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RESEARCH ARTICLE

The competitive and welfare effects of long-term contracts with network externalities and bounded rationality

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Abstract
This paper compares the long-term and short-term contracts in terms of their competitive and welfare effects in a dynamic nonlinear pricing model with network externalities and bounded rationality. Contrary to the existing literature and traditional treatments adopted by competition authorities, we find that a long-term contract is at least as competition-friendly and socially efficient as a sequence of short-term contracts. If the consumers have constant types and pessimistic expectation regarding the network size, then for a certain range of parameters, a long-term contract facilitates entry of more efficient competitors and is socially more efficient than the short-term contracts. If the consumers' types are independent across time, a long-term contract leads to the same competitive outcome as, but gives a higher social surplus than, its short-term counterpart.

Keywords Nonlinear pricing · Long-term contract · Entry deterrence · Network effect · Bounded rationality

JEL Classification D82 · D62 · L12 · L44

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1 Introduction

As a form of exclusive dealings (EDs), long-term contracts (LTCs) are pervasive in business practice.[1] When a seller signs an LTC with buyers, he actually becomes an exclusive supplier within a specified time horizon. One traditionally objects to this form of contract due to the anticompetitive concerns it raises. Beginning in the 1950s, this view was challenged by some "Chicago School" scholars on the grounds that exclusive contracts must be efficient, otherwise they will not be signed by buyers (Director and Levi 1956; Posner 1976; Bork 1978). Since the "Chicago critique," the debate over EDs has lasted for several decades, and it is still a matter of disagreement so far.[2] Being responsive to the twists and turns of economic theories, most legal provisions only advocate a circumspect treatment rather than an outright prohibition of LTCs.[3] In practice, however, competition authorities are more inclined to categorize LTCs, particularly those signed by dominant firms, as anticompetitive. They usually caution against or impose severe scrutiny on contracts of this form.[4] These treatments might err on the side of an overly strict approach. The commitments obtained from the defendants may have gone further than what is necessary to remedy the competition problems at stake, possibly preventing some efficiencies from being realized.

In this paper, we give a new procompetitive and pro-efficiency rationale for an LTC in comparison with a sequence of short-term contracts (STCs), and thus invite a more cautious attitude toward it. We develop our explanations in a dynamic entry-deterrence nonlinear pricing model with network effects and bounded rationality. An incumbent firm provides repeatedly a network good to a continuum of consumers. Each consumer's utility depends on the individual consumption and the aggregate consumption of all consumers (network size) as well. Facing a threat of entry by a set of competitors who enjoy a lower cost but provide no network value, the incumbent protects his monopolistic position through an entry-deterrence nonlinear pricing contract. Each consumer decides whether to accept the contract or reject it. Their participation decisions are based on both the stand-alone utilities they derive and their expectation regarding the network size. Consumers are assumed to be either fully or boundedly rational, depending on whether or not they could forecast beforehand the network size accurately. A consumer is called "*optimistic*" (resp. "*pessimistic*") if his

[1] EDs can be classified into three forms: (i) naked exclusion (NE): agreements unabashedly meant to exclude rivals without offering any efficiency justification; (ii) softer version of exclusive dealing (SE): clauses such as minimum-share requirement (i.e., the buyer is required to book a minimum share of his total purchases from the seller), market share discounts (a seller provides a buyer with a discount if the latter commits to buying a certain percentage of his total purchases from that seller), and a tying arrangement (through a contractual or technological requirement, a seller conditions the sale of one product on the customer's agreement to take a second product); and (iii) long-term contract (LTC): clauses of long duration specifically designed to lock in customers and exclude rivals.

[2] We will discuss the literature in greater detail in Sect. 2.

[3] For example, the European Commission's guidance paper on Article 101, 102, and Guidelines on Vertical Restraints.

[4] Such treatments can be found in a number of famous antitrust cases in Europe and North America. For example, European Commission's Decision on Distrigaz case on 11 October 2007 (Case COMP/37.966), the decision on GDF Suez case on 3 December 2009 (Case COMP/39.316), and the Electricité de France (EDF) case on 17 March 2010 (Case COMP/39.386).

expectation regarding the network size is higher (resp. lower) than the real value. The consumers' types have different degrees of persistence over time: they may be either constant or independent.

The main focus of our paper is to compare the long-term and short-term contracts in terms of their competitive and efficiency effects under different degrees of rationality and type persistence. Our main findings are threefold. Firstly, an LTC is at least as competition-friendly as its short-term counterpart, regardless of the degrees of rationality and whether types are constant or independent. Secondly, with constant types and full rationality, an LTC leads to the same competitive outcome as, but gives a higher social surplus than, a sequence of STCs. It is more procompetitive and efficient than STCs under bounded rationality when the entrants' cost is within a certain range. Finally, with independent types, an LTC leads to the same competition outcome as, but gives a higher social surplus than, STCs, regardless of whether or not the consumers are fully rational.

The intuitions behind these results are as follows. Our first result hinges on the incumbent's different degrees of freedom conferred by contracts. A sequence of STCs endows the incumbent seller with greater pricing flexibility. Even when facing a deteriorating situation of customer loss, the incumbent could turn the tables by a temporary price cutting. It is desperately difficult for new competitors to gain a footing in the market. In contrast, an LTC deprives the incumbent of all the flexibilities of adjusting the initial clauses, but it does not affect an individual customer's freedom to escape the contractual relationship whenever he prefers to do so. This is because committing to an LTC and upholding a trustworthy reputation is more important for a firm than for an individual consumer. Hence, in all situations where an LTC could foreclose entry, STCs could also do that. In this sense, an LTC is at least as competition-friendly as a sequence of STCs.[5]

When the consumers' types are constant over time, it is optimal for the incumbent to offer an optimal LTC constituting repetition of the optimal static contract period after period.[6] Under full rationality, an LTC and a sequence of STCs lead to the same competitive results. By entering into an LTC, the incumbent is able to avoid the transaction costs of reaching new agreements. An LTC is thus more efficient than a sequence of STCs. Things are different for bounded rationality. The quitting of a cohort of consumers upon a pessimistic expectation will in turn dampen the remaining consumers' confidence regarding the incumbent's product. The incumbent is unable to reverse this unfavorable situation since he has to commit to a prespecified LTC. Then for a certain range of the entrants' costs, an avalanche-like process, in which a lower expectation and a lower participation rate reinforce each other alternatingly, is triggered. As a result, the incumbent will cede all his market share to the entrants. When the entrants have such a cost advantage *vis-à-vis* the incumbent as to outweigh the network value he provides, this result is also socially efficient.

5 Here, a natural question may arise: now that an LTC deprives the incumbent of all flexibilities, why would he sign it ex ante? As we will show later, an LTC allows the incumbent to save transaction costs of reaching new agreements.

6 Constant types do not alter the nature of dynamic contract, since the incentive constraints beyond the initial period are indispensable for the principal to elicit truthtelling in subsequent periods.

When types are statistically independent across time, the incumbent can construct, without loss of generality, an optimal contract that keeps only one type (either the lowest or the highest type depending on specific parameters) at his reservation utility in the starting period, and confers on all types sufficiently large information rents in all subsequent periods. This LTC has the same competitive feature as its short-term counterpart since they can both foreclose entry and are both robust to pessimistic expectation. But the LTC gives a higher social surplus than a chain of STCs. The efficiency comes from two sources: alleviating asymmetric information and saving transaction costs. When the LTC is signed at the outset, all consumers do not know their future types. Signing up consumers before they become better informed will keep the incumbent from distorting the allocations away from their efficient levels. Moreover, an LTC avoids the expense of reaching a new agreement in each round of spot transaction. Therefore, the social surplus under an LTC is higher than that under a sequence of STCs.

A well-known example of LTC in the network industry is the per-processor license instituted by Microsoft in 1988. Microsoft, as an incumbent owning a large installed user base and technical incompatibility in operating system (OS) market, offered licensing contract to some original equipment manufacturers (OEMs). The latter, if they wanted to use Microsoft's OS products at a very substantial discount, would have to pay a per-unit fee for every PC shipped, regardless of whether a Microsoft OS was installed on all machines. Per-processor contracts were often for a duration of at least three years, a period exceeding the product life cycle of most OS products. The main complaint from competing OSs was the difficulty in convincing OEMs to escape the LTC and offer a non-Microsoft OS with a small installed base. The sales data showed, however, that the per-processor contract did not have a material impact on the competition of OS market. In fiscal year 1993, five years after its launch, per-processor contract accounted for only 60% of Microsoft's MS-DOS sales and only 43% of Windows sales to PC makers. Many small manufacturers quitted or refused to sign the contracts. In the new era when the market was shifting toward mobile devices, Microsoft was ousted from the dominant position by a new generation of non-Microsoft OSs (such as iOS and Android) at an amazing speed. The exact reasons for Microsoft's failure to achieve a full foreclosure of the OS market are very complex. Nevertheless, the network effects, LTCs[7] and the bounded rationality of customers undoubtedly play important roles, which are the main concerns of this paper.

The rest of this paper proceeds as follows. The next section surveys the related literature. Section 3 sets up the baseline static contracting model. Section 4 analyzes the dynamic model with constant types. Section 5 analyzes the case with independent types. Finally, concluding remarks are given in Sect. 6.

[7] After the *consent decree* that prohibited per-processor contracts was issued in 1994, Microsoft agreed to stop using such contracts. It was, however, accused of maintaining its per-processor pricing scheme, and adopting some alternative long-term licensing agreements (the licensing basis moved from processors to physical cores) despite the consent.

2 Related literature

Since the mid-1980s, a number of studies raise their objections to the "Chicago critique," and show that the courts' traditional treatment was not ill-founded. Rasmusen et al. (1991) and Segal and Whinston (2000) (RRW-SW) show that increasing returns make a minimum scale of operation necessary for profitable entry; an incumbent can achieve full exclusion cheaply by exploiting the dispersed consumers' lack of coordination or by discriminating between them.

Some other scholars have countered the skepticism about the "Chicago critique" by posing challenges to the view of RRW-SW. The Chicago view is thus resuscitated in some environments. Innes and Sexton (1994) show that buyers are willing to elicit entry in order to create competition, even if such entry is socially inefficient. Exclusive contracts may facilitate the foreclosure of an inefficient entry that would otherwise occur. Fumagalli and Motta (2006) show that with strong downstream competition, a single deviating buyer who buys from the entrant at a lower cost can induce entry since he can monopolize the whole retail market. This undermines the coordination failure argument for exclusion of RRW-SW. The progress of theoretical literature in recent years does not settle but rather escalates the debate. Both hostile and lenient attitudes toward EDs find their respective supports (see, among many others, Simpson and Wickelgren 2007; De Meza and Selvaggi 2007; Abito and Wright 2008; Wright 2009; Doganoglu and Wright 2010; Kitamura 2010; Spector 2011; Calzolari and Denicolò 2013; Gratz and Reisinger 2013; Vasconcelos 2014; Chen and Shaffer 2014, for detailed discussions).

Whereas the above-mentioned studies focus mainly on naked and/or softer exclusions, some others restrict their attention to the impacts of LTCs. Aghion and Bolton (1987), in a classic contribution, show that even simple LTCs can be anticompetitive if breach penalty provisions are allowed and there exists uncertainty about the potential entrant's costs. Bedre-Defolie and Biglaiser (2017) also investigate the anticompetitive effects of an LTC with breakup fees.[8] More recently, Gratz and Reisinger (2013) discuss the anticompetitive features of an LTC when the products provided by the incumbent and the entrant are horizontally differentiated.

Contrary to the conventional wisdom, this paper finds that an LTC, in comparison with its short-term counterpart, is more procompetitive and efficiency-enhancing under the setting we consider. It differs from the existing literature on LTCs cited above in some respects. First, we incorporate network effects and bounded rationality in our model. In previous studies, a consumer's participation decision is based on the incumbent versus entrant price comparison, its own switching cost, and/or a prespecified damages payment. This paper assumes away switching cost and damages payment but focuses on another two elements affecting the participation decision: the network value provided by the incumbent and the consumers' bounded rationality in forecasting this network value. Second, previous studies assume that consumers'

[8] Although similar anticompetitive outcomes arise in these two papers, the mechanisms behind are different. In Aghion and Bolton (1987), inefficient foreclosure arises because breakup fees can be used as a tool to shift rent from the more efficient entrant to the incumbent. The result of Bedre-Defolie and Biglaiser (2017), however, hinges on the ability of the incumbent using the terms of an LTC, in particular breakup fees and most favored nation clauses (MFNs), to alter the consumers' outside option of not signing it.

valuations of the good are homogeneous and publicly observable, so they consider only uniform price contracts. In contrast, our paper considers consumer heterogeneity and private information. The analysis is thus presented in a mechanism design framework. Finally, in the existing studies, the incumbent firm is assumed to compete with a unique strategic entrant. In our paper, however, there exist a vast number of potential entrants, whose pricing flexibilities are deprived by the fierce competition in outside market. We show that even if the entrants are that disadvantageous, efficient entry may still happen in some circumstances.

Since we build a long-term contracting model and adopt the first-order approach (FOA) to solve it, this paper is also related to a stream of literature on dynamic contract. We briefly refer to the work that is most related to ours. Bedre-Defolie and Biglaiser (2017) is the first paper using FOA to study the dynamic contracting models. Some extensions are presented by Bose and Deltas (2007), Laffont and Tirole (1990) and Battaglini (2005), etc. More recently, Battaglini and Lamba (2019) explore the general applicability of FOA in dynamic contracting models. They show that FOA is usually not applicable in general settings. Two notable exceptions are cases with constant and independent types. In this paper, we analyze these two cases following the methodology of Battaglini and Lamba (2019). Our main innovation relative to this literature is to incorporate network effects, bounded rationality and type-dependent outside options.

Another strand of literature that is closely related to this paper deals with platform competition. A platform's success depends not only on its quality but also on consumers' beliefs that other consumers will patronize it. Caillaud and Jullien (2001, 2003) discuss the issue of static platform competition in a two-sided market and introduce the notion of favorable beliefs as a tool of equilibrium selection.[9] This concept was used in subsequent research on static platform competition. Recently, Halaburda et al. (2020) extend to a dynamic context the concept of favorable expectations. They give conditions under which a high-quality nonfocal platform could overcome the market's unfavorable expectations and capture focality from its low-quality focal opponent. In these papers, the customers are assumed to have time-invariant favorable/optimistic, or unfavorable/pessimistic beliefs with respect to a platform. In contrast to their time-invariant assumptions, we depict an adaptively evolving process of the consumers' beliefs. We show that a small forecasting error may sometimes trigger an avalanche-like process leading the consumers' belief to change from extreme optimism to extreme pessimism.

3 The baseline model

We consider a repeated buyer-seller relationship in discrete time indexed by $t = 0, 1, \ldots, \infty$. In each period, an incumbent firm produces a good with network effects at a constant marginal cost c_I. A continuum of consumers enjoying a per-period utility $\theta_t v(q_t) + \Omega(Q_t) - \tau_t$ purchase from the incumbent repeatedly, where q_t is the per-

[9] Favorable (resp. unfavorable) beliefs mean that if there is an equilibrium in which all customers (resp. no customer) join a platform, such an equilibrium is selected.

period individual consumption, Q_t is the total consumption of all consumers in period t (referred to as network size), and τ_t is the per-period price charged by the incumbent. $\beta \in (0, 1)$ denotes the discount factor of both the incumbent and consumers. $\{\theta_t\}_{t\geq 0}$ are private information to the consumers in each period, and their distribution is common knowledge to both parties. The initial type θ_0 is distributed according to a distribution function $F(\theta)$ with density $f(\theta)$ supported on $[\underline{\theta}, \overline{\theta}]$. The following monotone hazard rate properties hold:

$$\frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right) \leq 0 \leq \frac{d}{d\theta}\left(\frac{F(\theta)}{\theta f(\theta)}\right), \forall \theta \in [\underline{\theta}, \overline{\theta}]. \tag{1}$$

The first inequality is standard in the literature, while the second one is stronger than the usual monotone hazard rate condition that only requires $F(\theta)/f(\theta)$ to be nondecreasing. We further assume that $\underline{\theta} - 1/f(\underline{\theta}) > 0$, which ensures a positive virtual valuation over $[\underline{\theta}, \overline{\theta}]$, i.e., $\theta - [1 - F(\theta)]/f(\theta) > 0, \forall \theta \in [\underline{\theta}, \overline{\theta}]$. θ_t evolves over time according to a Markov process.

The intrinsic utility function is increasing and concave (i.e., $v' > 0$, $v'' < 0$), and it also satisfies the Inada conditions $\lim_{q\to 0} v'(q) = \infty$ and $\lim_{q\to +\infty} v'(q) = 0$. The network value term satisfies $\Omega(0) = 0$, $\Omega(Q) > 0$ and $\Omega''(Q) < 0, \forall Q \in [0, +\infty)$. We also assume that $\Omega'(0) = \sup_{Q\in[0,\infty)} \Omega'(Q) < c_I$; the incumbent may otherwise wish to produce an unbounded output level.

There is an outside market composed of a large number of dispersed potential entrants. They produce an intrinsically perfect substitute for the incumbent's product. The entrants are homogeneous and enjoy a lower marginal cost $c_E < c_I$, but their products are technically incompatible with the incumbent's network. The entrants do not have their own network because they are of vast number and are highly dispersed.[10] The homogeneous Bertrand competition in the outside market drives the price down to the marginal cost c_E. Then, a θ-type obtains a type-dependent reservation utility $U^0(\theta) := \max_{q\in[0,\infty)}[\theta v(q) - c_E q]$ in every period, $q^0(\theta) := \arg\max_{q\in[0,\infty)}[\theta v(q) - c_E q]$ denotes the optimal amount purchased from the outside market.

The seller discriminates among different types of customers through a long-term nonlinear pricing contract. It can be represented as $\{q_t(\hat{\theta}_t|h^{t-1}), \tau_t(\hat{\theta}_t|h^{t-1})\}$, where h^{t-1} and $\hat{\theta}_t$ are, respectively, the public history up to period $t-1$ and the type revealed at time t. In general, h^t can be defined recursively as $h^t := \{h^{t-1}, \hat{\theta}_t\}$, $h^{-1} = \emptyset$. h^t_s, $s \leq t$ denotes the s^{th} element of vector h^t. We denote by $\mathcal{H}^t$ the set of all possible histories at time t. The set of full histories at time $T \geq t$ that follow h^t until time t is given by $\mathcal{H}^T(h^t) := \{h \in \mathcal{H}^T : h_s = h^t_s, \forall s \leq t\}$.

Let $U_t(\theta_t|h^{t-1}) := \theta_t v(q_t(\theta_t|h^{t-1})) - \tau_t(\theta_t|h^{t-1}) + \Omega(Q_t(h^{t-1})) - U^0(\theta_t)$ denote a θ_t-type's instantaneous utility net of its reservation utility upon a truthful declaration and history h^{t-1}, where $Q_t(h^{t-1}) = \mathbb{E}_{\theta_t}[q_t(\theta_t|h^{t-1})]$ is the per-period network size contingent on history h^{t-1}. $\widetilde{U}_t(\theta_t, \hat{\theta}_t|h^{t-1}) := \theta_t v(q_t(\hat{\theta}_t|h^{t-1})) - \tau_t(\hat{\theta}_t|h^{t-1}) + \Omega(Q_t(h^{t-1})) - U^0(\theta_t)$ is the per-period net utility of type θ_t reporting to be of type

[10] Note that some small brands cannot provide efficient after-sales services network due to their limited volume of sales and sparsely distributed outlets.

$\hat{\theta}_t$ after history h^{t-1}.[11] The expected continuation payoff of type θ_t upon truthful declaration and a history h^{t-1} is

$$\mathcal{U}_t(\theta_t|h^{t-1}) := \sum_{s=t}^{\infty} \beta^{s-t} \mathbb{E}_{h^s \in \mathcal{H}^s(h^{t-1},\theta_t)} \left[U_s(\theta_s|h^{s-1}) \Big| h^{t-1}, \theta_t \right],$$

or recursively, $\mathcal{U}_t(\theta_t|h^{t-1}) = U_t(\theta_t|h^{t-1}) + \beta\mathbb{E}\left[\mathcal{U}_{t+1}(\theta_{t+1}|h^{t-1},\theta_t)\big|\,\theta_t\right]$.[12]

$$\widetilde{\mathcal{U}}_t(\theta_t, \hat{\theta}_t|h^{t-1}) := \widetilde{U}_t(\theta_t, \hat{\theta}_t|h^{t-1}) + \beta\mathbb{E}\left[\mathcal{U}_{t+1}(\theta_{t+1}|h^{t-1}, \hat{\theta}_t)\Big|\,\theta_t\right]$$

is the continuation payoff of a θ_t agent reporting to be $\hat{\theta}_t$.

The incumbent's objective from the perspective of date 0 is the expected discounted surplus net of the buyer's expected rents and a fixed transaction cost:

$$\Pi_\ell := \mathbb{E}\left\{ \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} \theta_t v(q_t(\theta_t|h^{t-1})) - c_I q_t(\theta_t|h^{t-1}) \\ +\Omega(Q_t(h^{t-1})) - U^0(\theta_t) \end{bmatrix} - \mathcal{U}_0(\theta_0) \right\} - C, \tag{2}$$

C denotes the transaction cost paid by the incumbent in a contracting process.[13] To guarantee an agent's participation and truthful report after any history, the following constraints must be satisfied:

$$IC_t : \mathcal{U}_t(\theta_t|h^{t-1}) \geq \widetilde{\mathcal{U}}_t(\theta_t, \hat{\theta}_t|h^{t-1}), \forall h^{t-1} \in \mathcal{H}^{t-1}, \forall \theta_t, \hat{\theta}_t \tag{3}$$

$$IR_t : \mathcal{U}_t(\theta_t|h^{t-1}) \geq 0, \quad \forall h^{t-1} \in \mathcal{H}^{t-1}, \quad \forall \theta_t. \tag{4}$$

The optimal long-term contracting problem is represented as:

$$[\mathcal{P}_\ell] : \max_{\{q_t(\theta_t|h^{t-1}), \tau_t(\theta_t|h^{t-1})\}_{t=0}^{\infty}} \Pi_\ell, s.t. : IC_t, IR_t, Q_t(h^{t-1}) = \mathbb{E}_{\theta_t}[q_t(\theta_t|h^{t-1})]. \tag{5}$$

To facilitate our further analysis of $[\mathcal{P}_\ell]$, we set aside, for the moment, the issue of long-term contract and focus on a benchmark case where the seller and buyers interact for only one period. In this case, the seller's optimization problem simplifies to:

$$[\mathcal{P}_s] : \max_{\{q(\theta), U(\theta)\}} \pi, s.t. : U'(\theta) = v(q(\theta)) - v(q^0(\theta)), q(\theta) \text{ is nondecreasing}, U(\theta) \geq 0, \tag{6}$$

[11] Since every individual consumer is of zero measure, his misreport will not affect the whole network size $Q_t(h^{t-1})$.

[12] It follows from our Markovian assumption that the distribution of θ_{t+1} depends only on θ_t rather than $\{\theta_s\}_{s<t}$.

[13] Say, costs of organizing an auction, hosting a trade fair, drawing up the contract clauses, printing and distributing the brochures, searching cost such as broker's fee, etc.

where $\pi := \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta v(q(\theta)) - c_I q(\theta) - U^0(\theta) - U(\theta)\right] f(\theta)d\theta + \Omega\left(\int_{\underline{\theta}}^{\overline{\theta}} q(\theta) f(\theta)d\theta\right) - C$. Again, C is the per-round transaction cost of contracting. The first constraint is the usual envelope condition implied by the local incentive constraint. The second constraint is the monotonic implementability condition. The third one is the usual IR constraint. We denote by $[\mathcal{P}_s^f]$ the full-information problem when only IR constraint is considered, and by $[\mathcal{P}_s^r]$ the relaxed problem subject only to the envelope and IR conditions.

Let $q^i(\theta, Q), i \in \{-, f, +\}$ be functions defined, respectively, by the following expressions (7) to (9):

$$q^-(\theta, Q) : \left[\theta - \frac{1 - F(\theta)}{f(\theta)}\right] v'(q) + \Omega'(Q) = c_I, \tag{7}$$

$$q^f(\theta, Q) : \theta v'(q) + \Omega'(Q) = c_I, \tag{8}$$

$$q^+(\theta, Q) : \left[\theta + \frac{F(\theta)}{f(\theta)}\right] v'(q) + \Omega'(Q) = c_I. \tag{9}$$

As is well known in adverse selection models, when information is complete, the principal concedes no information rent, and the efficient allocation $q^f(\theta, Q)$ is obtained ("f" denotes "full information"). With incomplete information, the principal faces a tradeoff between rent extraction and efficiency. When the information rent is increasing (resp. decreasing), a distorted quantity $q^-(\theta, Q)$ (resp. $q^+(\theta, Q)$) is obtained. Let $\varphi^i(Q) := \mathbb{E}_\theta[q^i(\theta, Q)]$. It then follows directly from conditions $v'(0) = +\infty$, $v'(+\infty) = 0$ and $\Omega'(x) < \Omega'(0) \leq c_I, \forall x \geq 0$ that $\varphi^i(0) > 0$, $\varphi^i(+\infty) < +\infty$. Since $\Omega''(Q) < 0$, $\varphi^i(Q)$ is strictly decreasing on $[0, \infty)$, it thus has a unique fixed point, denoted by Q^i. We also denote, with a slight abuse of notation, $q^i(\theta) := q^i(\theta, Q^i), \forall i \in \{-, f, +\}$. $\theta^-(Q)$ (resp. $\theta^+(Q)$) denotes the intersection of curves $q^0(\theta)$ and $q^-(\theta, Q)$ (resp. $q^+(\theta, Q)$). Let

$$\varphi^*(Q) := \begin{cases} \int_{\underline{\theta}}^{\theta^+(Q)} q^+(\theta, Q) f(\theta)d\theta + \int_{\theta^+(Q)}^{\overline{\theta}} q^0(\theta) f(\theta)d\theta & \text{if } q^0(\theta) \geq q^f(\theta) \\ \int_{\theta^-(Q)}^{\overline{\theta}} q^-(\theta, Q) f(\theta)d\theta + \int_{\underline{\theta}}^{\theta^-(Q)} q^0(\theta) f(\theta)d\theta & \text{if } q^0(\theta) < q^f(\theta) \end{cases}.$$

$$c_L^* := \frac{c_I - \Omega'(Q^+)}{1 + \frac{1}{\overline{\theta} f(\overline{\theta})}}, \quad c_M^* := c_I - \Omega'(Q^f), \quad c_H^* := \frac{c_I - \Omega'(Q^-)}{1 - \frac{1}{\underline{\theta} f(\underline{\theta})}}$$

denote three cutoffs of cost. Here, we assume implicitly that $\underline{\theta} f(\underline{\theta})$ is sufficiently large so that $c_H^* < c_I$. The following lemma ranks these cutoffs.

Lemma 1 $c_L^* < c_M^* < c_H^*$.

Proof See Appendix A. □

The following proposition describes the solution to $[\mathcal{P}_s]$.

Proposition 1 *Suppose that consumers are fully rational. Then the optimal entry-deterrence static contract is* $\{q^*(\theta), \tau^*(\theta)\}$, *where* $\tau^*(\theta) = \theta v(q^*(\theta)) + \Omega(Q^*) - U^0(\theta) - U^*(\theta)$, $q^*(\theta)$ *and* $U^*(\theta)$ *are given as follows:*

- *if* $c_E \in \left[c_H^*, c_I\right)$, *then* $Q^* = Q^-$, $q^*(\theta) = q^-(\theta)$, *and*

$$U^*(\theta) = \int_{\underline{\theta}}^{\theta} \left[v(q^-(\theta)) - v(q^0(\theta))\right] d\theta, \forall\theta \in [\underline{\theta}, \overline{\theta}];$$

- *if* $c_E \in \left(c_M^*, c_H^*\right)$, *then* $Q^* \in (Q^-, Q^f)$,

$$q^*(\theta) = \begin{cases} q^0(\theta) & \text{if } \theta \in [\underline{\theta}, \theta^-(Q^*)) \\ q^-(\theta, Q^*) & \text{if } \theta \in [\theta^-(Q^*), \overline{\theta}] \end{cases},$$

$$U^*(\theta) = \begin{cases} 0 & \text{if } \theta \in [\underline{\theta}, \theta^-(Q^*)) \\ \int_{\theta^-(Q^*)}^{\theta} \left[v(q^-(\theta, Q^*)) - v(q^0(\theta))\right] d\theta & \text{if } \theta \in [\theta^-(Q^*), \overline{\theta}] \end{cases};$$

- *if* $c_E = c_M^*$, *then* $Q^* = Q^f$, $q^*(\theta) = q^0(\theta) = q^f(\theta)$, $U^*(\theta) \equiv 0, \forall\theta \in [\underline{\theta}, \overline{\theta}]$;
- *if* $c_E \in \left(c_L^*, c_M^*\right)$, *then* $Q^* \in (Q^f, Q^+)$,

$$q^*(\theta) = \begin{cases} q^0(\theta) & \text{if } \theta \in [\theta^+(Q^*), \overline{\theta}] \\ q^+(\theta, Q^*) & \text{if } \theta \in [\underline{\theta}, \theta^+(Q^*)) \end{cases},$$

$$U^*(\theta) = \begin{cases} \int_{\theta^+(Q^*)}^{\theta} \left[v(q^+(\theta, Q^*)) - v(q^0(\theta))\right] d\theta & \text{if } \theta \in [\underline{\theta}, \theta^+(Q^*)) \\ 0 & \text{if } \theta \in [\theta^+(Q^*), \overline{\theta}] \end{cases};$$

- *if* $c_E \in \left[0, c_L^*\right]$, *then* $Q^* = Q^+$, $q^*(\theta) = q^+(\theta)$,

$$U^*(\theta) = \int_{\theta}^{\overline{\theta}} \left[v(q^0(\theta)) - v(q^+(\theta))\right] d\theta, \forall\theta \in [\underline{\theta}, \overline{\theta}].$$

Proof See Appendix B. □

In the standard nonlinear pricing model where the reservation utility is normalized to zero, participation constrains of all but the lowest type are not binding, the traditional efficiency versus rent extraction tradeoff calls for a consumption scheme $q^-(\theta)$. This standard monopoly pricing contract remains optimal in our model if the potential entrants are sufficiently inefficient (i.e., $c_E \geq c_H^*$). As the entrants become more efficient such that $c_E \in (c_M^*, c_H^*)$, the low-end consumers prefer bypassing to staying in the network. The high-end consumers will not switch to the outside market but have incentive to underreport their types. Thus, it is optimal for the incumbent to provide the former a quantity $q^0(\theta)$ to match their outside option, and the latter a quantity $q^-(\theta, Q^*)$ to extract the information rents.

If $c_E = c_M^*$, all types have incentive to exit the network, and the incumbent finds it is optimal to provide the first-best consumption $q^f(\theta)$ to all consumers. As the entrants' cost continues to decrease such that $c_E \in \left(c_L^*, c_M^*\right)$, the outside market becomes attractive to the high-end consumers, and the low-end consumers now have incentive to overstate their types. The incumbent has to offer a quantity $q^0(\theta)$ to retain the high-end consumers and a quantity $q^+(\theta, Q^*)$ to the low-end customers to reduce the information rents. When the potential entrants become highly efficient with $c_E \leq c_L^*$, all but the highest-type have incentive to overstate their valuations, a scheme $q^+(\theta)$ is thus provided.

We now come back to the setup of long-term interactions. Two contracting ways are now available to the incumbent: a sequence of STCs signed in every period, and an LTC signed ex ante. In the former, the incumbent pays a transaction cost C repeatedly and is free to adjust the contract clauses in every period. In the latter, the seller pays a C only in the initial period, but he has to commit not to change the prespecified clauses at any later date. In the rest of this paper, we focus on comparing the competitive and efficiency effects of these two types of contracts in various setups. Under full rationality and a chain of STCs, the incumbent faces repeatedly the same problem $[\mathcal{P}_s]$ in every period.[14] He can thus achieve full exclusion by offering the optimal static contract $\{q^*(\theta), \tau^*(\theta)\}$ at a cost C in every period.

Proposition 2 *If consumers are fully rational, then the incumbent can foreclose entry by a sequence of STCs* $\{q^*(\theta), \tau^*(\theta)\}$ *given by Proposition* 1.

4 The case with constant types

In this section, we discuss the optimal LTC when consumers' types are constant over time. In this case, only histories $\theta^t := \{\theta, \cdots, t-times, \cdots, \theta\}, \forall t \geq 0$ in which the type remains equal to the initial type θ can be reached along the path of truthful reports.[15] To economize on notation, we write $\{q_t(\theta), \tau_t(\theta), U_t(\theta), \mathcal{U}_t(\theta)\}$, instead of $\{q_t(\theta|\theta^t), \tau_t(\theta|\theta^t), U_t(\theta|\theta^t), \mathcal{U}_t(\theta|\theta^t)\}$. Then the incumbent's expected payoff is represented as

$$\Pi_\ell = \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} \theta v(q_t(\theta) - c_I q_t(\theta) \\ +\Omega\left(\int_{\underline{\theta}}^{\overline{\theta}} q_t(\theta) f(\theta) d\theta\right) - U^0(\theta) \end{bmatrix} - \mathcal{U}_0(\theta) \right\} f(\theta) d\theta - C.$$

Since $\theta_{t+1} = \theta_t$ with probability one, we have $\mathbb{E}[\mathcal{U}_{t+1}(\theta_{t+1}|h^{t-1}, \hat{\theta}_t)|\theta_t] = \mathcal{U}_{t+1}(\theta_t|h^{t-1}, \hat{\theta}_t)$ IR_t and IC_t then take the forms

$$\mathcal{U}_t(\theta_t|h^{t-1}) = \begin{bmatrix} \theta_t v(q_t(\theta_t|h^{t-1})) - \tau_t(\theta_t|h^{t-1}) + \Omega(Q_t(t^{t-1})) \\ -U^0(\theta_t) + \beta \mathcal{U}_{t+1}(\theta_t|h^{t-1}, \theta_t) \end{bmatrix} \geq 0, \forall \theta_t, h^{t-1}; \tag{10}$$

$$\mathcal{U}_t(\theta_t|h^{t-1}) \geq \begin{bmatrix} \theta_t v(q_t(\hat{\theta}_t|h^{t-1})) - \tau_t(\hat{\theta}_t|h^{t-1}) + \Omega(Q_t(h^{t-1})) \\ -U^0(\theta_t) + \beta \mathcal{U}_{t+1}(\theta_t|h^{t-1}, \hat{\theta}_t) \end{bmatrix}, \forall \theta_t, \hat{\theta}_t, h^{t-1}. \tag{11}$$

Even though the types of buyers are revealed in the first period and keep constant across time, the subsequent incentive constraints $IC_t, t \geq 1$ cannot be neglected

[14] Given the large number of consumers, we implicitly assume that the short-term contracts are anonymous. Otherwise, the seller could infer a consumer's future type and then deprive some of his information rents from his current report when types are persistent.

[15] It is defined that $\theta^0 := \emptyset$.

because they confer the incumbent firm instruments to punish inconsistent reports. Similar to the well-known approach for solving the static screening problem, we need to give the necessary (envelope) and sufficient (implementability) conditions for IC constraints.

Lemma 2 *When types are constant, IC_t (11) implies a dynamic version of envelope formula:*

$$EN_t : \frac{\partial \mathcal{U}_t(\theta_t|h^{t-1})}{\partial \theta_t} = \sum_{s=t}^{\infty} \beta^{s-t} \left[v(q_s(\theta_t|h^{s-1})) - v(q^0(\theta_t)) \right], \forall h^{t-1}, \theta_t, \quad (12)$$

where $h^{s-1} := \{h^{t-1}, \theta_t, \cdots, (s-t)-times, \cdots, \theta_t\}, \forall s \geq t$. EN_t and the following monotonic condition MON_t are sufficient for dynamic implementability:

$$MON_t : q_t(h^t) \geq q_t(\hat{h}^t), \text{ if } h_s^t \geq \hat{h}_s^t, \forall s \leq t. \quad (13)$$

Proof See Appendix C. □

In static environments, the envelope condition and monotonicity of the allocation are necessary and sufficient for implementability. Our result here is, however, weaker because the monotonicity condition compares quantities along all histories instead of just the current quantities, and the result is only sufficient. Next, we show that the optimal LTC obtained from $[\mathcal{P}_\ell]$ is a repetition of the static contract.

Proposition 3 *Suppose that consumers are fully rational and their types are constant over time, then we have the following results.*

(i) The optimal long-term entry-deterrence contract constitutes a repetition of the optimal static contract $\{q^*(\theta), \tau^*(\theta)\}$, and gives consumers a continuation rent of $\mathcal{U}^*(\theta) = U^*(\theta)/[1-\beta]$ in every period, where $q^*(\theta)$, $\tau^*(\theta)$ and $U^*(\theta)$ are given in Proposition 1.
(ii) The incumbent gets a higher surplus under an LTC than under a sequence of STCs.
(iii) An LTC is socially more efficient than its short-term counterpart.

Proof See Appendix D. □

Propositions 2 and 3 show that when consumers are fully rational, an LTC leads to the same competitive outcomes as STCs. The incumbent could achieve full exclusion via either of them. Nevertheless, an LTC confers a higher profit to the incumbent, and is socially more efficient, than its short-term counterpart because it allows the incumbent to circumvent the additional costs of reaching new agreements with buyers. This result provides the incumbent a possible motivation for the adoption of an LTC.

The assumption of full rationality is, however, usually unrealistic since it confers on consumers unlimited cognitive and computing capacities. In the sequel, we assume instead that consumers are boundedly rational in the sense that they are unable to forecast accurately the network size, but can update their beliefs from the observation of the past realizations. The bounded rationality is caused by some exogenous shocks, which cannot be forecasted accurately by the incumbent and are not contractible ex ante.

When consumers are boundedly rational, the short-term and long-term contracts lead to different competitive results. There is a one-period time lag between the pessimistic market sentiments and the awareness of the incumbent. Under STCs, the competitors have a chance to erode part of the incumbent's market in this *unawareness* period, but their good luck will end once the incumbent realizes what has happened and then employs a countermeasure of price adjustment to recover his monopolistic position.

Proposition 4 *With bounded rationality, constant types and a sequence of short-term contracts, entry may happen temporarily. The entrants, however, will be expelled from the network in the next period.*

Proof See Appendix E. □

Things are different under an LTC. The incumbent believes the consumers to be fully rational at the outset, and offers a fully rational LTC. Once this contract is signed ex ante, the incumbent has to commit not to change it at later dates. Two forms of adjustment, i.e., raising and reducing the initial price, are both banned. The former way will naturally be boycotted by consumers since an increased price will harm them. Reducing the initial price is also prohibited though it seems beneficial to the customers. If this kind of behavior is allowed, the incumbent would attract back the customers who have quitted the network by a one-shot price reduction. He could then recover the price and recoup the loss when his monopolistic position is regained. An LTC allowing ex-post price reduction has exactly the same competitive result as a sequence of STCs given in Proposition 4. Such a temporary price cutting is, however, regarded as a sort of predatory pricing behavior, which is strictly banned by antitrust laws of both U.S. and Europe. A firm who has signed an LTC and then engage in predatory price cutting will be sued by his rivals and competition authorities, especially when he has significant market power.

When the incumbent has to abide by the LTC signed ex ante, a door is open to the new competitors. In the remainder of the paper, we assume the following: (i) the agents' beliefs are homogeneous, i.e., all agents share the same expectation Q_t^e; and (ii) the forecasting error may occur for only one period beforehand, and expectations with respect to the subsequent dates are still on the fully rational path. Therefore, the continuation payoff under a one-shot expectation deviation is $\theta v(q^*(\theta)) - \tau^*(\theta) + \Omega(Q_t^e) - U^0(\theta) + \frac{\beta}{1-\beta}\left[\theta v(q^*(\theta)) - \tau^*(\theta) + \Omega(Q^*) - U^0(\theta)\right] = \mathcal{U}^*(\theta) + \Omega(Q_t^e) - \Omega(Q^*)$.

The consumers' adaptive learning process under an LTC works as follows. At time t, all agents form a common expectation Q_t^e of the network size; then, consumers with nonnegative continuation utilities, i.e., $\theta \in \Theta(Q_t^e) := \left\{\theta \in [\underline{\theta}, \overline{\theta}] : \mathcal{U}^*(\theta) \geq \Omega(Q^*) - \Omega(Q_t^e)\right\}$, accept the contract; the rest will reject it and quit the network. Committing to the LTC, the incumbent firm still implements a repetition of $\{q^*(\theta), \tau^*(\theta)\}$ even after observing the loss of some customers, a network size of $Q_t = \rho(Q_t^e) := \int_{\theta\in\Theta(Q_t^e)} q^*(\theta) f(\theta) d\theta$ is then realized. If $Q_t = Q_t^e$, the equilibrium is reached; otherwise, consumers use the most recent realization as their present forecast, i.e., $Q^e_{t+1} = Q_t$. This process repeats until either an expectation is self-fulfilled, i.e., $Q^e_t = Q_t$, or the entire market is occupied by the entrants, i.e., $Q_t = 0$. The fully rational network size Q^* is obviously a fixed point of the feedback function $\rho(\cdot)$. In what follows, we discuss the stability of this fixed point. That is, when the incumbent commits to an LTC, whether or not the adaptive learning procedures will lead the economy to converge to a fully rational equilibrium Q^*.

If the network value is decreasing at the optimum, i.e., $\Omega'(Q^*) \leq 0$, consumers will stay in (resp. quit) the network upon an initial expectation $Q^e_0 < (resp. >) Q^*$. Then, the adaptive learning rule forms a negative feedback that leads the expectation back to Q^* within at most two periods. The rivals have no chance to stay in the market permanently (see Panel (a) of Fig. 1). In the sequel, we discuss the evolving properties of network size Q_t in the joint presence of increasing network effect (i.e., $\Omega'(Q^*) > 0$) and an LTC.

Proposition 5 *Suppose that $\rho(\cdot)$ has a unique fixed point Q^* on $(0, \infty)$.*[16] *Then, the evolving properties of Q_t under a long-term contract depend on the comparison between $\Omega'(Q^*)$ and*

$$\Delta(Q^*) := \frac{|v(q^*(\theta^*(Q^*))) - v(q^0(\theta^*(Q^*)))|}{(1-\beta) f(\theta^*(Q^*)) q^*(\theta^*(Q^*))} : \tag{14}$$

– *if $\Omega'(Q^*) > \Delta(Q^*)$, then*

$$\lim_{t\to\infty} Q_t = \begin{cases} Q^* & \text{if } Q^e_0 \geq Q^* \\ 0 & \text{if } Q^e_0 < Q^* \end{cases};$$

– *if $0 < \Omega'(Q^*) < \Delta(Q^*)$, then $\lim_{t\to\infty} Q_t = Q^*$, where $\theta^*(Q^*)$ in (14) satisfies*

$$\theta^*(Q^*) \in \begin{cases} \{\underline{\theta}\} & \textit{if } c_E \in \left[c^*_H, c_I\right) \\ \left[\underline{\theta}, \theta^-(Q^*)/(1-\beta)\right] & \text{if } c_E \in \left(c^*_M, c^*_H\right) \\ \left[\underline{\theta}, \overline{\theta}\right] & \text{if } c_E = c^*_M \\ \left[\theta^+(Q^*)/(1-\beta), \overline{\theta}\right] & \text{if } c_E \in \left[c^*_L, c^*_M\right) \\ \{\overline{\theta}\} & \textit{if } c_E \in \left[0, c^*_L\right) \end{cases}.$$

Proof See Appendix F. □

This proposition leaves out the case with a neutral fixed point, i.e., $\Omega'(Q^*) = \Delta(Q^*)$, wherein the stability of Q^* could not be determined until further information regarding the higher-order terms of the Taylor expansion of $\rho(\cdot)$ is available. An overoptimistic expectation encourages consumers' participation, but the goods are not overconsumed because the incumbent controls the total amount of goods that

[16] The procompetitive results do not change qualitatively without this unique assumption. If $\rho(\cdot)$ has another fixed point $Q_1 \in (0, Q^*)$, we have $\lim_{t\to+\infty} Q_t = Q_1 < Q^*$ provided that $\Omega'(Q^*) > \Delta(Q^*)$ and $Q^e_0 < Q^*$; the incumbent thus has part of his market eroded by the entrants.

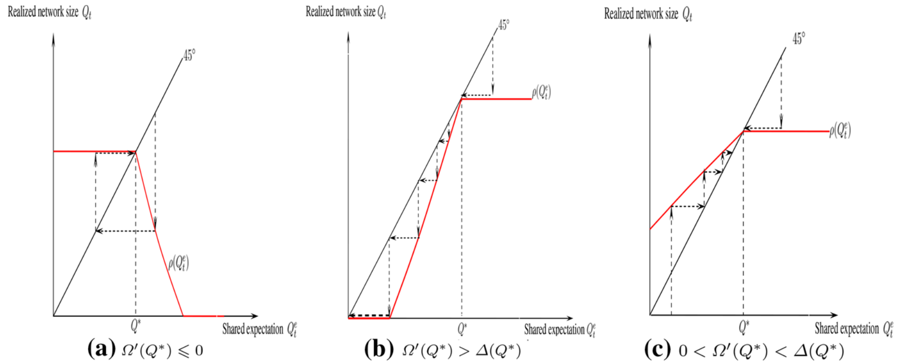

Fig. 1 Stability of fixed point Q^*

enter the market. So Q^* is always stable from above. Its stability from below hinges on the comparison between $\Omega'(Q^*)$ and $\Delta(Q^*)$. $\Omega'(Q^*)$ reflects the degree of network effect at the optimum. $\Delta(Q^*)$ measures the rate at which the information rent $\mathcal{U}^*(\theta)$ changes with respect to θ.

The larger $\Delta(Q^*)$ is, the fewer customers will quit upon pessimism, and thus, the less sensitive $\rho(Q_t^e)$ is to Q_t^e. If the network effect is relatively weak at the optimum ($\Omega'(Q^*) < \Delta(Q^*)$), an upward pressure will be exerted on the actual consumption. In every period, the realized network size is larger than the expected value, i.e., $Q_t^e < Q_t = Q_{t+1}^e, \forall t \geq 0$. Then the adaptive learning rule will gradually lead the economy back to the fully rational equilibrium. If network effect is relatively strong at the optimum ($\Omega'(Q^*) > \Delta(Q^*)$), an initial pessimistic expectation will trap the economy into a vicious cycle in which pessimism (lower expectation) and depression (lower participation) alternatingly reinforce each other. The incumbent firm will eventually lose the entire market to the entrants. Panels (b) and (c) of Fig. 1 depict the cases with, respectively, strongly and weakly increasing network effects.

Notice that Q^* is more stable for a larger discount factor β, because the more patient the consumers are, the less likely they will quit the incumbent network under a pessimistic expectation, and therefore the stronger the self-recovery ability of the market is. If $c_E \in \left(c_H^*, c_I\right)$, $\Delta(Q^*) = \Delta(Q^-) = [v(q^-(\underline{\theta})) - v(q^0(\underline{\theta}))]/[(1-\beta)q^-(\underline{\theta})f(\underline{\theta})]$; if $c_E \in \left[0, c_L^*\right)$, $\Delta(Q^*) = \Delta(Q^+) = [v(q^0(\overline{\theta})) - v(q^+(\overline{\theta}))]/[(1-\beta)q^+(\overline{\theta})f(\overline{\theta})]$. In both cases, $\Delta(Q^*)$ is strictly positive. The most interesting case arises when $c_E \in \left[c_L^*, c_H^*\right]$, where $q^*(\theta^*) = q^0(\theta^*)$ and $\Omega'(Q^*) > 0 = \Delta(Q^*)$. Q^* is thus unstable from below. We then have the following corollary.

Corollary 1 *With a long-term contract, a strictly increasing network effect at the optimum, i.e.,* $\Omega'(Q^*) > 0$*, a pessimistic initial expectation, i.e.,* $Q_0^e < Q^*$*, and an intermediate value of* c_E *belonging to* $\left[c_L^*, c_H^*\right]$*, the incumbent firm will gradually lose his entire market to the entrants, i.e.,* $\lim_{t\to+\infty} Q_t = 0$.

Figure 2 depicts the consumers segmentations in different settings. With an intermediate $c_E \in [c_L^*, c_H^*]$, a strictly positive measure set of consumers earn zero rents and feel indifferent between accepting and rejecting the contract under the rational

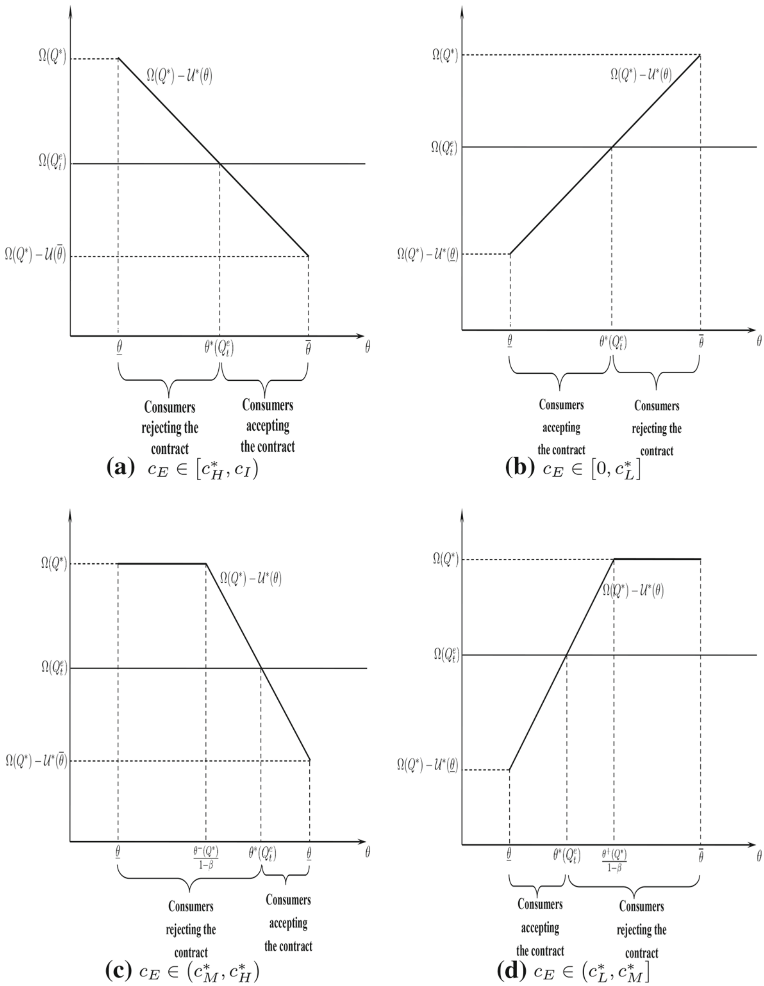

Fig. 2 Consumers segmentation under bounded rationality

expectation $Q_0^e = Q^*$ (see Panels (c) and (d) of Fig. 2). They will quit the network even under a very slightly pessimistic initial expectation. Then, a positive network feedback is started. The entire market may eventually be eroded by the entrants (see Fig. 3).

Proposition 5 and Corollary 1 show that under an LTC, initial market sentiment is very important for the incumbent to maintain his monopolistic position. With an intermediate value of cost c_E, the firm will not successfully foreclose entry unless the population's expectation for the network size is above or right at a critical point Q^*. Starting even slightly below this point and hoping to grow slowly is unlikely to succeed. Indeed, this phenomenon is ubiquitous in network industries. Unless a product is widely used, the population has no confidence in its success; it thus has little value to any potential purchaser.

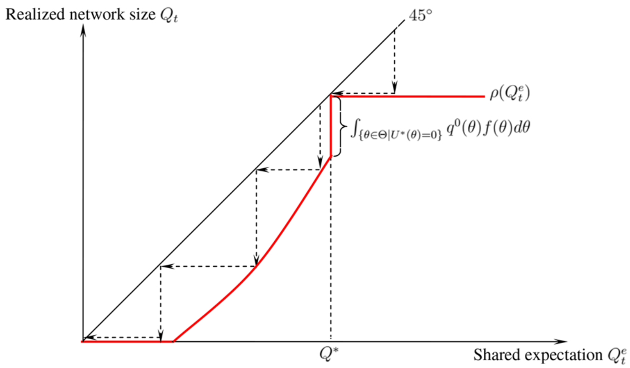

Fig. 3 The case with $\Omega'(Q^*) > 0$ and $c_E \in [c_L^*, c_H^*]$

Although whether an LTC could facilitate entry (i.e., $Q_t \to 0$) hinges crucially on the uncontrollable market sentiments, it is at least as competition-friendly as its short-term counterpart. An LTC allows potential rivals and competition authorities to monitor and discipline the incumbent firm more effectively than under STCs, since the firm's pricing behavior is now more transparent and predictable. Banning LTCs does not do any good to market competition, because the incumbent will resort instead to the more anticompetitive STCs.

Even though an LTC facilitating efficient entry has a procompetitive effect, its implication for social welfare is not immediately obvious since the entrants provide no network value. The social welfare comparison between LTC and STCs hinges on three elements: the entrants' cost advantage over the incumbent, the equilibrium network value $\Omega(Q^*)$ provided by the incumbent, and the transaction costs saved by an LTC. Let

$$\mathcal{W}_\ell^{bc}(c_E) := \frac{1}{1-\beta}\left[\int_{\underline{\theta}}^{\overline{\theta}} \left[\theta v(q^0(\theta, c_E)) - c_E q^0(\theta, c_E)\right] f(\theta)d\theta\right] - C \tag{15}$$

and

$$\mathcal{W}_s^{bc}(c_E) := \frac{1}{1-\beta}\left[\int_{\underline{\theta}}^{\overline{\theta}} \left[\theta v(q^*(\theta, c_E)) - c_I q^*(\theta, c_E)\right] f(\theta)d\theta + \Omega(Q^*(c_E)) - C\right] \tag{16}$$

denote, respectively, the discounted social surpluses conferred by LTC (ℓ) and STCs (s) under bounded rationality (b) and constant type (c).[17] Applying Proposition 1, we obtain:

– if $c_E \in [0, c_L^*]$, then $Q^*(c_E) = Q^+$ and $q^*(\theta, c_E) = q^+(\theta)$;

[17] When calculating $\mathcal{W}_\ell^{bc}(c_E)$ and $\mathcal{W}_s^{bc}(c_E)$, we neglect the finite transition periods when the incumbent and the entrants coexists with each having a positive market share.

– if $c_E \in (c_L^*, c_M^*)$, then $Q^*(c_E)$ is determined by

$$Q^* = \int_{\underline{\theta}}^{\overline{\theta}} \min\{q^+(\theta, Q^*), q^0(\theta, c_E)\} f(\theta) d\theta,$$

and $q^*(\theta, c_E) = \min\{q^+(\theta, Q^*(c_E)), q^0(\theta, c_E)\}$;
– if $c_E = c_M^*$, then $Q^*(c_E) = Q^f$, and $q^*(\theta, c_E) = q^f(\theta) = q^0(\theta, c_E)$;
– if $c_E \in (c_M^*, c_H^*]$, then $Q^*(c_E)$ is determined by

$$Q^* = \int_{\underline{\theta}}^{\overline{\theta}} \max\{q^-(\theta, Q^*), q^0(\theta, c_E)\} f(\theta) d\theta$$

, and $q^*(\theta, c_E) = \max\{q^-(\theta, Q^*(c_E)), q^0(\theta, c_E)\}$;
– if $c_E \in (c_H^*, c_I]$, then $Q^*(c_E) = Q^-$ and $q^*(\theta, c_E) = q^-(\theta, Q^-)$.

The difference of social welfare is then represented as $\Delta\mathcal{W}^{bc}(c_E) \equiv \mathcal{W}_s^{bc}(c_E) - \mathcal{W}_\ell^{bc}(c_E) = \Delta W(c_E)/(1-\beta)$, where

$$\Delta W(c_E) \equiv \left[\begin{array}{c} \int_{\underline{\theta}}^{\overline{\theta}} [\theta v(q^*(\theta, c_E)) - c_I q^*(\theta, c_E)] f(\theta) d\theta + \Omega(Q^*(c_E)) \\ - \int_{\underline{\theta}}^{\overline{\theta}} [\theta v(q^0(\theta, c_E)) - c_E q^0(\theta, c_E)] f(\theta) d\theta - \beta C \end{array} \right].$$

If $\Delta W(c_E) < 0$, an LTC is more efficient than a sequence of STCs. One can easily check that it holds for sufficiently small c_E, since $\Delta W(c_E) \to -\infty$ as $c_E \to 0$. A meaningful problem is under what conditions these c_Es guarantee a procompetitive outcome as well.

Proposition 6 *Suppose that $\Delta W(c_E^d) < 0$. Then there exists a $c_E^* \in (c_E^d, c_M^*]$ such that an LTC is procompetitive under pessimism, and is more efficient than its short-term counterpart whenever $c_E \in [c_E^d, c_E^*]$, where c_E^d is given implicitly by*

$$\Omega'(Q^+) = \frac{v(q^0(\overline{\theta}, c_E)) - v(q^+(\overline{\theta}))}{(1-\beta) f(\overline{\theta}) q^+(\overline{\theta})}.$$

Proof See Appendix G. □

Example 1 (A Numerical Example)

Consider an example where θ follows a uniform distribution over $[\underline{\theta}, \overline{\theta}]$, $\overline{\theta} = 105, \underline{\theta} = 100, \beta = 0.9, \epsilon = 1/5, k = 7, c_I = 70, C = 3,\ V(x) = x^{1-\epsilon}/(1-\epsilon), \Omega(x) = kx - ke^{-kx} + k$. We obtain $Q^+ = 12.96,\ Q^f = 11.42,\ Q^- = 10.16, c_H^* = 66.32, c_M^* = 63.00, c_L^* = 60.14$. c_E^* is the zero point of $\Delta W(\cdot)$. c_E^d and c_E^u are cutoffs of c_E given implicitly by $\Omega'(Q^+) = [v(q^0(\overline{\theta}, c_E)) - v(q^+(\overline{\theta}))]/[(1-\beta) f(\overline{\theta}) q^+(\overline{\theta})]$ and $\Omega'(Q^-) = [v(q^-(\underline{\theta})) - v(q^0(\underline{\theta}, c_E))]/[(1-\beta) f(\underline{\theta}) q^-(\underline{\theta})]$, respectively. Figure 4 depicts curve $\Delta W(c_E)$. If $c_E \le (resp. >) c_E^* = 62.62$, $\Delta W(c_E) \le (resp. >) 0$, then the LTC is more (resp. less) efficient than STCs. It follows from Proposition 5 that, when $c_E \in [c_E^d, c_E^u] = [57.51, 69.45]$ (resp. $c_E \in [0, c_E^d) \cup (c_E^u, c_I] = [0, 57.51) \cup$

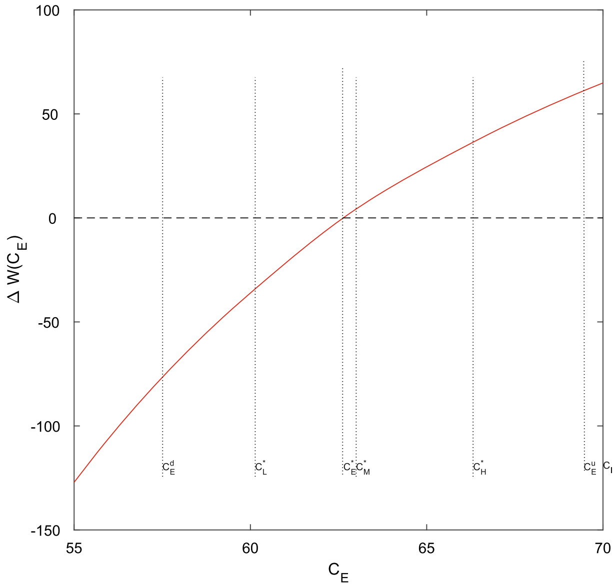

Fig. 4 Social welfare gap in Example 1

Table 1 Competitive and efficiency outcomes of an LTC in Example 1

c_E	Efficiency enhancing ($\Delta W(c_E) \leq 0$)	Efficiency reducing ($\Delta W(c_E) > 0$)
Procompetitive ($Q_t \to 0, \forall Q_t^e < Q^*$)	$(c_E^d, c_E^*] = (57.51, 62.62]$	$(c_E^*, c_E^u) = (62.62, 69.45)$
Anticompetitive ($Q_t \to Q^*, \forall Q_t^e$)	$[0, c_E^d) = [0, 57.51)$	$(c_E^u, c_I] = (69.45, 70]$

$(69.45, 70])$, $\Omega'(Q^*(c_E)) > \Delta(Q^*(c_E))$, a procompetitive (resp. anticompetitive) outcome arises. Table 1 summarizes all possible cases.

5 The case with independent types

In this section, we discuss the case with independent types. θ_t is now drawn independently by "nature" according to a continuous density $f_t(\cdot)$ and a c.d.f $F_t(\cdot)$ over a support $\Theta_t := [\underline{\theta}_t, \overline{\theta}_t]$. Still, the initial distribution $F_0(\cdot)$ satisfies the monotone hazard rate condition (1). Let $\boldsymbol{\theta}_t := (\theta_0, \cdots, \theta_t)$ be the whole history of type realizations up to date t. The incumbent's objective (2) now takes the form:

$$\Pi_\ell = \left\{ \begin{array}{l} \int_{\underline{\theta}_0}^{\bar{\theta}_0} \left[\theta v(q_0(\theta)) - c_I q_0(\theta) - U^0(\theta) - \mathcal{U}_0(\theta)\right] f_0(\theta)d\theta + \Omega\left(\int_{\underline{\theta}_0}^{\bar{\theta}_0} q_0(\theta) f_0(\theta)d\theta\right) - C \\ + \sum_{t=1}^{\infty} \beta^t \int\limits_{\prod_{s=0}^{t-1}\Theta_s} \left[\begin{array}{l} \int_{\underline{\theta}_t}^{\bar{\theta}_t} [\theta_t v(q_t(\boldsymbol{\theta}_t)) - c_I q_t(\boldsymbol{\theta}_t) - U^0(\theta_t)] f_t(\theta_t)d\theta_t \\ + \Omega\left(\int_{\underline{\theta}_t}^{\bar{\theta}_t} q_t(\boldsymbol{\theta}_t) f_t(\theta_t)d\theta_t\right) \end{array} \right] \prod_{s=0}^{t-1} f_s(\theta_s)d\boldsymbol{\theta}_{t-1} \end{array} \right\}. \tag{17}$$

Since types are independent across time, the current type bears no information about future types. So term $\mathbb{E}\left[\mathcal{U}_{t+1}(\theta_{t+1}|h^{t-1}, \hat{\theta}_t)|\theta_t\right]$ in IC_t (3) is independent of θ_t. We have the following lemma.

Lemma 3 *With independent types, IC_t condition holds if and only if the following EN_t and MON_t conditions hold:*

$$EN_t : \frac{\partial \mathcal{U}_t(\theta_t|h^{t-1})}{\partial \theta_t} = v(q_t(\theta_t|h^{t-1})) - v(q^0(\theta_t)), \forall t \geq 0, \forall \theta_t, \forall h^{t-1}; \tag{18}$$

$$MON_t : q_t(\theta_t|h^{t-1}) \textit{ is nondecreasing in } \theta_t, \forall t \geq 0, \forall h^{t-1}. \tag{19}$$

The proof is standard in adverse selection problem and is therefore omitted.

The principal's optimization problem $[\mathcal{P}_\ell]$ is then represented as

$$\max_{\{q_t(\boldsymbol{\theta}_t), \mathcal{U}_t(\boldsymbol{\theta}_t)\}_{t=0}^{\infty}} \Pi_\ell(17), s.t. : IR_t(4), EN_t(18), MON_t(19), \forall t \geq 0. \tag{20}$$

Obviously, this problem can be decomposed into a static contracting problem $[\mathcal{P}_s]$ in the initial period $t = 0$ and a repetition of the full information/first-best optimization problem $[\mathcal{P}_s^f]$ from $t = 1$ onwards. In analogy to Proposition 1, we can obtain the optimal quantities $\{q_t(\boldsymbol{\theta}_t)\}_{t=0}^{\infty}$ and the initial rent $\mathcal{U}_0(\theta_0)$. The subsequent continuation rents $\{\mathcal{U}_t(\boldsymbol{\theta}_t)\}_{t=1}^{\infty}$, however, are not uniquely determined since they do not enter the seller's objective Π_ℓ. These rents can be chosen arbitrarily to meet constraints IR_t and IC_t.

Proposition 7 *Suppose that consumers are fully rational and have independent types. Then the optimal entry-deterrence LTC constitutes an optimal static allocation in the first period and the full-information allocation in all the following periods: $q_0(\theta_0) = q^*(\theta_0)$[18], $q_t(\boldsymbol{\theta}_t) = q^f(\theta_t), \forall t \geq 1$. The initial continuation rent is $\mathcal{U}_0(\theta_0) = U^*(\theta_0)$; the subsequent continuation rents $\{\mathcal{U}_t(\boldsymbol{\theta}_t)\}_{t=1}^{\infty}$ are, however, not uniquely identified.*

The underlying intuition is simple. Since types are independent across periods, an agent has no private information about his own future type beyond the initial period when the contract is signed. The quantities offered after $t = 0$ are therefore efficient because there is no asymmetric information between the seller and buyers. However, the quantities of period $t = 0$ are not efficient because at that point, the agent is privately informed about his own type. The traditional rent extraction versus efficiency tradeoff thus calls for an inefficient allocation.

[18] Notice that $f(\cdot)$ and $F(\cdot)$ in the expressions of $q^*(\cdot)$ are now replaced by $f_0(\cdot)$ and $F_0(\cdot)$.

The fully rational agents could forecast accurately the network size beforehand. The boundedly rational agents, however, will make some errors in their initial expectation and then update it in accordance with an adaptive learning rule. Similar to the analysis of constant-type case, we discuss first the role of STCs. If the incumbent is free to sign an STC in every period, a temporary entry may happen under bounded rationality, but the incumbent will regain his monopolistic position via a one-shot price reduction.

Proposition 8 *With independent types and a chain of STCs, entry occurs upon pessimistic expectation. The entrants, however, will lose their market share again in the next period.*

The proof is analogous to that of Proposition 4 and is therefore omitted. Next, we come to the competitive effect of an LTC under bounded rationality.

Proposition 9 *With independent types and bounded rationality, the LTC given in Proposition 7 is robust to pessimistic expectations if the subsequent rents* $\{\mathcal{U}_t(\boldsymbol{\theta}_t)\}_{t=1}^{\infty}$ *are sufficiently large.*

Proof See Appendix H. □

The seller can provide an optimal LTC with binding IR constraints in every period. He can also construct contracts in which only IR_0 binds. Under full rationality, these two contracting ways are equivalent, since they can both foreclose entry and give an equal ex-ante payoff to the incumbent. With bounded rationality, however, the latter way is more robust to pessimistic sentiments than the former. It could ensure the incumbent a greater safety. In the proof given in Appendix H, we show that if all types are conferred a continuation rent larger than the network value they derive in each period $t \geq 1$, then even in the worst situation where consumers have an extremely pessimistic expectation (i.e., $Q_t^e = 0$), the nonnegative stand-alone utilities are enough to guarantee participation.

In business practice, a seller often offers a contract in which a sufficiently high price is charged in the first period, and he commits to paying consumers a rebate in installments in each subsequent period to guarantee their participation. A notable example is the membership rewarding program adopted by some retailers (say, Costco). To secure customers' loyalty, the firm requires them to sign as members by paying an initial membership fee. In return, members can shop at highly competitive prices. Expecting to be able to recoup their membership fees, buyers will accept this contract from the outset, and will not quit even when they face unfavorable market sentiments toward the incumbent.

Propositions 8 and 9 show that when types are independent, both STCs and LTC can achieve full exclusion since they are both immune to pessimistic expectations. Nevertheless, their efficiency outcomes are different. We denote by $\mathcal{W}_\ell^i$ and $\mathcal{W}_s^i$ the social surpluses under, respectively, LTC (ℓ) and STCs (s) when types are independent (i); by Π_ℓ^i and Π_s^i the incumbent's profits under, respectively, LTC and STCs when types are independent. We have the following results.

Proposition 10 *With independent types, an LTC confers a higher profit to the incumbent, and gives a higher social surplus, than its short-term counterpart, i.e.,* $\Pi_\ell^i > \Pi_s^i$ *and* $\mathcal{W}_\ell^i > \mathcal{W}_s^i$.

Proof See Appendix I. □

With a sequence of STCs, the seller faces the same adverse selection problem repeatedly, and an allocative distortion due to the asymmetry of information exists in every period. With an LTC, however, the seller confronts only asymmetric information problem in the first period. He could sign up consumers before they know their own future types. This mitigates the efficiency loss caused by asymmetric information. Moreover, an LTC allows the incumbent to avoid transaction costs of reaching new agreements with the customers repeatedly. As a result, an LTC gives a higher profit to the incumbent, and a higher social surplus than a sequence of STCs.

6 Conclusion

In this paper, we compare the competitive and efficiency outcomes of LTC and STCs in the presence of network effects and bounded rationality. If a chain of STCs are provided in every period, the incumbent firm has to pay a fixed transaction cost repeatedly. He could achieve full foreclosure of the market, regardless of whether or not the consumers are fully rational, and whether types are constant or independent. If an LTC is signed ex ante, the seller only needs to pay the transaction cost once. But he is locked into the bilateral relationship with consumers, who are still allowed to exit freely. The competitive outcome of LTC depends crucially on the rationality of consumers and the statistical persistence of their types.

With full rationality and constant types, an LTC constituting a repetition of the optimal static contract fully forecloses entry of more efficient competitors. It also gives a higher social surplus than its short-term counterpart because it saves transaction costs of contracting. The exclusionary power of an LTC is, however, dramatically weakened by the bounded rationality of consumers. When consumers are boundedly rational, the marginal cost of entrants falls into some intermediate range, a small pessimistic forecasting error will trigger an avalanche-like process leading the incumbent to cede the entire market to the entrants. This result is also socially efficient when the cost advantage of the entrants over the incumbent, plus a saving of transaction expense, outweighs the equilibrium network value provided by the incumbent.

In the case of independent types and full rationality, the incumbent could foreclose entry by an LTC providing consumers the optimal static allocation in the initial period and the first-best allocations in all the following periods. Moreover, this LTC is robust to pessimistic expectations when the incumbent chooses, without loss of generality, sufficiently large continuation rents from the second period onwards. Though an LTC and STCs lead to the same competitive result, the former gives a higher social surplus than the latter because it mitigates the asymmetry of information and also saves transaction costs of contracting. Our findings are summarized in Table 2.

It can be found that an LTC is at least as procompetitive and socially efficient as its short-term counterpart, regardless of the degree of rationality and whether types are constant or independent over time. These results thus invite a more tolerant attitude toward LTCs in antitrust practice.

Table 2 The main results of this paper (AC: anticompetitive; PC: procompetitive)

	Full Rationality		Bounded Rationality	
	Constant types	Independent types	Constant types	Independent types
STC	AC (Prop. 2)	AC (Prop. 2)	AC (Prop. 4)	AC (Prop. 8)
LTC	AC and more efficient than STCs (Prop. 3)	AC and more efficient than STCs (Props. 7, 10)	PC and more efficient than STCs for some c_Es (Props. 5, 6 Coro.1)	AC and more efficient than STCs (Props. 9, 10)

In this paper, we consider only cases with constant and independent types. It raises a natural question of how general stochastic process governing the evolution of types will affect the results of this article. Although intriguing, we believe that this question deserves a separate and more extensive analysis that goes beyond the scope of this paper, and we therefore leave it for further research.

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Appendix

A. Proof of Lemma 1

Expressions (7) to (9) imply $q^-(\theta, Q) \leq q^f(\theta, Q) \leq q^+(\theta, Q), \forall\theta \in [\underline{\theta}, \overline{\theta}]$, with equalities for, respectively, $\overline{\theta}$ and $\underline{\theta}$. Therefore, we can easily obtain $Q^- < Q^f < Q^+$. From the monotone hazard rate condition (1), we have

$$\frac{v'(q^-(\theta))}{v'(q^f(\theta))} = \frac{c_I - \Omega'(Q^-)}{[c_I - \Omega'(Q^f)]\left(1 - \frac{1-F(\theta)}{\theta f(\theta)}\right)}$$

is decreasing in θ. If $q^-(\underline{\theta}) \geq q^f(\underline{\theta})$, then $v'(q^-(\theta))/v'(q^f(\theta)) \leq v'(q^-(\underline{\theta}))/v'(q^f(\underline{\theta})) \leq 1, \forall\theta \in (\underline{\theta}, \overline{\theta}]$. Therefore, $q^-(\theta) \geq q^f(\theta)$ $\forall\theta \in (\underline{\theta}, \overline{\theta}]$, which contradicts $Q^- < Q^f$. So we must have $q^-(\underline{\theta}) < q^f(\underline{\theta})$.

It follows from condition (1) that

$$\frac{v'(q^+(\theta))}{v'(q^f(\theta))} = \frac{c_I - \Omega'(Q^+)}{[c_I - \Omega'(Q^f)]\left(1 + \frac{F(\theta)}{\theta f(\theta)}\right)}$$

is decreasing in θ. If $q^{+}(\overline{\theta}) \leq q^{f}(\overline{\theta})$, then $v'(q^{+}(\theta))/v'(q^{f}(\theta)) \geq v'(q^{+}(\overline{\theta}))/v'(q^{f}(\overline{\theta})) \geq 1$. Therefore, $q^{+}(\theta) \leq q^{f}(\theta), \forall \theta \in [\underline{\theta}, \overline{\theta})$, which contradicts $Q^{+} > Q^{f}$. So we must have $q^{+}(\overline{\theta}) > q^{f}(\overline{\theta})$.

Consequently, we have

$$\begin{aligned} c_H^* :=& \frac{c_I - \Omega'(Q^-)}{1 - \frac{1}{\underline{\theta} f(\underline{\theta})}} = \underline{\theta} v'(q^-(\underline{\theta})) > \underline{\theta} v'(q^f(\underline{\theta})) \\ =& c_M^* := c_I - \Omega'(Q^f) = \overline{\theta} v'(q^f(\overline{\theta})) > \overline{\theta} v'(q^+(\overline{\theta})) = \frac{c_I - \Omega'(Q^+)}{1 + \frac{1}{\overline{\theta} f(\overline{\theta})}} := c_L^*. \end{aligned}$$

B. Proof of Proposition 1

The principal's relaxed problem $[\mathcal{P}_s^r]$ is a standard control problem with U as a state variable and q as a control variable. The Hamiltonian function for this control problem is

$$\mathcal{H}(U, q, \mu, \theta) = \begin{bmatrix} \left[\theta v(q(\theta)) - c_I q(\theta) - U^0(\theta) - U(\theta) - F\right] f(\theta) + \\ \Omega\left(\int_{\underline{\theta}}^{\overline{\theta}} q(\theta) f(\theta) d\theta\right) + \mu(\theta)[v(q(\theta)) - v(q^0(\theta))] \end{bmatrix}, \tag{21}$$

where μ is the costate variable. The Lagrangian function is $\mathcal{L} = \mathcal{H} + \gamma(\theta) U(\theta)$, where $\gamma(\theta)$ is the multiplier of IR constraint. The first-order condition for the maximization of $\mathcal{H}(U, q, \mu, \theta)$ with respect to q is

$$\left[\theta + \frac{\mu(\theta)}{f(\theta)}\right] v'(q(\theta)) + \Omega'(Q) = c_I. \tag{22}$$

Since $v(\cdot)$ and $\Omega(\cdot)$ are both concave, this first-order condition characterizes the optimum. For a fixed network size Q, all variables (q, μ, U, γ) are represented as functions of θ and Q, then the following conditions must be satisfied:

$$\text{costate equation: } \mu_\theta = -\frac{\partial \mathcal{L}}{\partial U} = f(\theta) - \gamma(\theta, Q) \tag{23}$$

$$\text{state equation: } U_\theta = v(q(\theta, Q)) - v(q^0(\theta)) \tag{24}$$

$$\text{complementary slackness: } \gamma(\theta, Q) U(\theta, Q) = 0, \gamma(\theta, Q) \geq 0, U(\theta, Q) \geq 0 \tag{25}$$

$$\text{transversality conditions: } \begin{bmatrix} \mu(\underline{\theta}, Q) U(\underline{\theta}, Q) = \mu(\overline{\theta}, Q) U(\overline{\theta}, Q) = 0, \\ \mu(\underline{\theta}, Q) \leq 0, \mu(\overline{\theta}, Q) \geq 0 \end{bmatrix}. \tag{26}$$

Let $\hat{q}(\mu, \theta, Q)$ denote the value of q defined implicitly by (22), and let $\hat{\mu}(\theta, Q)$ be the solution in μ to equation $\hat{q}(\mu, \theta, Q) = q^0(\theta)$. It follows that $\hat{\mu}(\theta, Q) = \left[\left(c_I - \Omega'(Q)\right)/c_E - 1\right] \theta f(\theta)$, which is the value of costate variable such that the agent's utility is constant (i.e., $U_\theta = 0$). From (23) and (25), we have $\gamma(\theta, Q) = f(\theta) - \mu_\theta(\theta, Q) \geq 0$. If the IR is binding on a nondegenerate interval, then $\mu(\theta, Q)$ must be equal to $\hat{\mu}(\theta, Q)$ and $\mu_\theta = \hat{\mu}_\theta < f(\theta)$ on that interval.

To solve the problem $[\mathcal{P}_s^r]$, we conjecture a solution and then verify whether it satisfies conditions (23) to (26). Consider the following function,

$$\mu^*(\theta, Q) = \begin{cases} F(\theta) & \text{if } \hat{\mu}(\theta, Q) \geq F(\theta) \\ \hat{\mu}(\theta, Q) & \text{if } F(\theta) - 1 < \hat{\mu}(\theta, Q) < F(\theta) \\ F(\theta) - 1 & \text{if } \hat{\mu}(\theta, Q) \leq F(\theta) - 1. \end{cases} \tag{27}$$

For simplicity, we define Θ_1,Θ_2, and Θ_3 as the subintervals of $\Theta := [\underline{\theta}, \overline{\theta}]$ where $\mu^*(\theta, Q)$ is equal to $F(\theta)$, $\hat{\mu}(\theta, Q)$ and $F(\theta) - 1$, respectively. For $\mu^*(\theta, Q)$ to satisfy the conditions (23) to (26), we need to check the condition $\mu_\theta^* \leq f(\theta)$. It is obviously satisfied on Θ_1 and Θ_3. So we only need to check $\mu_\theta^* = \hat{\mu}_\theta(\theta, Q) < f(\theta)$ for $\theta \in \Theta_2$. Notice that $\hat{\mu}_\theta(\theta, Q) = [f(\theta) + \theta f'(\theta)]\hat{\mu}(\theta, Q)/\theta f(\theta)$. If $f + \theta f' \geq 0$, then we have

$$\hat{\mu}_\theta(\theta, Q) < \frac{F(\theta)[f(\theta) + \theta f'(\theta)]}{\theta f(\theta)} \leq f(\theta),$$

where the last inequality follows directly from $d\left[\frac{F(\theta)}{\theta f(\theta)}\right]/d\theta \geq 0$ in condition (1). If $f + \theta f' < 0$, then we also have

$$\begin{aligned} \hat{\mu}_\theta(\theta, Q) &< \frac{[F(\theta) - 1][\theta f'(\theta) + f(\theta)]}{\theta f(\theta)} \\ &\leq \frac{\theta f^2(\theta) - [1 - F(\theta)] f(\theta)}{\theta f(\theta)} \\ &= f(\theta) - \frac{1 - F(\theta)}{\theta} \\ &\leq f(\theta). \end{aligned}$$

The second inequality follows from condition $d\left[\frac{1-F(\theta)}{f(\theta)}\right]/d\theta \leq 0$ in condition (1). Therefore, the optimal quantity is

$$\begin{aligned} q^*(\theta, Q) &= \hat{q}(\mu^*(\theta, Q), Q, \theta) \\ &= \begin{cases} q^+(\theta, Q) & \text{if } \frac{c_I - \Omega'(Q)}{c_E} \geq 1 + \frac{F(\theta)}{\theta f(\theta)} \\ q^0(\theta) & \text{if } 1 - \frac{1-F(\theta)}{\theta f(\theta)} < \frac{c_I - \Omega'(Q)}{c_E} < 1 + \frac{F(\theta)}{\theta f(\theta)} \\ q^-(\theta, Q) & \text{if } \frac{c_I - \Omega'(Q)}{c_E} \leq 1 - \frac{1-F(\theta)}{\theta f(\theta)} \end{cases}. \end{aligned} \tag{28}$$

To determine the optimal $q^*(\theta, Q)$, we need to distinguish the following cases depending on the values of Q and c_E (Fig. 5):

– *case 1* : $\frac{c_I - \Omega'(Q)}{c_E} < 1 - \frac{1}{\underline{\theta} f(\underline{\theta})}$. In this case, $\hat{\mu}(\theta, Q) < F(\theta) - 1$, and thus, $q^*(\theta, Q) = q^-(\theta, Q)$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$;

– *case 2*: $1-\frac{1}{\underline{\theta} f(\underline{\theta})} \leq \frac{c_I-\Omega'(Q)}{c_E} < 1$. In this case, $F(\theta)-1 \leq \hat{\mu}(\theta, Q) < 0, \forall \theta \in [\underline{\theta}, \theta^-(Q))$ and $\hat{\mu}(\theta, Q) < F(\theta)-1, \forall \theta \in (\theta^-(Q), \overline{\theta}]$. Therefore, we have

$$q^*(\theta, Q)=\begin{cases} q^0(\theta) & \text{if } \theta \in [\underline{\theta}, \theta^-(Q)) \\ q^-(\theta, Q) & \text{if } \theta \in [\theta^-(Q), \overline{\theta}] \end{cases};$$

– *case 3*: $\frac{c_I-\Omega'(Q)}{c_E}=1$. In this case, $\hat{\mu}(\theta, Q)=0$, and $q^*(\theta, Q)=q^f(\theta, Q)$, $\forall \theta \in [\underline{\theta}, \overline{\theta}]$;
– *case 4*: $1<\frac{c_I-\Omega'(Q)}{c_E}<1+\frac{1}{\overline{\theta} f(\overline{\theta})}$. In this case, $\hat{\mu}(\theta, Q) > F(\theta)$ for $\theta \in [\underline{\theta}, \theta^+(Q))$; $0<\hat{\mu}(\theta, Q) < F(\theta)$ for $\theta \in [\theta^+(Q), \overline{\theta}]$. Therefore, we have

$$q^*(\theta, Q)=\begin{cases} q^+(\theta, Q) & \text{if } \theta \in [\underline{\theta}, \theta^+(Q)) \\ q^0(\theta) & \text{if } \theta \in [\theta^+(Q), \overline{\theta}] \end{cases};$$

– *case 5*: $\frac{c_I-\Omega'(Q)}{c_E} \geq 1+\frac{1}{\overline{\theta} f(\overline{\theta})}$. In this case, $\hat{\mu}(\theta, Q) \geq F(\theta)$, and thus, $q^*(\theta, Q)=q^+(\theta, Q)$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$.

$\varphi^*(Q) \equiv \mathbb{E}_\theta[q^*(\theta, Q)]$ is therefore given by

$$\varphi^*(Q)=\begin{cases} \varphi^+(Q) & \text{if } \frac{c_I-\Omega'(Q)}{c_E} \geq 1+\frac{1}{\overline{\theta} f(\overline{\theta})} \\ \int_{\underline{\theta}}^{\theta^+(Q)} q^+(\theta, Q) f(\theta) d\theta+\int_{\theta^+(Q)}^{\overline{\theta}} q^0(\theta) f(\theta) d\theta & \text{if } 1<\frac{c_I-\Omega'(Q)}{c_E}<1+\frac{1}{\overline{\theta} f(\overline{\theta})} \\ \varphi^f(Q) & \text{if } \frac{c_I-\Omega'(Q)}{c_E}=1 \\ \int_{\underline{\theta}}^{\theta^-(Q)} q^0(\theta) f(\theta) d\theta+\int_{\theta^-(Q)}^{\overline{\theta}} q^-(\theta, Q) f(\theta) d\theta & \text{if } 1>\frac{c_I-\Omega'(Q)}{c_E} \geq 1-\frac{1}{\underline{\theta} f(\underline{\theta})} \\ \varphi^-(Q) & \text{if } \frac{c_I-\Omega'(Q)}{c_E}<1-\frac{1}{\underline{\theta} f(\underline{\theta})} \end{cases}. \tag{29}$$

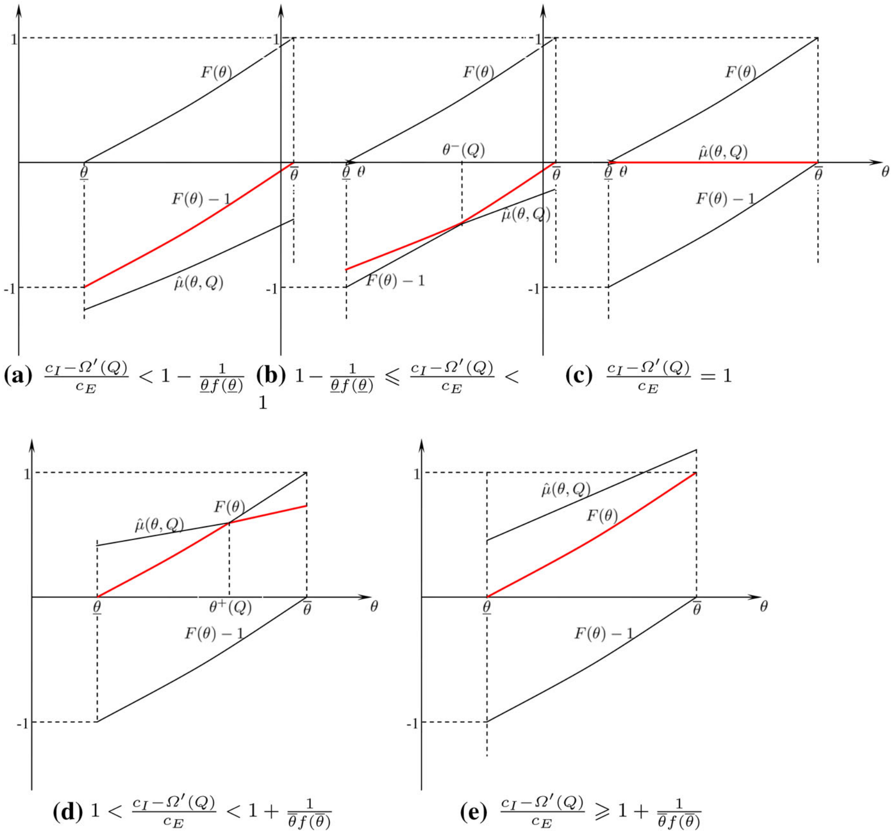
We now proceed to determine the fixed point Q^* of $\varphi^*(Q)$ and then characterize $q^*(\theta, Q^*)$.

Let Q_1, Q_0, Q_2 be points defined as follows:

$$\frac{c_I-\Omega'(Q_1)}{c_E}=1+\frac{1}{\overline{\theta} f(\overline{\theta})}, \frac{c_I-\Omega'(Q_0)}{c_E}=1, \frac{c_I-\Omega'(Q_2)}{c_E}=1-\frac{1}{\underline{\theta} f(\underline{\theta})}.$$

It is clear that $Q_1>Q_0>Q_2$ since $\Omega''(\cdot)<0$.

- If $c_E \in \left[c_H^*, c_I\right)$, then $[c_I-\Omega'(Q^-)]/c_E \leq 1-1/\underline{\theta} f(\underline{\theta})=[c_I-\Omega'(Q_2)]/c_E$, $Q^- \leq Q_2$, $(Q_2, \varphi^-(Q_2))$ is below the 45° line. The 45° line intersects $\varphi^*(Q)$ at point $(Q^-, \varphi^-(Q^-))$. Consequently, we have $Q^*=Q^- \leq Q_2$ (see Fig. 6a). From the above analysis of **Case** 1, it is clear that $q^*(\theta)=q^-(\theta, Q^-)=q^-(\theta)$ and $U^*(\theta)=\int_{\underline{\theta}}^{\theta}\left[v(q^-(\theta))-v(q^0(\theta))\right] d\theta, \forall \theta \in [\underline{\theta}, \overline{\theta}]$.
- If $c_E \in \left(c_M^*, c_H^*\right)$, we have $1=[c_I-\Omega'(Q_0)]/c_E>[c_I-\Omega'(Q^f)]/c_E$ and $1-1/[\underline{\theta} f(\underline{\theta})]=[c_I-\Omega'(Q_2)]/c_E<[c_I-\Omega'(Q^-)]/c_E$. It follows that $Q_0>Q^f, Q_2<Q^-$. Then the intersection of the 45° line and $\varphi^*(Q)$ lies between $(Q_0, \varphi^f(Q_0))$ and $(Q_2, \varphi^-(Q_2))$, and $Q^* \in (Q^-, Q^f) \subset (Q_2, Q_0)$. $Q_2<Q^*<Q_0$ implies $1=\left[c_I-\Omega'(Q_0)\right]/c_E>\left[c_I-\Omega'(Q^*)\right]/c_E>$



(a) $\frac{c_I-\Omega'(Q)}{c_E} < 1-\frac{1}{\underline{\theta} f(\underline{\theta})}$ **(b)** $1-\frac{1}{\underline{\theta} f(\underline{\theta})} \leqslant \frac{c_I-\Omega'(Q)}{c_E} < 1$ **(c)** $\frac{c_I-\Omega'(Q)}{c_E} = 1$

(d) $1 < \frac{c_I-\Omega'(Q)}{c_E} < 1+\frac{1}{\overline{\theta} f(\overline{\theta})}$ **(e)** $\frac{c_I-\Omega'(Q)}{c_E} \geqslant 1+\frac{1}{\overline{\theta} f(\overline{\theta})}$

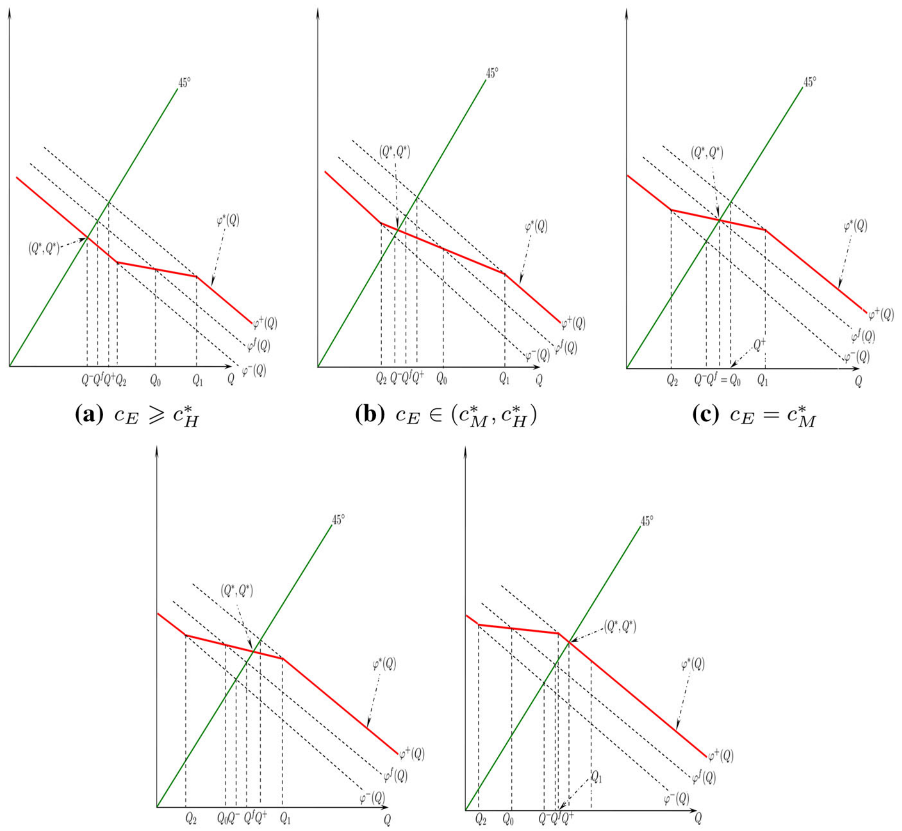
Fig. 5 The determination of $\mu^*(\theta, Q)$

$\left[c_I - \Omega'(Q_2)\right]/c_E = 1 - 1/\underline{\theta} f(\underline{\theta})$ (see Fig. 6b). It falls into **Case** 2. Therefore, we have

$$q^*(\theta) = \begin{cases} q^0(\theta) & \text{if } \ \theta \in [\underline{\theta}, \theta^-(Q^*)) \\ q^-(\theta, Q^*) & \text{if } \ \theta \in [\theta^-(Q^*), \overline{\theta}] \end{cases},$$

$$U^*(\theta) = \begin{cases} 0 & \text{if } \ \theta \in [\underline{\theta}, \theta^-(Q^*)) \\ \int_{\theta^-(Q^*)}^{\theta} \left[v(q^-(\theta, Q^*)) - v(q^0(\theta))\right] d\theta & \text{if } \ \theta \in [\theta^-(Q^*), \overline{\theta}] \end{cases}.$$

- If $c_E = c_M^*$, then the 45° line passes through the point $(Q_0, \varphi^f(Q_0))$ (see Fig. 6c). It is clear that $Q^* = Q_0 = Q^f$. As shown in **Case** 3 of the preceding discussions, $q^*(\theta) = q^f(\theta, Q^f) = q^f(\theta)$ and $U^*(\theta) \equiv 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$.
- If $c_E \in \left(c_L^*, c_M^*\right)$, we have $c_E = c_I - \Omega'(Q_0) < c_I - \Omega'(Q^f) \equiv c_M^*$ and $1 + 1/[\overline{\theta} f(\overline{\theta})] = [c_I - \Omega'(Q_1)]/c_E > [c_I - \Omega'(Q^+)]/c_E$. It follows that $Q_0 < Q^f$, $Q_1 > Q^+$. Then the 45° line intersects curve $\varphi^*(Q)$ at a point between $(Q_1, \varphi^+(Q_1))$ and $(Q_0, \varphi^f(Q_0))$. Therefore, we have $Q^* \in (Q^f, Q^+) \subset (Q_0, Q_1)$ (see Fig. 6d). Then, $1 \equiv \left[c_I - \Omega'(Q_0)\right]/c_E < \left[c_I - \Omega'(Q^*)\right]/c_E <$



(a) $c_E \geqslant c_H^*$ **(b)** $c_E \in (c_M^*, c_H^*)$ **(c)** $c_E = c_M^*$

(d) $c_E \in (c_L^*, c_M^*)$ **(e)** $c_E \leqslant c_L^*$

Fig. 6 The determination of Q^*

$\left[c_I - \Omega'(Q_1)\right]/c_E \equiv 1 + 1/\overline{\theta} f(\overline{\theta})$. Our discussion of **Case** 4 shows that

$$q^*(\theta) = \begin{cases} q^0(\theta) & \text{if } \theta \in [\theta^+(Q^*), \overline{\theta}] \\ q^+(\theta, Q^*) & \text{if } \theta \in [\underline{\theta}, \theta^+(Q^*)) \end{cases},$$

$$U^*(\theta) = \begin{cases} \int_{\theta^+(Q^*)}^{\theta} \left[v(q^+(\theta, Q^*)) - v(q^0(\theta))\right] d\theta & \text{if } \theta \in [\underline{\theta}, \theta^+(Q^*)) \\ 0 & \text{if } \theta \in [\theta^+(Q^*), \overline{\theta}] \end{cases}.$$

- If $c_E \in \left[0, c_L^*\right]$, then the 45° line intersects $\varphi^*(Q)$ at point $(Q^+, \varphi^+(Q^+))$ (see Fig. 6e). It follows from the preceding discussion of **Case** 5 that $q^*(\theta) = q^+(\theta)$, $U^*(\theta) = \int_{\theta}^{\overline{\theta}} \left[v(q^0(\theta)) - v(q^+(\theta))\right] d\theta, \forall \theta \in [\underline{\theta}, \overline{\theta}]$.

The optimal transfer $\tau^*(\theta) = \theta v(q^*(\theta)) + \Omega(Q^*) - U^0(\theta) - U^*(\theta)$ is then obtained accordingly. Given condition (1), it is easy to verify that $q^*(\theta)$ is nondecreasing in θ. Therefore, $\{q^*(\theta), \tau^*(\theta)\}$ is indeed the solution to the original problem $[\mathcal{P}_s]$.

C. Proof of Lemma 2

Applying an envelope theorem to IC_t (11) recursively, we get

$$
\begin{aligned}
EN_t: \quad & \frac{\partial \mathcal{U}_t(\theta_t|h^{t-1})}{\partial \theta_t} \\
&= v(q_t(\theta_t|h^{t-1})) - v(q^0(\theta_t)) + \beta \frac{\partial \mathcal{U}_{t+1}(\theta_t|h^{t-1}, \theta_t)}{\partial \theta_{t+1}} \\
&= v(q_t(\theta_t|h^{t-1})) - v(q^0(\theta_t)) + \beta[v(q_{t+1}(\theta_t|h^t)) - v(q^0(\theta_t))] \\
&\quad + \beta^2 \frac{\partial \mathcal{U}_{t+2}(\theta_t|h^{t+1})}{\partial \theta_{t+2}} \\
&= \cdots = \sum_{s=t}^{\infty} \beta^{s-t} \left[v(q_s(\theta_t|h^{s-1})) - v(q^0(\theta_t)) \right], h^{s-1} = \{h^{t-1}, \theta_t, \cdots, \theta_t\}.
\end{aligned}
$$

Differentiating

$$
\begin{aligned}
\widetilde{\mathcal{U}}_t(\theta_t, \hat{\theta}_t|h^{t-1}) := \theta_t v(q_t(\hat{\theta}_t|h^{t-1})) - \tau_t(\hat{\theta}_t|h^{t-1}) \\
+ \Omega(Q_t) - U^0(\theta_t) + \beta \mathcal{U}_{t+1}(\theta_t|h^{t-1}, \hat{\theta}_t)
\end{aligned}
$$

with respect to θ_t yields:

$$
\begin{aligned}
& \frac{\partial \widetilde{\mathcal{U}}_t(\theta_t, \hat{\theta}_t|h^{t-1})}{\partial \theta_t} \\
&= v(q_t(\hat{\theta}_t|h^{t-1})) - v(q^0(\theta_t)) + \beta \frac{\partial \mathcal{U}_{t+1}(\theta_t|h^{t-1}, \hat{\theta}_t)}{\partial \theta_{t+1}} \\
&= v(q_t(\hat{\theta}_t|h^{t-1})) - v(q^0(\theta_t)) \\
&\quad + \sum_{s=t+1}^{\infty} \beta^{s-t} \left[v(q_s(\theta_t|h^{t-1}, \hat{\theta}_t, \theta_t^{s-t-1})) - v(q^0(\theta_t)) \right].
\end{aligned}
$$

The marginal rent received by type θ_t for truthfully reporting her type rather than reporting type $\hat{\theta}_t$ after history h^{t-1} is

$$
\begin{aligned}
\Delta &:= \mathcal{U}_t(\theta_t|h^{t-1}) - \widetilde{\mathcal{U}}_t(\theta_t, \hat{\theta}_t|h^{t-1}) \\
&= \mathcal{U}_t(\theta_t|h^{t-1}) - \mathcal{U}_t(\hat{\theta}_t|h^{t-1}) + \widetilde{\mathcal{U}}_t(\hat{\theta}_t, \hat{\theta}_t|h^{t-1}) - \widetilde{\mathcal{U}}_t(\theta_t, \hat{\theta}_t|h^{t-1}) \\
&= \int_{\hat{\theta}_t}^{\theta_t} \left[\frac{\partial \mathcal{U}_t(x|h^{t-1})}{\partial \theta_t} - \frac{\partial \widetilde{\mathcal{U}}_t(x, \hat{\theta}_t|h^{t-1})}{\partial \theta_t} \right] dx \\
&= \int_{\hat{\theta}_t}^{\theta_t} \left\{ \begin{array}{l} \left[v(q_t(x|h^{t-1})) - v(q_t(\hat{\theta}_t|h^{t-1})) \right] \\ \quad + \sum_{s=t+1}^{\infty} \beta^{s-t} \left[v(q_s(x|h^{t-1}, x^{s-t})) - v(q_s(x|h^{t-1}, \hat{\theta}_t, x^{s-t-1})) \right] \end{array} \right\} dx.
\end{aligned}
$$

It is easy to see that $\Delta \geq 0$ under MON_t, therefore IC_t holds.

D. Proof of Proposition 3

– As usual in the mechanism design literature, we first consider a relaxed problem

$$[\mathcal{P}_\ell^r] : \max_{\{q_t(\theta),\tau_t(\theta)\}} \Pi_\ell, s.t. : EN_0, IR_0,$$

then verify that the solution obtained is indeed the solution to the original problem $[\mathcal{P}_\ell]$. $[\mathcal{P}_\ell^r]$ is an optimal control problem with a state variable $\mathcal{U}_0$ and control variables $\{q_t\}_{t\geq 0}$. The Hamiltonian function is

$$\begin{aligned}&\mathcal{H}(\mathcal{U}_0, \{q_t\}_{t=0}^{\infty}, \mu, \theta)\\ &= \left\{ \begin{array}{c} \left\{\sum_{t=0}^{\infty} \beta^t \left[\theta v(q_t(\theta)) - c_I q_t(\theta) - U^0(\theta)\right]\right\} f(\theta) + \sum_{t=0}^{\infty} \beta^t \Omega(Q_t) \\ -\mathcal{U}_0(\theta) f(\theta) + \mu(\theta) \sum_{t=0}^{\infty} \beta^t [v(q_t(\theta)) - v(q^0(\theta))] - C \end{array} \right\},\end{aligned}$$

where μ is the costate variable associated with EN_0. The first-order condition with respect to q_t is

$$\left[\theta + \frac{\mu(\theta)}{f(\theta)}\right] v'(q_t(\theta)) + \Omega'(Q_t) = c_I. \tag{30}$$

It follows from (30) and $Q_t = \int_{\underline{\theta}}^{\overline{\theta}} q_t(\theta) f(\theta) d\theta$ that the optimal quantities q_t are identical across t, which allows us to drop the subscript t and write the individual and aggregate consumptions as, respectively, q and Q. Substituting $q_t(\theta) \equiv q(\theta), \forall t \geq 0$ and $U(\theta) \equiv (1-\beta)\mathcal{U}_0(\theta)$ into the objective function Π_ℓ, we can rewrite $[\mathcal{P}_\ell^r]$ as its static counterpart $[\mathcal{P}_s^r]$ except a multiplication of the first term by $\frac{1}{1-\beta}$:

$$\begin{aligned}&\max_{\{q(\theta),U(\theta)\}} \frac{1}{1-\beta} \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta v(q(\theta)) - c_I q(\theta) + \Omega(Q) - U^0(\theta) - U(\theta)\right] f(\theta) d\theta - C\\ &s.t. : U'(\theta) = v(q(\theta)) - v(q^0(\theta)), U(\theta) \geq 0.\end{aligned}$$

Analogous to the proof of Proposition 1, we get $\{q^*(\theta), U^*(\theta)\}$ from this optimization problem. The optimal LTC is therefore $q_t^*(\theta) \equiv q^*(\theta)$ and $\tau_t^*(\theta) \equiv \tau^*(\theta), \forall t \geq 0$. The corresponding continuation payoff is $\mathcal{U}_t^*(\theta) \equiv \mathcal{U}^*(\theta) = \frac{U^*(\theta)}{1-\beta}, \forall t \geq 0$.

The only work left is to check the monotonic implementability condition MON_t (13). Obviously, along the truthful path, it holds since $q_t^*(\theta|\theta^t) = q^*(\theta) \geq q^*(\theta') = q_t^*(\theta'|(\theta')^t)$ whenever $\theta \geq \theta'$. For histories in which types are not constant, quantities can be chosen arbitrarily as long as the constraints IR_t and IC_t are satisfied, since these quantities do not affect the principal's objective Π_ℓ.

– Under full rationality (f) and constant types (c), the incumbent's profits under an LTC (ℓ) and a sequence of STCs (s) are, respectively,

$$\Pi_\ell^{fc} = \frac{1}{1-\beta} \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta v(q^*(\theta)) - c_I q^*(\theta) - U^*(\theta) - U^0(\theta) + \Omega(Q^*)\right] f(\theta) d\theta - C$$

and

$$\Pi_s^{fc} = \frac{1}{1-\beta}\int_{\underline{\theta}}^{\overline{\theta}} \left[\theta v(q^*(\theta)) - c_I q^*(\theta) - U^*(\theta) - U^0(\theta) + \Omega(Q^*) - C\right] f(\theta) d\theta.$$

Therefore, $\Delta\Pi^{fc} \equiv \Pi_\ell^{fc} - \Pi_s^{fc} = \frac{\beta C}{1-\beta} > 0$.

– Similarly, the fully-rational social surpluses under two contracting ways are, respectively

$$\mathcal{W}_\ell^{fc} = \frac{1}{1-\beta}\int_{\underline{\theta}}^{\overline{\theta}} \left[\theta v(q^*(\theta)) - c_I q^*(\theta) + \Omega(Q^*)\right] f(\theta) d\theta - C$$

and

$$\mathcal{W}_s^{fc} = \frac{1}{1-\beta}\int_{\underline{\theta}}^{\overline{\theta}} \left[\theta v(q^*(\theta)) - c_I q^*(\theta) + \Omega(Q^*) - C\right] f(\theta) d\theta$$

Therefore, $\Delta\mathcal{W}^{fc} \equiv \mathcal{W}_\ell^{fc} - \mathcal{W}_s^{fc} = \frac{\beta C}{1-\beta} > 0$.

E. Proof of Proposition 4

Under a sequence of STCs, taking the consumers to be fully rational initially, the incumbent will offer a contract $\{q^*(\theta), \tau^*(\theta)\}$ repeatedly in every period. If the realized consumption is smaller than the fully rational value at some point t, i.e., $Q_t < Q^* = \mathbb{E}[q^*(\theta)]$, the incumbent firm will be conscious of the fact that pessimism prevails among consumers. Also, the incumbent is able to infer that the pessimism carries over to the next period (i.e., $Q_{t+1}^e = Q_t < Q^*$) because the consumers use the most recent realization as their present forecast. He will offer a new short-term contract $\{q^*(\theta), \tau(\theta)\}$ with a lower price $\tau(\theta) = \tau^*(\theta) - [\Omega(Q^*) - \Omega(Q_{t+1}^e)]$ in period $t+1$. Then all consumers will accept the contract since $U(\theta) = \theta v(q^*(\theta)) - \tau(\theta) + \Omega(Q_{t+1}^e) - U^0(\theta) = \theta v(q^*(\theta)) - \tau^*(\theta) + \Omega(Q^*) - U^0(\theta) = U^*(\theta) \geq 0, \forall\theta \in [\underline{\theta}, \overline{\theta}]$. By such a one-shot price reduction, the incumbent could restore and stabilize the market expectation at Q^* from periods $t+1$ onwards. The rivals who have acquired a market share at period t will lose it again.

F. Proof of Proposition 5

When $c_E < c_M^*$, the acceptance set $\Theta(x) := \left\{\theta \in [\underline{\theta}, \overline{\theta}] : \Omega(Q^*) - \Omega(x) \leq \mathcal{U}^*(\theta)\right\}$ and the feedback function $\rho(x) := \int_{\Theta(x)} q^*(\theta) f(\theta) d\theta$ are given as follows:

– if $\Omega(Q^*) \geq \mathcal{U}^*(\underline{\theta})$, then Fig. 7a shows that

$$\Theta(x) = \begin{cases} [\underline{\theta}, \overline{\theta}] & \text{if } \ \Omega(x) \geq \Omega(Q^*) \\ \left[\underline{\theta}, \theta^*(x)\right] & \text{if } \ \Omega(Q^*) - \mathcal{U}^*(\underline{\theta}) \leq \Omega(x) < \Omega(Q^*) \\ \emptyset & \text{if } \ 0 \leq \Omega(x) < \Omega(Q^*) - \mathcal{U}^*(\underline{\theta}) \end{cases}$$

$$\rho(x)=\begin{cases} Q^* & \text{if } \ \Omega(x)\ge\Omega(Q^*) \\ \int_{\underline{\theta}}^{\theta^*(x)} q^*(\theta)f(\theta)d\theta & \text{if } \ \Omega(Q^*)-\mathcal{U}^*(\underline{\theta})\le\Omega(x)<\Omega(Q^*) \\ 0 & \text{if } \ 0\le\Omega(x)<\Omega(Q^*)-\mathcal{U}^*(\underline{\theta}) \end{cases};$$

– if $\Omega(Q^*)<\mathcal{U}^*(\underline{\theta})$, then Fig. 7b shows that

$$\rho(x)=\begin{cases} Q^* & \text{if } \ \Omega(x)\ge\Omega(Q^*) \\ \int_{\underline{\theta}}^{\theta^*(x)} q^*(\theta)f(\theta)d\theta & \text{if } \ 0\le\Omega(x)<\Omega(Q^*) \end{cases}.$$

In summary, we have

$$\rho(x)=\begin{cases} Q^* & \text{if } \ \Omega(x)\in\big[\Omega(Q^*),+\infty\big) \\ \int_{\underline{\theta}}^{\theta^*(x)} q^*(\theta)f(\theta)d\theta & \text{if } \ \Omega(x)\in\big[\max\{0,\Omega(Q^*)-\mathcal{U}^*(\underline{\theta})\},\Omega(Q^*)\big) \\ 0 & \text{if } \ \Omega(x)\in\big[0,\max\{0,\Omega(Q^*)-\mathcal{U}^*(\underline{\theta})\}\big) \end{cases}, \tag{31}$$

where $\theta^*(x)$ is given implicitly by $\Omega(Q^*)-\mathcal{U}^*(\theta^*)=\Omega(x)$, and $\mathcal{U}^*(\theta^*)=\frac{U^*(\theta^*)}{1-\beta}=\frac{\int_{\theta^*}^{\overline{\theta}}[v(q^0(\theta))-v(q^*(\theta))]d\theta}{1-\beta}$.

Analogously, Fig. 8 shows that when $c_E>c_M^*$

$$\rho(x)=\begin{cases} Q^* & \text{if } \ \Omega(x)\in\big[\Omega(Q^*),+\infty\big) \\ \int_{\theta^*(x)}^{\overline{\theta}} q^*(\theta)f(\theta)d\theta & \text{if } \ \Omega(x)\in\big[\max\{0,\Omega(Q^*)-\mathcal{U}^*(\overline{\theta})\},\Omega(Q^*)\big) \\ 0 & \text{if } \ \Omega(x)\in\big[0,\max\{0,\Omega(Q^*)-\mathcal{U}^*(\overline{\theta})\}\big) \end{cases}, \tag{32}$$

where $\theta^*(x)$ is given implicitly by

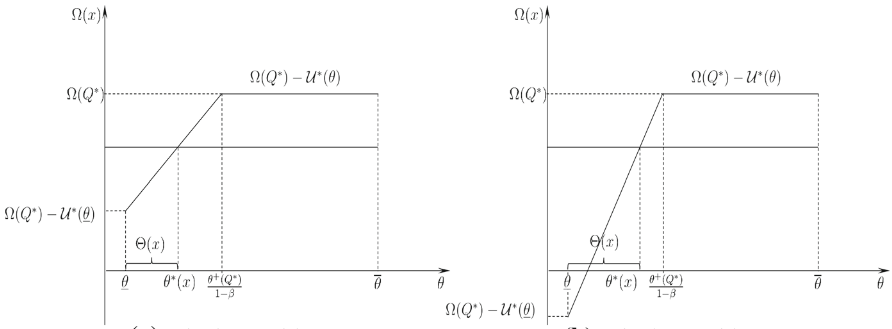
$$\frac{\int_{\underline{\theta}}^{\theta^*}[v(q^*(\theta))-v(q^0(\theta))]d\theta}{1-\beta}+\Omega(x)=\Omega(Q^*).$$

In the borderline case of $c_E=c_M^*$, we have

$$\rho(x)=\begin{cases} Q^* & \text{if } \ \Omega(x)\in\big[\Omega(Q^*),+\infty\big) \\ 0 & \text{if } \ \Omega(x)\in\big[0,\Omega(Q^*)\big) \end{cases}. \tag{33}$$

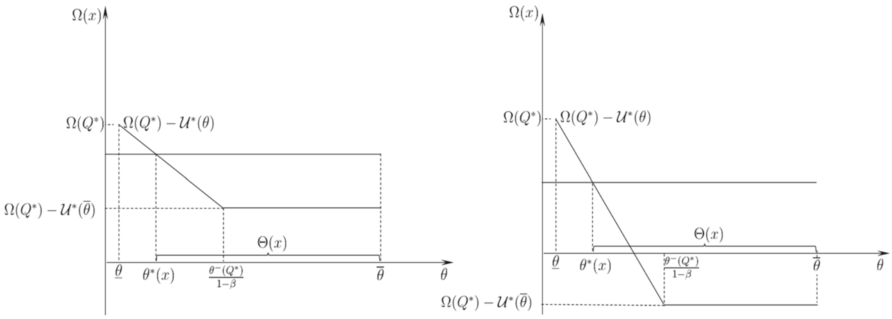
It follows from (31) to (33) that $\rho'_+(Q^*)=0$, and

$$\rho'_-(Q^*)=\begin{cases} \frac{(1-\beta)q^*(\theta^*(Q^*))f(\theta^*(Q^*))\Omega'(Q^*)}{v(q^*(\theta^*(Q^*)))-v(q^0(\theta^*(Q^*)))} & \text{if } \ c_E>c_M^* \\ +\infty & \text{if } \ c_E=c_M^* \\ \frac{(1-\beta)q^*(\theta^*(Q^*))f(\theta^*(Q^*))\Omega'(Q^*)}{v(q^0(\theta^*(Q^*)))-v(q^*(\theta^*(Q^*)))} & \text{if } \ c_E<c_M^* \end{cases}.$$



(a) $\Omega(Q^*) > \mathcal{U}^*(\underline{\theta})$ **(b)** $\Omega(Q^*) \leqslant \mathcal{U}^*(\underline{\theta})$

Fig. 7 $c_E < c_M^*$



(a) $\Omega(Q^*) > \mathcal{U}^*(\overline{\theta})$ **(b)** $\Omega(Q^*) \leqslant \mathcal{U}^*(\overline{\theta})$

Fig. 8 $c_E > c_M^*$

The stability of Q^* for the dynamical system $Q^e_{t+1} = \rho(Q^e_t)$ depends on $\rho'(Q^*)$. $\rho'_+(Q^*) = 1$,

$$\rho'_-(Q^*) = \frac{(1-\beta)\Omega'(Q^*)q^*(\theta^*(Q^*))f(\theta^*(Q^*))}{|v(q^*(\theta^*(Q^*))) - v(q^0(\theta^*(Q^*)))|} = \frac{\Omega'(Q^*)}{\Delta(Q^*)}.$$

Next, we only prove the case from below, i.e., $Q^e_0 < Q^*$; the case from above is analogous. Linearizing $\rho(\cdot)$ at Q, which is around and smaller than Q^*, with the Taylor expansion:

$$\rho(Q) = \rho(Q^*) + \rho'_-(Q^*)(Q - Q^*) + o(Q - Q^*).$$

Then, we obtain the following cases:

- if $0 \le \Omega'(Q^*) < \Delta(Q^*)$, then $0 \le \rho'_-(Q^*) < 1$. Hence, $\exists b \in (\rho'_-(Q^*), 1)$ and $\epsilon > 0$, such that for $Q \in (Q^* - \epsilon, Q^*)$, we have $\rho(Q) - \rho(Q^*) > b(Q - Q^*)$. So starting with $Q \in (Q^* - \epsilon, Q^*)$ and iterating $Q_{t+1} = \rho(Q_t)$ gives a sequence of points $\rho^{(t)}(Q)$ with $0 > \rho^{(t)}(Q) - Q^* > b^t(Q - Q^*)$. Therefore, $\lim_{t\to\infty} \rho^{(t)}(Q) = Q^*$ whenever the starting point is within $(Q^* - \epsilon, Q^*)$. Since

Q^* is the unique fixed point of $\rho(\cdot)$, we have $\rho(Q) > Q, \forall Q \in (0, Q^*)$. It is easy to check that $Q_t < \rho(Q_t) = Q_{t+1}, \forall t \geq 0, \forall Q_0^e < Q^*$. Therefore, for an arbitrary starting point $Q_0^e < Q^*$, Q_t may eventually fall into $(Q^* - \epsilon, Q^*)$ and thus converge to Q^* upon iteration.

– if $\Omega'(Q^*) > \Delta(Q^*)$, then $\rho'_-(Q^*) > 1$. We must have $\rho(Q) < Q, \forall Q \in (0, Q^*)$, since Q^* is the unique fixed point. Therefore, $Q_{t+1} = \rho(Q_t) < Q_t, \forall t \geq 0$. It follows that for an arbitrary $Q_0^e < Q^*$, Q_t diverges from Q^* upon iteration, and the iteration will not stop until $Q_t = 0$. We thus have $\lim_{t\to\infty} Q_t = 0$.

G. Proof of Proposition 6

- *We first prove that* $\Delta W'(c_E) > 0, \forall c_E \leq c_M^*$. If $c_E \in (0, c_L^*]$, then it is obvious that $\Delta W'(c_E) = \int_{\underline{\theta}}^{\overline{\theta}} q^0(\theta, c_E) f(\theta) d\theta > 0$. If $c_E \in (c_L^*, c_M^*)$,

$$\begin{aligned}
&\Delta W'(c_E) \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta v'(q^*(\theta, c_E)) - c_I\right] \frac{\partial q^*(\theta, c_E)}{\partial c_E} f(\theta) d\theta \\
&\quad + \Omega'(Q^*) \frac{dQ^*}{dc_E} + \int_{\underline{\theta}}^{\overline{\theta}} q^0(\theta, c_E) f(\theta) d\theta \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta v'(q^*(\theta, c_E)) - c_I + \Omega'(Q^*)\right] \frac{\partial q^*(\theta, c_E)}{\partial c_E} f(\theta) d\theta \\
&\quad + \int_{\underline{\theta}}^{\overline{\theta}} q^0(\theta, c_E) f(\theta) d\theta \\
&= \begin{bmatrix} \int_{\underline{\theta}}^{\theta^+(Q^*)} \left[\theta v'(q^+(\theta, Q^*)) - c_I + \Omega'(Q^*)\right] \frac{\partial q^+(\theta, Q^*)}{\partial Q^*} \frac{dQ^*}{dc_E} f(\theta) d\theta \\ + \int_{\theta^+(Q^*)}^{\overline{\theta}} \left[\theta v'(q^0(\theta, c_E)) - c_I + \Omega'(Q^*)\right] \frac{\partial q^0(\theta, c_E)}{\partial c_E} f(\theta) d\theta \end{bmatrix} + \int_{\underline{\theta}}^{\overline{\theta}} q^0(\theta, c_E) f(\theta) d\theta \\
&\geq \begin{bmatrix} \int_{\underline{\theta}}^{\theta^+(Q^*)} \left[\theta v'(q^0(\theta, c_E)) - c_I + \Omega'(Q^*)\right] \frac{\partial q^+(\theta, Q^*)}{\partial Q^*} \frac{dQ^*}{dc_E} f(\theta) d\theta \\ + \int_{\theta^+(Q^*)}^{\overline{\theta}} \left[\theta v'(q^0(\theta, c_E)) - c_I + \Omega'(Q^*)\right] \frac{\partial q^0(\theta, c_E)}{\partial c_E} f(\theta) d\theta \end{bmatrix} + \int_{\underline{\theta}}^{\overline{\theta}} q^0(\theta, c_E) f(\theta) d\theta \\
&= \begin{Bmatrix} [c_E - c_I + \Omega'(Q^*)] \frac{d}{dc_E} \left[\int_{\underline{\theta}}^{\theta^+(Q^*)} q^+(\theta, Q^*) f(\theta) d\theta + \int_{\theta^+(Q^*)}^{\overline{\theta}} q^0(\theta, c_E) f(\theta) d\theta\right] \\ + \int_{\underline{\theta}}^{\overline{\theta}} q^0(\theta, c_E) f(\theta) d\theta \end{Bmatrix} \\
&= [c_E - c_I + \Omega'(Q^*)] \frac{dQ^*}{dc_E} + \int_{\underline{\theta}}^{\overline{\theta}} q^0(\theta, c_E) f(\theta) d\theta > 0.
\end{aligned}$$

The first inequality follows from $\theta v'(q^+(\theta, Q^*)) > \theta v'(q^0(\theta, c_E)) = c_E, \forall \theta \in [\underline{\theta}, \theta^+(Q^*)]$ and

$$\frac{\partial q^+}{\partial Q^*} = \frac{-\Omega''(Q^*)}{\left[\theta + \frac{F(\theta)}{f(\theta)}\right] v''(q^+)} < 0, \quad \frac{dQ^*}{dc_E} = \frac{\int_{\theta^+(Q^*)}^{\overline{\theta}} \frac{\partial q^0}{\partial c_E}(\theta, c_E) f(\theta) d\theta}{1 - \int_{\underline{\theta}}^{\theta^+(Q^*)} \frac{\partial q^+}{\partial Q^*}(\theta, Q^*) f(\theta) d\theta} < 0.$$

The last inequality follows from $c_E - c_I + \Omega'(Q^*) < c_E - [c_I - \Omega'(Q^f)] = c_E - c_M^* < 0$ and $dQ^*/dc_E < 0$ for $c_E \in [c_L^*, c_M^*]$.[19]

- $\exists c_E^* \in (c_E^d, c_M^*]$ *such that* $\Delta W(c_E) \leq 0, \forall c_E \in [c_E^d, c_E^*]$. If $\Delta W(c_M^*) = \Omega(Q^f) - Q^f\Omega'(Q^f) - \beta C \leq 0$, then using the monotonicity of $\Delta W(c_E)$, we have $\Delta W(c_E) \leq 0, \forall c_E \in [c_E^d, c_M^*]$. If $\Delta W(c_M^*) = \Omega(Q^f) - Q^f\Omega'(Q^f) - \beta C > 0$, then $\Delta W(c_E^d) < 0$ and monotonicity of $\Delta W(c_E)$ imply: $\exists c_E^* \in (c_E^d, c_M^*)$ such that $\Delta W(c_E^*) = 0$, and $\Delta W(c_E) \leq 0, \forall c_E \in [c_E^d, c_E^*]$.
- *An LTC is procompetitive if* $Q_0^e < Q^*$ *and* $c_E \in [c_E^d, c_E^*]$. We have

$$\Omega'(Q^*) = \Omega'(Q^+) \geq \Delta(Q^*) = \Delta(Q^+) = \frac{v(q^0(\overline{\theta}, c_E)) - v(q^+(\overline{\theta}))}{(1-\beta)f(\overline{\theta})q^+(\overline{\theta})}$$

if $c_E \in [c_E^d, c_L^*)$; and $\Omega'(Q^*) > \Delta(Q^*) = 0$ if $c_E \in [c_L^*, c_E^*]$. It follows from Proposition 5 that $\lim_{t\to\infty} Q_t = 0$ whenever $Q_0^e < Q^*$ and $c_E \in [c_E^d, c_E^*]$.

H. Proof of Proposition 9

The optimal long-term contract confers an information rent $\mathcal{U}_0(\theta) = U^*(\theta)$ in the first period. Since $\{\mathcal{U}_t(\theta_t)\}_{t=1}^{\infty}$ do not enter the seller's objective Π_ℓ, we assume, without loss of generality, a history-independent rent $\mathcal{U}_t(\theta_t) = \underline{\mathcal{U}}(\theta_t) + P, \forall t \geq 1$, where

$$\underline{\mathcal{U}}(\theta) := \begin{cases} \int_{\underline{\theta}}^{\theta} \left[v(q^f(\theta)) - v(q^0(\theta))\right] d\theta & \text{if } \; c_E > c_I - \Omega'(Q_t^f) \\ 0 & \text{if } \; c_E = c_I - \Omega'(Q_t^f) \\ \int_{\theta}^{\overline{\theta}} \left[v(q^0(\theta)) - v(q^f(\theta))\right] d\theta & \text{if } \; c_E < c_I - \Omega'(Q_t^f) \end{cases}, \tag{34}$$

[19] Notice that, when $c_E \in [c_M^*, c_H^*]$, the sign of

$$\Delta W'(c_E) = \begin{bmatrix} \int_{\theta^-(Q^*)}^{\overline{\theta}} \left[\theta v'(q^-(\theta, Q^*)) - c_I + \Omega'(Q^*)\right] \frac{\partial q^-(\theta, Q^*)}{\partial Q^*} \frac{dQ^*}{dc_E} f(\theta) d\theta \\ + \int_{\underline{\theta}}^{\theta^-(Q^*)} \left[\theta v'(q^0(\theta, c_E)) - c_I + \Omega'(Q^*)\right] \frac{\partial q^0(\theta, c_E)}{\partial c_E} f(\theta) d\theta \end{bmatrix}$$
$$+ \int_{\underline{\theta}}^{\overline{\theta}} q^0(\theta, c_E) f(\theta) d\theta$$

is ambiguous, because the two terms have different signs:

$$\begin{bmatrix} \int_{\theta^-(Q^*)}^{\overline{\theta}} \left[\theta v'(q^-(\theta, Q^*)) - c_I + \Omega'(Q^*)\right] \frac{\partial q^-}{\partial Q^*} \frac{dQ^*}{dc_E} f(\theta) d\theta \\ + \int_{\underline{\theta}}^{\theta^-(Q^*)} \left[\theta v'(q^0(\theta, c_E)) - c_I + \Omega'(Q^*)\right] \frac{\partial q^0}{\partial c_E} f(\theta) d\theta \end{bmatrix} \leq [c_E - c_I + \Omega'(Q^*)] \frac{dQ^*}{dc_E} < 0,$$

$$\int_{\underline{\theta}}^{\overline{\theta}} q^0(\theta, c_E) f(\theta) d\theta > 0.$$

$Q_t^f := \int_{\Theta_t} q^f(\theta) f_t(\theta) d\theta$, $P \geq \max_{t \geq 1} \Omega(Q_t^f)$. These rents can be guaranteed by a sequence of history-independent monetary payments:

$$\tau_0(\theta) = \theta v(q^*(\theta)) + \Omega(Q_0^*) + \beta \mathbb{E}\left[\underline{\mathcal{U}}(\theta_1) + P\right] - U^*(\theta) - U^0(\theta)$$
$$\tau_t(\theta) = \theta v(q^f(\theta)) + \Omega(Q_t^f) + \beta \mathbb{E}\left[\underline{\mathcal{U}}(\theta_{t+1})\right] - \underline{\mathcal{U}}(\theta) - (1-\beta)P - U^0(\theta), \forall t \geq 1,$$

where $Q_0^* := \int_{\Theta_0} q^*(\theta) f_0(\theta) d\theta$. Obviously, IR_t, EN_t, and MON_t hold for $\forall t \geq 0$. If pessimism happens at the outset, i.e., $\Omega_0^e < Q_0^*$, some consumers will exit. Then an actual network size of

$$Q_0 = \rho_0(Q_0^e) := \int_{\{\theta \in \Theta_0 | \mathcal{U}_0(\theta) \geq \Omega(Q_0^*) - \Omega(Q_0^e)\}} q^*(\theta) f_0(\theta) d\theta \tag{35}$$

is realized, and a pessimistic expectation $Q_1^e = Q_0 < Q_1^f$ is formed. However, all consumers will accept the contract from $t \geq 1$ onwards even under this irrational expectation, because $\mathcal{U}_t(\theta_t) \geq P > \Omega(Q_t^f) - \Omega(Q_t^e), \forall Q_t^e, \forall t \geq 1, \forall \theta_t \in \Theta_t$. The entrants will lose their market share gained in the first period. If pessimism happens at $t \geq 1$, then even the temporary entry will not happen, because the continuation rents $\{\mathcal{U}_t(\theta_t)\}_{t=1}^{\infty}$ are all high enough to be immune to any pessimistic expectation.

I. Proof of Proposition 10

Let

$$\begin{aligned}
\pi_t^* \equiv & \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\theta v(q^*(\theta)) - c_I q^*(\theta) - U^0(\theta) - U^*(\theta)\right] f_t(\theta) d\theta \\
& + \Omega\left(\int_{\underline{\theta}_t}^{\overline{\theta}_t} q^*(\theta) f_t(\theta) d\theta\right), \\
\pi_t^f \equiv & \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\theta v(q^f(\theta)) - c_I q^f(\theta) - U^0(\theta)\right] f_t(\theta) d\theta + \Omega\left(\int_{\underline{\theta}_t}^{\overline{\theta}_t} q^f(\theta) f_t(\theta) d\theta\right).
\end{aligned}$$

It follows from

$$\begin{aligned}
& \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\theta v(q^f(\theta)) - c_I q^f(\theta)\right] f_t(\theta) d\theta + \Omega\left(\int_{\underline{\theta}_t}^{\overline{\theta}_t} q^f(\theta) f_t(\theta)\right) \\
& = \max_{q(\theta)} \left\{\int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\theta v(q(\theta)) - c_I q(\theta)\right] f_t(\theta) d\theta + \Omega\left(\int_{\underline{\theta}_t}^{\overline{\theta}_t} q(\theta) f_t(\theta)\right)\right\} \\
& \geq \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\theta v(q^*(\theta)) - c_I q^*(\theta)\right] f_t(\theta) d\theta + \Omega\left(\int_{\underline{\theta}_t}^{\overline{\theta}_t} q^*(\theta) f_t(\theta)\right) \\
& > \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\theta v(q^*(\theta)) - c_I q^*(\theta) - U^*(\theta)\right] f_t(\theta) d\theta + \Omega\left(\int_{\underline{\theta}_t}^{\overline{\theta}_t} q^*(\theta) f_t(\theta)\right), \forall t \geq 1
\end{aligned} \tag{36}$$

that $\pi_t^f > \pi_t^*, \forall t \geq 1$. Therefore,

$$\begin{aligned}\Pi_\ell^i - \Pi_s^i &= \left[\pi_0^* + \sum_{t=1}^{\infty} \beta^t \pi_t^f - C\right] - \left[\sum_{t=0}^{\infty} \beta^t (\pi_t^* - C)\right] \\ &= \sum_{t=1}^{\infty} \beta^t (\pi_t^f - \pi_t^*) + \frac{\beta C}{1-\beta} > 0.\end{aligned}$$

The aggregate social surpluses under a sequence of STCs and LTC are, respectively,

$$\mathcal{W}_s^i = \sum_{t=0}^{\infty} \beta^t \left\{ \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\theta v(q^*(\theta)) - c_I q^*(\theta)\right] f_t(\theta) d\theta + \Omega\left(\int_{\underline{\theta}_t}^{\overline{\theta}_t} q^*(\theta) f_t(\theta) d\theta\right) - C \right\}$$

and

$$\mathcal{W}_\ell^i = \left\{ \begin{array}{l} \int_{\underline{\theta}_0}^{\overline{\theta}_0} \left[\theta v(q^*(\theta)) - c_I q^*(\theta)\right] f_0(\theta) d\theta + \Omega\left(\int_{\underline{\theta}_0}^{\overline{\theta}_0} q^*(\theta) f_0(\theta) d\theta\right) - C \\ + \sum_{t=1}^{\infty} \beta^t \left\{ \int_{\underline{\theta}_t}^{\overline{\theta}_t} \left[\theta v(q^f(\theta)) - c_I q^f(\theta)\right] f_t(\theta) d\theta + \Omega\left(\int_{\underline{\theta}_t}^{\overline{\theta}_t} q^f(\theta) f_t(\theta) d\theta\right) \right\} \end{array} \right\}.$$

It follows from the first inequality of (36) that

$$\begin{aligned}&\mathcal{W}_\ell^i - \mathcal{W}_s^i \\ &= \sum_{t=1}^{\infty} \beta^t \left[\begin{array}{l} \int_{\underline{\theta}_t}^{\overline{\theta}_t} [\theta v(q^f(\theta)) - c_I q^f(\theta)] f_t(\theta) d\theta + \Omega\left(\int_{\underline{\theta}_t}^{\overline{\theta}_t} q^f(\theta) f_t(\theta) d\theta\right) \\ - \int_{\underline{\theta}_t}^{\overline{\theta}_t} [\theta v(q^*(\theta)) - c_I q^*(\theta)] f_t(\theta) d\theta + \Omega\left(\int_{\underline{\theta}_t}^{\overline{\theta}_t} q^*(\theta) f_t(\theta) d\theta\right) \end{array} \right] \\ &\quad + \frac{\beta C}{1-\beta} > 0.\end{aligned}$$

References

Abito, J., Wright, J.: Exclusive dealing with imperfect downstream competition. Int. J. Ind. Organ. **26**, 227–246 (2008)

Aghion, P., Bolton, P.: Contracts as a barrier to entry. Am. Econ. Rev. **77**, 388–401 (1987)

Anderson, S., Engers, M.: Preemptive entry in differentiated product markets. Econ. Theory **17**, 419–445 (2001)

Baron, D., Besanko, D.: Regulation and information in a continuing relationship. Inf. Econ. Policy **1**, 267–302 (1984)

Battaglini, M., Lamba, R.: Optimal dynamic contracting: the first-order approach and beyond. Theor. Econ. forthcoming (2019)

Battaglini, M.: Long-term contracting with Markovian consumers. Am. Econ. Rev. **95**, 637–658 (2005)

Bedre-Defolie, O., Biglaiser, G.: Contracts as a barrier to entry in markets with nonpivotal buyers. Am. Econ. Rev. **107**, 1–32 (2017)

Besanko, D.: Multi-period contracts between principal and agent with adverse selection. Econ. Lett. **17**, 33–37 (1985)

Besanko, D., Perry, M.: Equilibrium incentives for exclusive dealing in a differentiated products oligopoly. Rand. J. Econ. **24**, 646–667 (1993)

Bisin, A., Rampini, A.: Exclusive contracts and the institution of bankruptcy. Econ. Theor. **27**, 277–304 (2006)
Bork, R.: The Antitrust Paradox: A Policy at War with Itself. Basic Books, New York (1978)
Bose, S., Deltas, G.: Exclusive versus non-exclusive dealing in auctions with resale. Econ. Theory **31**, 1–17 (2007)
Caillaud, B., Jullien, B.: Competing cybermediaries. Eur. Econ. Rev. **45**, 797–808 (2001)
Caillaud, B., Jullien, B.: Chicken and egg: competition among intermediation service providers. Rand. J. Econ. **34**, 309–328 (2003)
Calzolari, G., Denicolò, V.: Competition with exclusive contracts and market-share discounts. Am. Econ. Rev. **103**, 2384–2411 (2013)
Chen, Z., Shaffer, G.: Naked exclusion with minimum share requirements. Rand. J. Econ. **45**, 64–91 (2014)
Choi, J., Stefanadis, C.: Sequential innovation, naked exclusion, and upfront lump-sum payments. Econ. Theor. **65**, 891–915 (2018)
De Meza, D., Selvaggi, M.: Exclusive contracts foster relationship-specific investment. Rand. J. Econ. **38**, 85–97 (2007)
Director, A., Levi, E.: Law and the future: trade regulation. Northwest Univ. Law Rev. **51**, 281–296 (1956)
Doganoglu, T., Wright, J.: Exclusive dealing with network effects. Int. J. Ind. Organ. **28**, 145–154 (2010)
Fumagalli, C., Motta, M.: Exclusive dealing and entry, when buyers compete. Am. Econ. Rev. **96**, 785–795 (2006)
Gavin, S., Ross, T.: Long-term contracts as barriers to entry with differentiated products. Int. J. Ind. Organ. **59**, 514–537 (2018)
Gratz, L., Reisinger, M.: On the competition enhancing effects of exclusive dealing contracts. Int. J. Ind. Organ. **31**, 429–437 (2013)
Halaburda, H., Yaron, Y.: Platform competition under asymmetric information. Am. Econ. J.: Microecon. **5**, 22–68 (2013)
Halaburda, H., Jullien, B., Yehezkel, Y.: Dynamic competition with network externalities: why history matters. Rand. J. Econ. **51**, 3–31 (2020)
Innes, R., Sexton, R.: Strategic buyers and exclusionary contracts. Am. Econ. Rev. **84**, 566–584 (1994)
Jullien, B.: Competition in multi-sided markets: divide-and-conquer. Am. Econ. J.: Microecon. **3**, 1–35 (2011)
Kitamura, H.: Exclusionary vertical contracts with multiple entrants. Int. J. Ind. Organ. **28**, 213–219 (2010)
Laffont, J., Tirole, J.: Adverse Selection and Renegotiation in Procurement. Rev. Econ. Stud. **57**, 597–625 (1990)
Lewis, T., Sappington, D.: Countervailing incentives in agency problems. J. Econ. Theory **49**, 294–313 (1989)
Liu, Q., Shuai, J.: Multi-dimensional product differentiation. Front. Econ. China **14**, 497–535 (2019)
Maggi, G., Rodriguez-Clare, A.: On countervailing incentives. J. Econ. Theory **66**, 238–263 (1995)
Maskin, E.: Uncertainty and entry deterrence. Econ. Theory **14**, 429–437 (1999)
Mitchell, M., Skrzypacz, A.: Network externalities and long-run market shares. Econ. Theory **29**, 621–648 (2006)
Muth, J.: Rational expectations and the theory of price movements. Econometrica **29**, 315–335 (1961)
Pavan, A., Segal, I., Toikka, J.: Dynamic mechanism design: a myersonian approach. Econometrica **82**, 601–653 (2014)
Posner, R.: Antitrust Law: An Economic Perspective. University of Chicago Press, Chicago (1976)
Rasmusen, E., Ramseyer, J., Wiley, J.: Naked exclusion. Am. Econ. Rev. **81**, 1137–1145 (1991)
Segal, I., Whinston, M.: Naked exclusion: comment. Am. Econ. Rev. **90**, 296–309 (2000)
Simpson, J., Wickelgren, A.: Naked exclusion, efficient breach, and downstream competition. Am. Econ. Rev. **97**, 1305–1320 (2007)
Spector, D.: Exclusive contracts and demand foreclosure. Rand. J. Econ. **42**, 629–638 (2011)
Spector, D.: The European commission's approach to long-term contracts: an economist's view. J. Eur. Compet. Law Pract. **5**, 492–497 (2014)
Tversky, A., Kahneman, D.: Judgment under uncertainty: heuristics and biases. Science **185**, 1124–1131 (1974)
Vasconcelos, H.: Contractual signaling, relationship-specific investment and exclusive agreements. Games Econ. Behav. **87**, 19–33 (2014)
Wright, J.: Exclusive dealing and entry, when buyers compete: comment. Am. Econ. Rev. **99**, 1070–1081 (2009)

Zhang, L.: Price-matching guarantees and entry deterrence under incomplete information. Front. Econ. China **7**, 246–262 (2012)

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