} \cdot \underbrace{\frac{z[\theta\phi_z + \Phi(\theta)\psi_z]}{\theta\phi + \Phi(\theta)\psi}}_{\varepsilon_{\theta\phi+\Phi(\theta)\psi,z}} > \underbrace{\frac{z[\phi_z + \Phi'(\theta)\psi_z]}{\phi + \Phi'(\theta)\psi}}_{\varepsilon_{\phi+\Phi'(\theta)\psi,z}}$$

which when applied to Eq. (58) concludes the proof of Part (ii). □

The key message conveyed by Proposition 5.1 can be summarized in two ways: (1) in the asymmetric-information optimum only the intertemporal allocation at the endpoints of type distribution is not distorted with respect to the first-best; (2) the presence of the asymmetric information between the center and regions prevents full insurance.

We now proceed to the implementation of the asymmetric-information optimum established in Proposition 5.1 through decentralized regional debt decisions. Note that decentralized debt issuance decisions require that the intertemporal rate of substitution must be equal to the intertemporal rate of transformation:

$$\frac{g_1'(\phi(b(\theta), z(\theta), \theta))}{g_2'(\theta\phi(b(\theta), z(\theta), \theta) + \Phi(\theta)\psi(b(\theta), z(\theta), \theta))} = \Phi(\theta)(1 + r) - \theta,$$

which is desired by each region for any given amount of federal transfers. The center is faced with the task of designing an intergovernmental grants scheme that guarantees incentive compatibility for all regions. Indeed, we can obtain the following result:

Proposition 5.2 The grant scheme $z^*(b)$ that decentralizes the asymmetric-information optimum $\{b^*(\theta), z^*(\theta)\}_{\theta\in\Theta}$ is a nonlinear nondecreasing function of b, almost everywhere differentiable, with the slope

$$\frac{dz^*}{db} \begin{cases} = & 0 \quad \text{for } b \in \{b^*(\underline{\theta}), b^*(\bar{\theta})\}; \\ > & 0 \quad \text{for } b \in (b^*(\underline{\theta}), b^*(\bar{\theta})). \end{cases}$$

Proof Making use of the first-order necessary condition for incentive compatibility, we reach

$$\frac{dz}{db} = \frac{dz}{d\theta}\frac{d\theta}{db} = -\frac{V_b}{V_z}.$$

Note that we have shown above that $V_z > 0$ always holds true. As we focus on the case without bunching, we get from (61) and (63) that

$$\begin{aligned} & - V_b(b(\theta), z(\theta), \theta) \\ & = \frac{\eta_1(\theta)}{\mu_1(\theta)f(\theta)}\{g_2'' \cdot [\theta\phi_b + \Phi(\theta)\psi_b][\phi + \Phi'(\theta)\psi] + g_2' \cdot [\phi_b + \Phi'(\theta)\psi_b]\} > 0 \end{aligned}$$

for all $\theta \in (\underline{\theta}, \bar{\theta})$. We thus have the following for all $b \in (b^*(\underline{\theta}), b^*(\bar{\theta}))$:

$$\frac{dz}{db} = \frac{\eta_1(\theta(b))\{g_2'' \cdot [\theta(b)\phi_b + \Phi(\theta(b))\psi_b][\phi + \Phi'(\theta(b))\psi] + g_2' \cdot [\phi_b + \Phi'(\theta(b))\psi_b]\}}{\mu_1(\theta(b))f(\theta(b))V_z} > 0$$

in which $\theta(b)$ denotes the inverse of $b(\theta)$ by a little abuse of notation. The inverse does exist by using Lemma 5.1. Finally, applying (62) immediately completes the proof. □

The main conclusion we obtain from Proposition 5.2 is that federal transfers and local debt generally play a complementary role for regional insurance provision. Excluding the bottom and top types whose intertemporal allocations are undistorted in the asymmetric information optimum, the optimal funding structure in the case with observable expenditure on IPGs exhibits the following feature: regions with a higher degree of intergenerational externality should issue more debt and receive more federal transfers than regions with a lower degree of intergenerational externality.

II. The case with observable physical output of public goods

Using Assumption 5.1, the value function given by (9) can be rewritten as:

$$\begin{aligned} V(b, z, \theta) \equiv & \max_{G_1, G_2} u_1(y_1 + b + z - G_1) + g_1(G_1) \\ & + u_2(y_2 - b(1 + r) - \Psi(\theta)G_2) + g_2(\theta G_1 + G_2). \end{aligned} \tag{64}$$

The first-order conditions are thus given by:

$$\begin{aligned} u_1'(y_1+b+z-G_1) &= g_1'(G_1)+\theta g_2'(\theta G_1+G_2) \quad \text{and} \\ \Psi(\theta)u_2'(y_2-b(1+r)-\Psi(\theta)G_2) &= g_2'(\theta G_1+G_2). \end{aligned} \tag{65}$$

Let $v(\theta) \equiv V(b(\theta), z(\theta), \theta)$. Applying the Envelope Theorem to (64) produces the following first-order necessary condition for incentive compatibility:

$$\begin{aligned} \dot{v}(\theta) = &-u_2'(y_2-b(\theta)(1+r)-\Psi(\theta)\psi(b(\theta),z(\theta),\theta))\Psi'(\theta)\psi(b(\theta),z(\theta),\theta) \\ &+ g_2'(\theta\phi(b(\theta),z(\theta),\theta)+\psi(b(\theta),z(\theta),\theta))\phi(b(\theta),z(\theta),\theta), \end{aligned} \tag{66}$$

in which $G_1(\theta) \equiv \phi(b(\theta), z(\theta), \theta)$ and $G_2(\theta) \equiv \psi(b(\theta), z(\theta), \theta)$.

Similar to Lemma 5.1, we now arrive at:

Lemma 5.2 Under Assumption 5.1, the global optimality of truth-telling strategy is guaranteed by the second-order condition, $\dot{b}(\theta) \leq 0$ for all $\theta \in \Theta$.

Proof Applying again the Envelope Theorem to the value function (64) gives us $V_b(b,z,\theta) = u_1' - (1+r)u_2'$ and $V_z(b,z,\theta) = u_1' > 0$ for all $\theta \in \Theta$. Under Assumption 5.1, we then arrive at

$$\frac{\partial}{\partial\theta}\left[\frac{V_b(b,z,\theta)}{V_z(b,z,\theta)}\right] \quad = \quad (1+r)\frac{u_2''\Psi'(\theta)G_2}{u_1'} < 0, \tag{67}$$

which therefore guarantees the Spence-Mirrlees property. The second-order condition for incentive compatibility can be expressed as:

$$\dot{b}(\theta) \cdot V_z(b(\theta),z(\theta),\theta) \cdot \frac{\partial}{\partial\tilde{\theta}}\left(\frac{V_b(b(\theta),z(\theta),\tilde{\theta})}{V_z(b(\theta),z(\theta),\tilde{\theta})}\right)\Bigg|_{\tilde{\theta}=\theta} \geq 0,$$

which when combined with (67) reveals that $\dot{b}(\theta) \leq 0$ must hold. □

We now exploit (66) and Lemma 5.2, and the optimization problem facing the center is formalized as follows:

$$\begin{aligned} &\max \int_{\underline{\theta}}^{\bar{\theta}} v(\theta)f(\theta)d\theta \\ &\quad s.t. \quad v(\theta) \; = V(b(\theta),z(\theta),\theta); \\ &\quad \int_{\underline{\theta}}^{\bar{\theta}} z(\theta)f(\theta)d\theta \; \leq 0; \\ &\quad \dot{v}(\theta) \; = -u_2'(y_2-b(\theta)(1+r)-\Psi(\theta)\psi(b(\theta),z(\theta),\theta))\Psi'(\theta)\psi(b(\theta),z(\theta),\theta) \\ &\quad + g_2'(\theta\phi(b(\theta),z(\theta),\theta)+\psi(b(\theta),z(\theta),\theta))\phi(b(\theta),z(\theta),\theta); \\ &\quad \dot{b}(\theta) \; \leq 0. \end{aligned}$$

Solving the above problem permits us to arrive at the following proposition:

Proposition 5.3 Suppose Assumption 5.1 holds. In the asymmetric-information case without bunching, the welfare optimum $\{b^*(\theta), z^*(\theta)\}_{\theta\in\Theta}$ satisfies:

(i) Suppose Assumption 4.2 holds. Concerning the relationship between the intertemporal rate of substitution between current and future public goods consumption and the intertemporal rate of transformation, we have:

$$\frac{u_1'(c_1^*(\theta))}{u_2'(c_2^*(\theta))}\begin{cases} = \ 1+r & \text{for } \theta\in\{\underline{\theta},\bar{\theta}\}; \\ < \ 1+r & \text{for } \theta\in(\underline{\theta},\bar{\theta}). \end{cases}$$

(ii) Let $\mu_1(\theta) > 0$ be the Lagrange multiplier on the value constraint $v(\theta) \equiv V(b(\theta), z(\theta), \theta)$ of any type-θ region that is reporting truthfully, then we have:

- $V_z(b^*(\theta), z^*(\theta), \theta) = \gamma/\mu_1(\theta)$ for $\theta\in\{\underline{\theta},\bar{\theta}\}$;
- If $|\varepsilon_{g_2',\theta G_1}| \leq 1$, in which the elasticity is given by $\varepsilon_{g_2',\theta G_1} \equiv g_2'' \cdot \theta\phi/g_2'$, then

$$V_z(b^*(\theta), z^*(\theta), \theta) < \gamma/\mu_1(\theta)$$

for $\theta\in(\underline{\theta},\bar{\theta})$.

Proof We shall complete the proof in four steps.
Step 1: Firstly, we let $\dot{b}(\theta) \equiv \beta(\theta)$ as before, and write the generalized Hamiltonian as:

$$\begin{aligned}\mathcal{H} &= v(\theta)f(\theta) + \mu_1(\theta)[V(b(\theta), z(\theta), \theta) - v(\theta)]f(\theta) - \mu_2(\theta)\beta(\theta) - \gamma z(\theta)f(\theta) \\ &\quad - \eta_1(\theta)u_2'(y_2 - b(\theta)(1+r) - \Psi(\theta)\psi(b(\theta), z(\theta), \theta))\Psi'(\theta)\psi(b(\theta), z(\theta), \theta) \\ &\quad + \eta_1(\theta)g_2'(\theta\phi(b(\theta), z(\theta), \theta) + \psi(b(\theta), z(\theta), \theta))\phi(b(\theta), z(\theta), \theta) + \eta_2(\theta)\beta(\theta),\end{aligned}$$

where $\mu_1(\theta)$, $\mu_2(\theta)$ and γ are non-negative Lagrange multipliers, and $\eta_1(\theta)$ and $\eta_2(\theta)$ are co-state variables. The first-order necessary conditions are

$$\begin{aligned}\mathcal{H}_z \ &= \mu_1(\theta)V_z(b(\theta), z(\theta), \theta)f(\theta) - \gamma f(\theta) \\ &\quad + \eta_1(\theta)\left[u_2'' \cdot \Psi(\theta)\Psi'(\theta)\psi\psi_z - u_2' \cdot \Psi'(\theta)\psi_z\right] \\ &\quad + \eta_1(\theta)\left[g_2'' \cdot (\theta\phi_z + \psi_z)\phi + g_2' \cdot \phi_z\right] = 0,\end{aligned} \tag{68}$$

$$\mathcal{H}_\beta \ = \ -\mu_2(\theta) + \eta_2(\theta) = 0, \tag{69}$$

$$\dot{\eta}_1(\theta) \ = \ -\mathcal{H}_v \ = \ [\mu_1(\theta) - 1]f(\theta), \tag{70}$$

and

$$
\begin{aligned}
\dot{\eta}_2(\theta) &= -\mathcal{H}_b \\
&= -\mu_1(\theta)[u_1' - (1+r)u_2']f(\theta) \\
&\quad + \eta_1(\theta)\{-u_2'' \cdot [1+r+\Psi(\theta)\psi_b]\Psi'(\theta)\psi + u_2' \cdot \Psi'(\theta)\psi_b\} \\
&\quad - \eta_1(\theta)[g_2'' \cdot (\theta\phi_b + \psi_b)\phi + g_2' \cdot \phi_b].
\end{aligned} \tag{71}
$$

In addition, we have the following transversality conditions:

$$
\eta_1(\theta) \;=\; \eta_2(\theta) \;=\; 0 \;\text{ for } \forall\theta \in \{\underline{\theta}, \bar{\theta}\}. \tag{72}
$$

Step 2: Following the first-order approach, we must have $\mu_2(\theta) = 0$ for all $\theta \in \Theta$. Exploiting (69) yields $\eta_2(\theta) = 0$ for all $\theta \in \Theta$, and hence we must have $\dot{\eta}_2 \equiv 0$. Application of $\dot{\eta}_2 \equiv 0$, $\mu_1(\theta)f(\theta) > 0$ and (72) to (71) reveals that $u_1' = (1+r)u_2'$ for $\theta \in \{\underline{\theta}, \bar{\theta}\}$, as desired in Part (i). Additionally, application of (72) to (68) leads to $\mu_1(\theta)V_z(b(\theta), z(\theta), \theta) = \gamma$ for $\theta \in \{\underline{\theta}, \bar{\theta}\}$, as desired in Part (ii).
Step 3: By applying the Implicit Function Theorem to the FOCs given by (65), we can still have those partial derivatives given by Eqs. (52) and (53). In consequence, (52) and (53) enable us to reach

$$
\theta\phi_b + \psi_b \;=\; \frac{\{[\theta\Psi(\theta) - (1+r)]u_1'' - (1+r)g_1''\}\Psi(\theta)u_2''}{Q} < 0 \tag{73}
$$

and

$$
1 + r + \Psi(\theta)\psi_b \;=\; \frac{[1 + r - \theta\Psi(\theta)]u_1''g_1'' + (1+r)g_1''g_2''}{Q} > 0. \tag{74}
$$

Applying $\dot{\eta}_2 \equiv 0$, Assumption 5.1, (52), (53), (73) and (74) to (71) yields that

$$
\begin{aligned}
&\mu_1(\theta)[u_1' - (1+r)u_2']f(\theta) \\
&= \eta_1(\theta)\{\underbrace{-u_2'' \cdot [1+r+\Psi(\theta)\psi_b]\Psi'(\theta)\psi}_{+} + \underbrace{u_2' \cdot \Psi'(\theta)\psi_b}_{-}\} \\
&\quad - \eta_1(\theta)\underbrace{[g_2'' \cdot (\theta\phi_b + \psi_b)\phi + g_2' \cdot \phi_b]}_{+}.
\end{aligned} \tag{75}
$$

As such, applying $\mu_1(\theta)f(\theta) > 0$, (54) (which uses Assumption 4.2), and Assumption 5.1 to (75) concludes the proof of Part (i).
Step 4: In addition, applying (52), (53) and Assumption 5.1 to (68) gives rise to

$$
\begin{aligned}
&[\mu_1(\theta)V_z(b(\theta), z(\theta), \theta) - \gamma]f(\theta) \\
&= -\eta_1(\theta)\underbrace{[u_2'' \cdot \Psi(\theta)\Psi'(\theta)\psi\psi_z - u_2' \cdot \Psi'(\theta)\psi_z]}_{+} \\
&\quad - \eta_1(\theta)\left\{[g_2' + g_2'' \cdot \theta\phi]\underbrace{\phi_z}_{+} + \underbrace{g_2'' \cdot \psi_z\phi}_{+}\right\},
\end{aligned}
$$

which when combined with

$$g_2' + g_2'' \cdot \theta\phi \geq 0 \;\Leftrightarrow\; \underbrace{-\frac{\theta\phi g_2''}{g_2'}}_{\varepsilon_{g_2',\theta G_1}} \leq 1$$

completes the proof of Part (ii). □

The main messages conveyed by Proposition 5.3 are as follows: (1) in the asymmetric-information optimum only the intertemporal allocation at the endpoints of type distribution is undistorted with respect to the first-best; (2) the presence of the asymmetric information between the center and regions prevents full insurance.

We now characterize the scheme of federal transfers than decentralizes the asymmetric-information optimum established in Proposition 5.3.

Proposition 5.4 Suppose Assumptions 4.2 and 5.1 hold. The grant scheme $z^*(b)$ that decentralizes the asymmetric-information optimum $\{b^*(\theta), z^*(\theta)\}_{\theta\in\Theta}$ is a nonlinear nondecreasing function of b, almost everywhere differentiable, with the slope

$$\frac{dz^*}{db}\begin{cases} = 0 & \text{for } b \in \{b^*(\underline{\theta}), b^*(\bar{\theta})\}; \\ > 0 & \text{for } b \in (b^*(\bar{\theta}), b^*(\underline{\theta})). \end{cases}$$

Proof Using the first-order necessary condition for incentive compatibility, we get

$$\frac{dz}{db} = \frac{dz}{d\theta}\frac{d\theta}{db} = -\frac{V_b}{V_z},$$

in which we have shown above that $V_z > 0$ always holds true. As we focus on the case without bunching, we get from (75) (which uses Assumption 5.1) and (54) (which uses Assumption 4.2) that

$$\begin{aligned} & - V_b(b(\theta), z(\theta), \theta) \\ = & -\frac{\eta_1(\theta)}{\mu_1(\theta)f(\theta)}\underbrace{\{-u_2'' \cdot [1 + r + \Psi(\theta)\psi_b]\Psi'(\theta)\psi + u_2' \cdot \Psi'(\theta)\psi_b\}}_{-} \\ & + \frac{\eta_1(\theta)}{\mu_1(\theta)f(\theta)}\underbrace{[g_2'' \cdot (\theta\phi_b + \psi_b)\phi + g_2' \cdot \phi_b]}_{+} \end{aligned}$$

for all $\theta \in (\underline{\theta}, \bar{\theta})$. We thus have for all $b \in (b^*(\bar{\theta}), b^*(\underline{\theta}))$ that:

$$\begin{aligned} \frac{dz}{db} = & -\frac{\eta_1(\theta(b))\{-u_2'' \cdot [1 + r + \Psi(\theta(b))\psi_b]\Psi'(\theta(b))\psi + u_2' \cdot \Psi'(\theta(b))\psi_b\}}{\mu_1(\theta(b))f(\theta(b))V_z} \\ & + \frac{\eta_1(\theta(b))[g_2'' \cdot (\theta(b)\phi_b + \psi_b)\phi + g_2' \cdot \phi_b]}{\mu_1(\theta(b))f(\theta(b))V_z} > 0. \end{aligned}$$

Following a little abuse of notation, $\theta(b)$ denotes the inverse of $b(\theta)$, which exists in light of Lemma 5.2. Finally, applying (72) immediately completes the proof. □

The main message conveyed by Proposition 5.4 is that federal transfers and local debt generally play a complementary role for regional insurance provision. Excluding the bottom and top types whose intertemporal allocations are undistorted in the asymmetric-information optimum, the following feature is exhibited by the optimal funding structure with regards to observable physical output of IPGs: regions with a higher degree of intergenerational externality should issue less debt and receive less federal transfers than regions with a lower degree of intergenerational externality.

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