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Two Differential Games Between Rent-Seeking Politicians and Capitalists: Implications for Economic Growth

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We comparatively study two differential games between politicians and capitalists in terms of reducing rent-seeking distortions and stimulating economic growth. These two games imply two relationships — top-down authority and rational cooperation — between politicians and capitalists. In the current context, we prove that cooperation leads to faster growth than does authority, and simultaneously satisfies individual rationality, group rationality, Pareto efficiency and sub-game consistency. We thus show that setting a bargaining table between capitalists and politicians may create desirable incentives for reducing rent-seeking distortions, developing the spirit of capitalism and stimulating economic growth.

Keywords: Rent-seeking politicians; stochastic differential game; capital income tax; endogenous growth.

JEL Classification: C70, O43, P50

1. Introduction

We comparatively study two differential games between rent-seeking politicians and capitalists from the perspective of growth performance. These two games imply two relationships, namely top-down authority and rational cooperation, between government and capitalists. We attempt to identify the one having advantage in reducing rent-seeking activities and promoting economic growth. As demonstrated by historical facts and theoretical arguments (see, Murphy *et al.*, 1993; Mauro, 1995; Aidt, 2003), rent-seeking activities are quite costly to economic growth. It is therefore economically meaningful to search for some mechanisms to reduce such distortions.

To achieve our goal, we construct a simple model in which economic agents pursue utility maximization and are divided into three groups: self-interested politicians who have power to levy taxes on capital income, capitalists who own capital, and entrepreneurs who own technology. The number of capitalists and entrepreneurs evolves following a geometric Brownian motion, and matching between capital and technology through market search is the major engine of economic growth.

We first show that efficient capital-income tax rate should be zero when the government is benevolent. However, it just represents an ideal case because rent-seeking politicians always exist, i.e., a politician's preference may diverge from those of his constituents to pursue his self-interest [e.g., Buchanan and Tullock, 1962; Barro, 1973; Ferejohn, 1986]. Here, politicians are game players rather than game designers.

We then compare growth rates under a noncooperative differential game and a cooperative differential game that can be regarded as two different types of institutional arrangements [e.g., North, 1990; Hurwicz, 1996]. The result reveals that cooperation reduces more rent-seeking distortions and induces more investments, hence leading to faster economic growth.

In addition, for stimulating economic growth, our result implies that there should be a complementary rather than substitutive relationship between competitive market mechanism and the cooperative mechanism which emphasizes the complementarity between politicians and capitalists by maximizing their encompassing interests (see, Olson [2000]). Since cooperation arises from rational economic agents without appealing to third-party enforcement, it is incentive compatible so that it is essentially different from central planning.

Our work is related to the existing literature in several aspects. Since we focus on the basic idea that different institutional arrangements produce different incentive structures among economic agents, induce different levels of investment, and hence yield different speeds of economic growth, we are in line with North [1990] who argues that institutions are the underlying determinants of economic performance.[a] Nevertheless, we follow a different approach by using stochastic differential games to identify microeconomic details based on which much faster speed of economic growth can be achieved and sustained.

Murphy *et al.* [1993] indicate that rent-seeking activities exhibit increasing returns and hurt innovative activities more than everyday production, thereby becoming so costly to economic growth. As a necessary complement, we prove

[a]Recently, this viewpoint has been empirically proved by Acemoglu *et al.* [2005].

that rent-seeking activities hurt economic growth through negatively distorting the savings motive of capitalists. It thus hurts the development of the spirit of capitalism emphasized by Max Weber.

Acemoglu *et al.* [2008] and Yared [2010] analyze distortions induced by self-interested politicians who have the power to allocate some of the tax revenue to themselves as rents. The current model departs from these studies by employing a specific form of rent-seeking consumption such that politicians have economic incentives to discipline themselves. Also, we focus on the solution concept of Markovian-feedback equilibrium rather than perfect Bayesian Nash equilibrium, and hence we resolve the dynamic commitment issue by proving sub-game consistency while Acemoglu *et al.* [2008] deal with this by imposing a power-sustainability constraint on politicians.

Moreover, we use a continuous-time infinitely repeated game with aggregate shocks while Acemoglu *et al.* [2008] use a discrete-time repeated game with asymmetric information. As such, they solve their dynamic programming problem by using revelation principle while we rely on the general algorithm pioneered by Yeung and Petrosyan [2006].

As a remarkable point, while Yared [2010] derives conditions under which political-economy distortions disappear in the long run, distortions do persist in the current context. Typically, Acemoglu *et al.* [2008] prove that it may be beneficial for the society to tolerate political-economy distortions in exchange for the improvement in risk sharing, whereas here cooperation tolerates certain level of distortion for the sake of maximizing the encompassing interests between politicians and capitalists.

When discussing the issue of capitalism using differential games, some literatures are to be noticed. For example, Lancaster [1973] and Kaitala and Pohjola [1990] adopt a two-player deterministic differential game to prove that cooperation between government and firm will be more beneficial compared to noncooperation, resulting in dynamic inefficiency of capitalism. Later on, Seierstad [1993] uses a slight extension of the original finite-horizon model of Lancaster, proving the dynamic efficiency of capitalism. Inspired by recent financial crisis, Leong and Huang [2010] develop a stochastic differential game of capitalism to analyze the role of uncertainty. They demonstrate that cooperation is Pareto optimal relative to noncooperative Markovian Nash equilibrium. Different from us, government is assumed to be a vote-maximizer in their model.

In sum, the current paper distinguishes itself from these studies in five aspects. Firstly, we focus on reducing political-economy distortions resulted from rent-seeking activities that can be found in both capitalism and socialism. Secondly, the current model evaluates noncooperative mechanism and cooperative mechanism from the perspective of economic growth rather than welfare loss, class conflict or income redistribution. Thirdly, we construct the microfoundation of growth based on search and matching and suggest the reasonable coexistence of competitive market mechanism and cooperative mechanism. Fourthly, we impose risk-averse other than risk-neutral preferences on politicians and capitalists. Finally, to emphasize the distortion effect, we use linear tax rather than lump-sum tax.

The rest of the paper is organized as follows. Section 2 constructs the model and provides some basic assumptions. Section 3 derives equilibrium growth rates and shows the associated comparative statics. Section 4 proceeds to a comparative study of alternative governance mechanisms. Section 5 closes the paper with some concluding remarks, regarding the limitation and further extension of the current study. As usual, all mathematical derivations are shown in Appendix A.

2. The Model

2.1. *Bilateral matching*

Consider an economy with three types of agents: politicians, capitalists, and entrepreneurs. A bilateral matching occurs as long as a capitalist meets an entrepreneur and vice versa. For each capitalist and each entrepreneur, constants $\underline{u}^C > 0$ and $\underline{u}^E > 0$ stand for their initial endowments, respectively. Let a constant $\sigma_C \in (0, 1)$ be the search intensity of capitalists and correspondingly $\sigma_E \in (0, 1)$ of entrepreneurs with disutility $\varphi(\sigma) > 0$ for $\sigma = \sigma_C, \sigma_E$.

The population is divided into two groups with $M(t)$ politicians and $N(t)$ capitalists and entrepreneurs at period t. Let $C(t) \equiv \gamma N(t)$ and $E(t) \equiv (1-\gamma)N(t)$ denote the numbers of capitalists and entrepreneurs at t, respectively. The aggregate search intensity of capitalists is then $\sigma_C C(t) = \sigma_C \gamma N(t)$ and of entrepreneurs $\sigma_E E(t) = \sigma_E(1-\gamma)N(t)$ with the fraction $0 < \gamma < 1$ characterizing market composition. The total number of realized matches is defined by a matching function $\mathcal{M}(\sigma_C\gamma N(t), \sigma_E(1-\gamma)N(t))$, which exhibits constant returns to scale (CRS),[b] and is strictly increasing and concave.

The tightness of the market is defined by

$$\omega \equiv \frac{\sigma_E E(t)}{\sigma_C C(t)} = \frac{\sigma_E(1-\gamma)N(t)}{\sigma_C \gamma N(t)} = \left(\frac{1}{\gamma} - 1\right)\frac{\sigma_E}{\sigma_C}. \tag{1}$$

Intuitively, when ω is very big, the market is thick for capitalists and thin for entrepreneurs. By the CRS assumption, average matching rates per search intensity for capitalists and entrepreneurs at date t are respectively given by

$$\frac{\mathcal{M}(\sigma_C\gamma N(t), \sigma_E(1-\gamma)N(t))}{\sigma_C\gamma N(t)} = \mathcal{M}(1,\omega) \equiv \alpha(\omega)$$

and

$$\frac{\mathcal{M}(\sigma_C\gamma N(t), \sigma_E(1-\gamma)N(t))}{\sigma_E(1-\gamma)N(t)} = \frac{\sigma_C\gamma}{\sigma_E(1-\gamma)}\mathcal{M}(1,\omega) = \frac{1}{\omega}\alpha(\omega),$$

[b]This type of matching function is widely used in two-sided matching markets (see, e.g., Petrongolo and Pissarides [2001]). It is shown to be of technical convenience as well as empirical relevance. Here, we just follow the common practice because we do not find any evidences showing that CRS is not suitable for the capital market. Importantly, as shall be shown below, the main results of this paper do not depend on the specific functional form of the matching function.

where $\alpha(0) = 0, \alpha'(0) \geq 1, \alpha'(\omega) > 0, \alpha''(\omega) < 0$ and $\alpha(\omega) < \min\{1, \omega\}$ for all ω. The probabilities of getting involved in a match are then respectively given by

$$\sigma_C \frac{\mathcal{M}(\sigma_C \gamma N(t), \sigma_E(1-\gamma)N(t))}{\sigma_C \gamma N(t)} = \sigma_C \alpha(\omega), \tag{2}$$

$$\sigma_E \frac{\mathcal{M}(\sigma_C \gamma N(t), \sigma_E(1-\gamma)N(t))}{\sigma_E(1-\gamma)N(t)} = \frac{\sigma_E}{\omega}\alpha(\omega). \tag{3}$$

2.2. *Entrepreneurs*

Now, we proceed to the production activity.

Assumption 2.1 (Technology).[c] Entrepreneurs are equipped with a linear production technology, namely $y(t) = Ak(t)$ for each entrepreneur.

That is, with $k(t) > 0$ amounts of capital input, the entrepreneur can produce $y(t)$ amounts of output at time t. Meanwhile, entrepreneurs are assumed to exhibit risk neutral preferences. Then, Assumption 2.1 means that they are homogenous. By (3), the representative entrepreneur's utility-maximizing problem is

$$\max_{k \geq 0} \frac{\sigma_E}{\omega}\alpha(\omega)[\underline{u}^E + \underbrace{Ak - R(t)k}_{\text{profit}}] + \left[1 - \frac{\sigma_E}{\omega}\alpha(\omega)\right]\underline{u}^E - \varphi(\sigma_E), \tag{4}$$

where $A > 0$ denotes the productivity parameter, and $R(t)$ represents the gross capital rental rate that is competitively determined. Solving problem (4) gives rise to[d]

$$R(t) = A, \forall\, t \geq 0. \tag{5}$$

Without loss of generality, to make entrepreneurs have a neutral standpoint in the cooperative capitalism,[e] we put $\underline{u}^E \equiv \varphi(\sigma_E)$ so that their equilibrium utility is zero.

[c] Since the model emphasizes capital accumulation as the major engine of economic growth, AK production technology is our first choice for the sake of simplicity and tractability. In fact, one can introduce additional constraints to equivalently transfer Cobb–Douglas type production functions into an AK type [e.g., Turnovsky, 2000].

[d] Under perfect competition, capital rental rate, $R(t)$, must be equal to the marginal productivity of capital, A, leaving no arbitrage opportunities for all participants.

[e] We focus on the conflict between capitalists and politicians rather than between entrepreneurs and politicians just because we are currently interested in governance mechanisms that promote the spirit of capitalism emphasized by Max Weber, the celebrated German sociologist and political economist. Certainly, as a promising topic of independent interest for future research, one may emphasize the conflict between entrepreneurs and politicians and similarly study governance mechanisms that promote entrepreneurs' incentive of creative destruction emphasized by another well-known economist Joseph Schumpeter.

2.3. *Capitalists*

Capitalists are specialized in capital accumulation, and the law of motion of aggregate capital accumulation is expressed as

$$\dot{K}(t) \equiv \frac{dK(t)}{dt} = (1-\tau_k(t))(A-\delta)K(t) - (1-s(t))AK(t), \tag{6}$$

where $\delta > 0$ denotes a constant depreciation rate, and $\tau_k(t)$ and $s(t)$ stand for capital-income tax rate and savings rate, respectively.

Assumption 2.2 (Uncertainty).[f] The number $N(t)$ of capitalists and entrepreneurs follows a geometric Brownian motion.

We then set:

$$dN(t) = nN(t)dt + \sigma N(t)dB(t),$$

where $n, \sigma \in \mathbb{R}_0 \equiv \mathbb{R}\backslash\{0\}$ are constants, $B(t)$ stands for a standard Brownian motion defined on the (augmented) filtered probability basis $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{0\le t\le\infty}, P)$ with $B(0) = 0$ a.s.-P and the usual conditions fulfilled. Since $C(t) \equiv \gamma N(t)$, applying Itô formula results in

$$dC(t) = nC(t)dt + \sigma C(t)dB(t). \tag{7}$$

As a result, for $k(t) \equiv K(t)/C(t)$, combining (6) with (7) and applying Itô's rule again lead to

$$dk(t) = [(1-\tau_k(t))(A-\delta) - n + \sigma^2 - (1-s(t))A]k(t)dt - \sigma k(t)dB(t), \tag{8}$$

subject to a given initial condition $k(t_0) \equiv k_0 > 0$ for $0 \le t_0 \le t$.

The capitalist's utility-maximizing problem is then

$$\max_{0<s(t)<1} \mathbb{E}_{t_0}\left(\int_{t_0}^{\infty} e^{-\rho(t-t_0)}\{\sigma_C\alpha(\omega)(\underline{u}^C + \ln[c(t)]) + [1-\sigma_C\alpha(\omega)]\underline{u}^C - \varphi(\sigma_C)\}dt\right), \tag{9}$$

subject to individual consumption $c(t) = (1-s(t))Ak(t)$ and constraint (8). Here, $\mathbb{E}_{t_0}$ is the expectation operator conditional on information set $\mathcal{F}_{t_0}$, and $0 < \rho < 1$ is the subjective discount factor. For notational simplicity, we also let $\underline{u}^C \equiv \varphi(\sigma_C)$.

Thus, (9) can be rewritten as

$$\max_{0<s(t)<1} \mathbb{E}_{t_0}\left(\int_{t_0}^{\infty} e^{-\rho(t-t_0)}\{\sigma_C\alpha(\omega)\ln[(1-s(t))Ak(t)]\}dt\right), \tag{10}$$

subject to (8). Also, we just need to consider a representative capitalist.

2.4. *Politicians*

There is a self-interested other than benevolent politician in power at each period.

[f] The same assumption has been adopted by Merton [1975], and Leong and Huang [2010]. Following the common practice in economics literature, $N(t)$ is not necessarily an integer.

Assumption 2.3 (Preference).[g] The politician exhibits log preferences and has the same discount factor as capitalists.

His optimal control problem is then expressed as

$$\max_{0\leq\tau_k(t)\leq 1} \mathbb{E}_{t_0}\left(\int_{t_0}^{\infty} e^{-\rho(t-t_0)}\{\ln[\tau_k(t)(A-\delta)k(t)] + \phi\sigma_C\alpha(\omega)\ln[c(t)] + \theta g(t)\}dt\right), \tag{11}$$

subject to (8), $0 \leq \phi \leq 1, 0 \leq \theta \leq 1$ as well as

$$As(t) - (A-\delta)\tau_k(t) - \delta = \frac{\dot{K}(t)}{K(t)} = \frac{\dot{Y}(t)}{Y(t)} \equiv g(t), \tag{12}$$

in which $Y(t) = AK(t)$ denotes aggregate output, and hence $g(t)$ represents the economic growth rate. Here, $0 \leq \phi \leq 1$ stands for *welfare weight*, which characterizes the degree to which the politician cares about capitalist's welfare in the rent-seeking process. For instance, we can classify the governance type as: $\phi = 1, 0 < \phi < 1, \phi = 0$ represent democratic governance, compromised governance, and oligarchic/Leviathan governance, respectively. In addition, growth weight $0 \leq \theta \leq 1$ measures the contribution of GDP growth to his welfare.

Indeed, it follows from (11) that the politician faces a *dynamic tradeoff*: on the one hand, an increase of $\tau_k(t)$ implies a resulting increase of instantaneous utility (*ceteris paribus*); whereas, on the other hand, an increase of $\tau_k(t)$ produces a negative effect on the accumulation of $k(t)$, thereby inducing a reduction of the instantaneous utility (*ceteris paribus*). As such, we conjecture that there should be a critical value of $\tau_k(t)$ such that his utility is maximized.

Why is it possible that self-interested politicians also care about the rate of economic growth *per se*? First, fast economic growth and inequality can coexist *under certain institutional circumstance* and *during certain period*. For example, in the industrialization of the Soviet Union from the first Five-Year Plan in 1928 until the 1970s, the country was able to achieve eye-catching economic growth because it could use the absolute power of the state to reallocate resources from agriculture to industry (see, Acemoglu and Robinson [2012]). Similar episode happened in the industrialization process of China, resulting in great inequality between the rural and the urban. Therefore, inspired by these facts, self-interested politicians care about economic growth but not for improving the level of social equity and social justice.

[g] Logarithmic preference is usually adopted for establishing closed-form solutions in continuous-time stochastic maximization problems (see, e.g., He and Krishnamurthy [2012]). To make things easier, we are in line with Kaitala and Pohjola [1990] and Leong and Huang [2010] to let the representative capitalist and the self-interested politician share the same discount factor. One can certainly assume heterogeneous discount factors, but the computation is highly complicated, especially in the present cooperative stochastic differential game of capitalism. Since self-interested politicians are modeled as rational economic agents who just pursue utility maximization, letting politicians and capitalists be homogeneous along this dimension seems reasonable. Needless to say, we admit the limitation of this assumption.

Secondly, in autocratic societies, politicians focus on economic growth to extract more income and wealth [i.e., the grabbing hand rather than the helping hand, see, Frye and Shleifer, 1997], sustain their power and further consolidate their political dominance. For instance, one can refer to the time-honored Maya Classical Era and the Caribbean Islands between the sixteenth and eighteenth centuries (see, Acemoglu and Robinson [2012], for more historical details).

Last but not least, such kind of motive can also be driven by political and economic competition between local governments. Montinola *et al.* [1995] argue that China's remarkable economic success rests on a foundation of political reform that reflects a special type of institutionalized decentralization, i.e., it fosters the economic competition among local governments and hence constructs an efficient micro-incentive structure, particularly when noting that China has a vast amount of bureaucracy (see also, Xu [2011]). Moreover, one may find it thought-provoking that two famous and also adjacent provinces, Jiangsu and Zhejiang, in China have similar speed of GDP growth, whereas it is recognized that the people in Zhejiang are in average much richer than their counterparts in Jiangsu (see, Huang [2008]). As such, it is important to distinguish between growth weight and welfare weight in government's objective function.

2.5. *An ideal case: Zero distortion*

Although the current study emphasizes the unavoidability of rent-seeking activities in reality, there assumed to be a benevolent government in many benchmark models. That is, in view of current underpinnings, the maximization problem facing a politician should be

$$\max_{0\leq\tau_k(t)\leq 1} \mathbb{E}_{t_0}\left(\int_{t_0}^{\infty} e^{-\rho(t-t_0)}\{\sigma_C\alpha(\omega)\ln[(1-s(t))Ak(t)]\}dt\right),$$

subject to (8) for a given savings rate. Thus, the Bellman equation can be written as

$$\begin{aligned}\rho J^G(k(t)) &- \frac{1}{2}\sigma^2 k^2(t) J^G_{kk}(k(t)) \\ &= \max_{0\leq\tau_k(t)\leq 1}\{\sigma_C\alpha(\omega)\ln[(1-s(t))Ak(t)] + J^G_k(k(t))k(t) \\ &\quad \times [(1-\tau_k(t))(A-\delta) - n + \sigma^2 - (1-s(t))A]\},\end{aligned}$$

where $J^G(k(t))$ denotes the value function satisfying $J^G_k(k(t)) > 0$[h] for any available $k(t) > 0$. Thus the FOC is given by $-J^G_k(k(t))k(t)(A-\delta) < 0$ with $A > \delta$. We,

[h]It is immediate from the above objective function that the value function must be nondecreasing in capital. The only interesting case is that the value function is strictly increasing in capital, as shall be similarly shown in the following sections. If $J^G_k(k(t)) = 0$, then capital accumulation is no longer of economic relevance. That is, capitalists have no incentives to accumulate capital in such a case, and this is useless for the current study. As such, we just need to consider the case with $J^G_k(k(t)) > 0$.

by using the monotonicity, claim that efficient capital-income tax rate should be zero. Moreover, applying this result to (12) produces the efficient growth rate, i.e., $\overline{g}(t) = A\overline{s}(t) - \delta$, for the corresponding optimal savings rate $\overline{s}(t)$.

Capital-income taxation plays a crucial role in income redistribution and adjusting investment. In our model, any positive capital-income tax rate should result in some degree of rent-seeking distortion, reducing investment as well as slowing economic growth. Thus, any benevolent government should set it to zero. Actually, we will compute below to get that $\overline{s}(t) = 1 - \frac{\rho}{A}$, and hence $\overline{g}(t) = A - \rho - \delta$, a constant relying on the productivity, the degree of patience and the depreciation rate.

3. Equilibrium Growth Rates and Comparative Statics

3.1. *Noncooperative growth*

Under top-down authority, the capitalist and the politician are involved in a noncooperative differential game denoted by $\Gamma^{NM}(t_0, k_0)$ with given initial condition (t_0, k_0). Precisely, the capitalist chooses the best savings strategy s^* given the politician's best-response strategy τ_k^*, and simultaneously, the politician chooses the best rent-seeking strategy τ_k^* given the capitalist's best-response strategy s^*. In addition, we let $J^C(k(t))$ and $J^G(k(t))$ be value functions for the capitalist and the politician, respectively.

Definition 3.1 (Markovian-feedback Nash equilibrium). A set of strategies $\{s^*(t), \tau_k^*(t)\}$ constitutes a Markovian-feedback Nash equilibrium to $\Gamma^{NM}(t_0, k_0)$ if there exist continuously differentiable functions $J^C(k(t)) : \mathbb{R} \to \mathbb{R}$ and $J^G(k(t)) : \mathbb{R} \to \mathbb{R}$ satisfying Bellman equations

$$\begin{aligned}\rho J^C(k(t)) &- \frac{1}{2}\sigma^2 k^2(t) J^C_{kk}(k(t)) \\ &= \max_{0<s(t)<1} \{\sigma_C \alpha(\omega) \ln[(1-s(t))Ak(t)] + J^C_k(k(t))k(t) \\ &\quad \times [(1-\tau_k^*(t))(A-\delta) - n + \sigma^2 - (1-s(t))A]\}\end{aligned}$$

and

$$\begin{aligned}\rho J^G(k(t)) &- \frac{1}{2}\sigma^2 k^2(t) J^G_{kk}(k(t)) \\ &= \max_{0\le\tau_k(t)\le 1} \{\ln[\tau_k(t)(A-\delta)k(t)] + \theta[As^*(t) - (A-\delta)\tau_k(t) - \delta] \\ &\quad + \phi\sigma_C\alpha(\omega)\ln[(1-s^*(t))Ak(t)] + J^G_k(k(t))k(t)[(1-\tau_k(t))(A-\delta) \\ &\quad - n + \sigma^2 - (1-s^*(t))A]\},\end{aligned}$$

respectively, with

$$J^{(t_0)C}(t_0, k_0) \equiv \mathbb{E}_{t_0}\left(\int_{t_0}^{\infty} e^{-\rho(t-t_0)}\{\sigma_C\alpha(\omega)\ln[(1-s^*(t))Ak(t)]\}dt \,\middle|\, k(t_0) \equiv k_0\right)$$

and

$$J^{(t_0)G}(t_0, k_0) \equiv \mathbb{E}_{t_0}\left\{\int_{t_0}^{\infty} e^{-\rho(t-t_0)}\{\ln[\tau_k^*(t)(A-\delta)k(t)] + \theta[As^*(t) - (A-\delta)\tau_k^*(t) - \delta] + \phi\sigma_C\alpha(\omega)\ln[(1-s^*(t))Ak(t)]\}dt \,|\, k(t_0) \equiv k_0\right\}$$

representing the current-value payoffs for the capitalist and the politician, respectively.

Now we establish the first major result.

Proposition 3.1. *Suppose Assumptions* 2.1–2.3 *hold. Then, the Markovian-feedback Nash equilibrium is given by*

$$\{s^*(t), \tau_k^*(t)\} = \left\{1 - \frac{\rho}{A}, \frac{\rho}{(A-\delta)[\rho\theta + 1 + \phi\sigma_C\alpha(\omega)]}\right\}$$

for any $t \geq t_0$. *Meanwhile, the noncooperative growth rate amounts to*

$$g^*(t) = A - \rho - \delta - \frac{\rho}{\rho\theta + 1 + \phi\sigma_C\alpha(\omega)}.$$

In fact, the Markovian-feedback Nash equilibrium is a *dominant-strategy* equilibrium, which usually provides us with much stronger equilibrium predictions. Moreover, if we analyze the strategic interaction between capitalist and politician in a dynamic game, one can easily verify that it also defines a subgame perfect Nash equilibrium (SPNE) due to the dominant-strategy feature.

We then establish the following comparative statics.

Corollary 3.1. *For the noncooperative growth rate* $g^*(t)$,

$$\frac{\partial g^*(t)}{\partial \sigma_C} > 0, \frac{\partial g^*(t)}{\partial \sigma_E} > 0, \frac{\partial g^*(t)}{\partial \gamma} < 0, \frac{\partial g^*(t)}{\partial \theta} > 0, \frac{\partial g^*(t)}{\partial \phi} > 0, \frac{\partial g^*(t)}{\partial \rho} < 0.$$

In the Nash equilibrium, only through tax rate can the microfoundation affect the equilibrium growth rate. Thus, to know how the microfoundation imposes impacts on the equilibrium growth rate is equivalent to analyze how the equilibrium tax rate is endogenously determined by the microfoundation. In the proof, we show that the equilibrium tax rate is a decreasing function with respect to search intensities σ_C and σ_E, welfare weight ϕ and growth weight θ. Intuitively, search intensities positively affect the capitalist's utility by increasing the matching probability; growth weight and welfare weight impose a positive effect on growth rate and the capitalist's utility, respectively. Meanwhile, tax rate always plays a negative role in all of these dimensions. In addition, the equilibrium tax rate is an increasing function of the market fraction γ of capitalists in the capital market. Indeed, since an increase of this fraction implies that capital market becomes thinner for capitalists, a lower matching probability follows, which, accordingly, hurts the capitalist's welfare. As a result, market fraction indirectly imposes a negative effect on equilibrium growth rate through increasing capital-income tax rate.

Corollary 3.2. *Suppose the economy is under oligarchic governance, i.e., $\phi = 0$. Then agents' search intensity, bilateral matching technology and market composition do not affect the politician's best-response strategy $\tau_k^*(t)$, and thus the resulting non-cooperative growth rate $g^*(t)$. In particular, in terms of welfare weight $\phi \in [0,1]$ and growth weight $\theta \in [0,1]$, $g^*(t)$ reaches its maximum level when the economy is under democratic governance, namely $\phi = 1$, as well as completely growth-rate oriented, namely $\theta = 1$.*[i]

Proof. We just mention the fact that in terms of welfare weight $\phi \in [0,1]$ and growth weight $\theta \in [0,1]$,

$$A - 2\rho - \delta \leq g^*(t) \leq A - \rho - \delta - \frac{\rho}{\rho + 1 + \sigma_C \alpha(\omega)}$$

for any $t \geq t_0$. □

3.2. *Cooperative growth*

Under cooperation, the capitalist and the politician are involved in a cooperative differential game denoted by $\Gamma^{CM}(t_0, k_0)$ with given initial condition (t_0, k_0). That is, they are motivated to maximize their encompassing interests. Also, we set $J^{CM}(k(t))$ to be the value function.

Assumption 3.1 (Additivity). Payoffs/utilities are transferable across the capitalist and the politician, and over time.

Using (10)–(12) and Assumption 3.1, the maximization problem under cooperative mechanism can be written as

$$\max_{0 \leq \tau_k(t), s(t) \leq 1} \mathbb{E}_{t_0} \left\{ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \{\ln[\tau_k(t)(A-\delta)k(t)] + (1+\phi)\sigma_C \alpha(\omega) \ln[(1-s(t))Ak(t)] + \theta[As(t) - (A-\delta)\tau_k(t) - \delta]\} dt \,\middle|\, k(t_0) \equiv k_0 \right\}, \tag{13}$$

subject to constraint (8). That is, cooperative mechanism chooses a time path of tax rate and savings rate to maximize the summation of the capitalist's payoff and the politician's payoff.

Definition 3.2 (Markovian-feedback cooperative equilibrium). A set of strategies $\{s^{**}(t), \tau_k^{**}(t)\}$ constitutes a Markovian-feedback cooperative equilibrium

[i]See also Fig. 1.

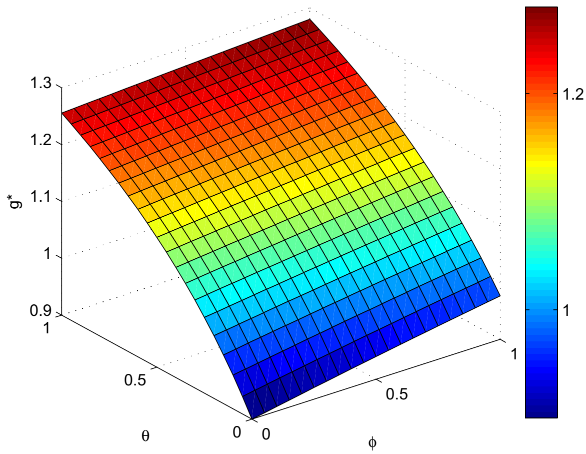

Fig. 1. Noncooperative growth rate as a function of welfare weight (ϕ) and growth weight (θ) (parameter values: $A = 3, \rho = 0.8, \delta = 0.5, \sigma_e = 0.2, \sigma_c = 0.6, \gamma = 0.8$).

to $\Gamma^{CM}(t_0, k_0)$ if there exists a continuously differentiable function $J^{CM}(k(t)) : \mathbb{R} \to \mathbb{R}$ satisfying Bellman equation

$$\begin{aligned}\rho J^{CM}(k(t)) - \frac{1}{2}\sigma^2 k^2(t) J^{CM}_{kk}(k(t)) \\ = \max_{0 \le \tau_k(t), s(t) \le 1} \{\ln[\tau_k(t)(A-\delta)k(t)] + \theta[As(t) - (A-\delta)\tau_k(t) - \delta] \\ + (1+\phi)\sigma_C \alpha(\omega) \ln[(1-s(t))Ak(t)] + J^{CM}_k(k(t))k(t) \\ \times [(1-\tau_k(t))(A-\delta) - n + \sigma^2 - (1-s(t))A]\}\end{aligned}$$

with

$$\begin{aligned}J^{(t_0)CM}(t_0, k_0) \equiv \mathbb{E}_{t_0}\left\{\int_{t_0}^{\infty} e^{-\rho(t-t_0)}\{\ln[\tau_k^{**}(t)(A-\delta)k(t)] + \theta[As^{**}(t) \right. \\ - (A-\delta)\tau_k^{**}(t) - \delta] + (1+\phi)\sigma_C\alpha(\omega) \\ \left. \times \ln[(1-s^{**}(t))Ak(t)]\}dt \,\middle|\, k(t_0) \equiv k_0 \right\}\end{aligned}$$

representing the current-value cooperative payoff.

Proposition 3.2. *Suppose Assumptions* 2.1–2.3 *and* 3.1 *hold. Then, the cooperative equilibrium* $\{s^{**}(t), \tau_k^{**}(t)\}$ *is given by*

$$\left\{1 - \frac{\rho(1+\phi)\sigma_C\alpha(\omega)}{A[\rho\theta + 1 + (1+\phi)\sigma_C\alpha(\omega)]}, \frac{\rho}{(A-\delta)[\rho\theta + 1 + (1+\phi)\sigma_C\alpha(\omega)]}\right\}$$

for any $t \ge t_0$. *And the cooperative growth rate is*

$$g^{**}(t) = A - \delta - \frac{\rho[1 + (1+\phi)\sigma_C\alpha(\omega)]}{\rho\theta + 1 + (1+\phi)\sigma_C\alpha(\omega)}.$$

By Proposition 3.2, we can proceed to comparative-static analyses, as one can see below.

Corollary 3.3. *For the cooperative growth rate* $g^{**}(t)$,

$$\frac{\partial g^{**}(t)}{\partial \sigma_C} < 0, \frac{\partial g^{**}(t)}{\partial \sigma_E} < 0, \frac{\partial g^{**}(t)}{\partial \gamma} > 0, \frac{\partial g^{**}(t)}{\partial \theta} > 0, \frac{\partial g^{**}(t)}{\partial \phi} < 0, \frac{\partial g^{**}(t)}{\partial \rho} < 0.$$

It follows from (12) that growth rate positively relies on savings rate while negatively relying on tax rate. Search intensities negatively affect equilibrium growth rate through savings rate on the one hand, whereas, on the other hand, they positively impact it via tax rate. Since the former negative effect overwhelms the latter positive effect, cooperative growth rate is a decreasing function of search intensities. Similar assertion follows for the welfare weight. In addition, since the market composition positively impacts equilibrium growth rate by savings rate while negatively affecting it via tax rate, it is an increasing function of market composition as the former positive effect outweighs the latter negative effect. For the growth weight, as it positively affects equilibrium growth rate through margins of savings rate and tax rate, the comprehensive effect is immediate.

Corollary 3.4. (i) *No matter the economy is under oligarchic governance with* $\phi = 0$, *compromised governance with* $0 < \phi < 1$, *or democratic governance with* $\phi = 1$, *agents' search intensity, bilateral matching technology and market composition always impose nontrivial economic effects on the cooperative equilibrium.*

(ii) *Regardless of the type of governance, cooperative growth rate is equal to* $A-\delta-\rho$ *whenever letting* $\theta = 0$. *Actually, in terms of welfare weight* $\phi \in [0,1]$ *and growth weight* $\theta \in [0,1]$, *oligarchic governance combined with* $\theta = 1$ *leads to the fastest speed of cooperative economic growth* (*ceteris paribus*).

Proof. This is a direct application of Corollary 3.3. In particular, in terms of welfare weight $\phi \in [0,1]$ and growth weight $\theta \in [0,1]$,

$$A - \rho - \delta \leq g^{**}(t) \leq A - \delta - \frac{\rho[1 + \sigma_C \alpha(\omega)]}{\rho + 1 + \sigma_C \alpha(\omega)}$$

for any $t \geq t_0$. □

Here, oligarchic governance means that tax rate is chosen wholly for seeking rent. Our result hence encompasses that cooperative mechanism can support a type of institutional arrangement involving oligarchic governance (i.e., $\phi = 0$) and growth-oriented policy (i.e., $\theta = 1$) that leads to the fastest speed of economic growth. Actually, this is a formal demonstration of the following views.

First, Baumol *et al.* [2007] argue that state-guided capitalism is a kind of system, which is however different from central planning, such that the government can typically take a regulation position to make the economy have the best way to maximize its economic growth.

Second, Acemoglu and Robinson [2012] illustrate with historical examples that there are two distinct but complementary ways in which growth under extractive political institutions can emerge. The first example is the rapid economic growth of the Soviet Union from the first Five-Year Plan in 1928 until 1970s. The second example is the rapid industrialization of South Korea under General Park. They further argue that Chinese economic growth has several commonalities with both Soviet Union and South Korean experiences.

Corollary 3.5. *In terms of welfare weight* $\phi \in [0,1]$ *and growth weight* $\theta \in [0,1], g^{**}(t) \geq A - \rho - \delta \equiv \overline{g}(t)$, *where* $\overline{g}(t)$ *is the efficient growth rate under a benevolent government.*

Proof. This is a corollary of Corollary 3.4. □

In other words, cooperative mechanism (compatible with equilibrium rent-seeking distortions) can dominate the traditional benevolent governance (with a benevolent government and hence without any equilibrium rent-seeking distortions) from the dimension of stimulating economic growth.

Since aggregate economic growth rate does not enter the capitalist's objective function, it does not enter the objective of a benevolent government. Under cooperative mechanism, however, growth rate *per se* enters the objective of the politician. Elaborating further, under benevolent governance, zero equilibrium tax rate implies that the negative effect placed on investment and hence growth vanished. In contrast, under cooperative mechanism, the positive equilibrium tax rate imposes a negative effect on growth, whereas there also exists a positive effect resulted from the fact that maximizing growth rate is a part of the politician's objective. Our result implies that the positive effect actually outweighs the negative effect, yielding a positive net effect on growth rate under cooperative mechanism. As a consequence, cooperative mechanism dominates benevolent governance in promoting economic growth.

4. Top-Down Authority versus Rational Cooperation

In what follows, we shall show that the proposed cooperative mechanism fulfills properties: group rationality, individual rationality, sub-game consistency and Pareto efficiency under certain cooperative equilibrium solution concept. Indeed, we will derive the payoff distribution procedure (PDP) of the cooperative differential game based upon sub-game consistent imputation and provided that the politician and the capitalist agree to act according to agreed-upon Pareto-optimal principles, say, Nash bargaining solution and Shapley value.

From Proposition 3.2, the cooperative-equilibrium trajectory of capital per capita can be expressed as

$$dk(t) = \left(A - \delta - n + \sigma^2 - \frac{\rho[1 + (1+\phi)\sigma_C\alpha(\omega)]}{\rho\theta + 1 + (1+\phi)\sigma_C\alpha(\omega)}\right) k(t)dt - \sigma k(t)dB(t),$$

subject to the given initial condition $k(t_0) \equiv k_0 > 0$. The strong solution can be written as the integral form

$$k^{**}(t) = k_0 + \int_{t_0}^{t} \left(A - \delta - n + \sigma^2 - \frac{\rho[1 + (1+\phi)\sigma_C\alpha(\omega)]}{\rho\theta + 1 + (1+\phi)\sigma_C\alpha(\omega)}\right) k^{**}(s)ds - \int_{t_0}^{t} \sigma k^{**}(s)dB(s). \quad (14)$$

Let Ξ_t^{**} denote the set of reliable values of $k^{**}(t)$ at time t generated by (14). In particular, we employ k_t^{**} to represent a generic element of set Ξ_t^{**}. Moreover, let vector $\eta(\tau) \equiv [\eta_C(\tau), \eta_G(\tau)]$, assigned respectively to the capitalist and the politician, denote the instantaneous payoff for $\Gamma^{CM}(t_0, k_{t_0}^{**})$ at time $\tau \in [t_0, \infty)$ with initial state $k_{t_0}^{**} \in \Xi_{t_0}^{**}$. Then, along trajectory $\{k^{**}(t)\}_{t=t_0}^{\infty}$ we put

$$\xi^{(t_0)i}(\tau, k_\tau^{**}) \equiv \mathbb{E}_\tau \left[\int_\tau^\infty e^{-\rho(\lambda-\tau)}\eta_i(\lambda)d\lambda \,\middle|\, k(\tau) = k_\tau^{**}\right]$$

and

$$\xi^{(t_0)i}(t, k_t^{**}) \equiv \mathbb{E}_t \left[\int_t^\infty e^{-\rho(\lambda-t)}\eta_i(\lambda)d\lambda \,\middle|\, k(t) = k_t^{**}\right],$$

for capitalist and/or politician abbreviated to an economic agent $i \in \{C, G\}$, $k_\tau^{**} \in \Xi_\tau^{**}$, $k_t^{**} \in \Xi_t^{**}$ and $t \geq \tau \geq t_0$. Accordingly, based on an agreed-upon Pareto principle, the vectors $\xi^{(t_0)}(\tau, k_\tau^{**}) \equiv [\xi^{(t_0)C}(\tau, k_\tau^{**}), \xi^{(t_0)G}(\tau, k_\tau^{**})]$ for $\tau \geq t_0$ are valid imputations in the sense of the following definition.

Definition 4.1 (Valid imputation). The vector $\xi^{(t_0)}(\tau, k_\tau^{**})$ is a valid imputation of $\Gamma^{CM}(\tau, k_\tau^{**})$ for $\tau \in [t_0, \infty)$ and $k_\tau^{**} \in \Xi_\tau^{**}$ if it satisfies

(1) $\xi^{(t_0)}(\tau, k_\tau^{**}) \equiv [\xi^{(t_0)C}(\tau, k_\tau^{**}), \xi^{(t_0)G}(\tau, k_\tau^{**})]$ is a Pareto optimal imputation vector;
(2) Individual rationality requirement, i.e., $\xi^{(t_0)i}(\tau, k_\tau^{**}) \geq J^{(t_0)i}(\tau, k_\tau^{**})$ for $i \in \{C, G\}$,

$$J^{(t_0)C}(\tau, k_\tau^{**}) \equiv \mathbb{E}_{t_0} \left[\int_\tau^\infty e^{-\rho(\lambda-t_0)}\sigma_C\alpha(\omega)\ln((1 - s^{**}(\lambda))Ak_\lambda^{**})d\lambda \,\middle|\, k^{**}(\tau) = k_\tau^{**}\right]$$

and also

$$J^{(t_0)G}(\tau, k_\tau^{**}) \equiv \mathbb{E}_{t_0}\left\{\int_\tau^\infty e^{-\rho(\lambda-t_0)}\{\ln[\tau_k^{**}(\lambda)(A-\delta)k_\lambda^{**}] + \theta[As^{**}(\lambda) - (A-\delta)\tau_k^{**}(\lambda) - \delta] + \phi\sigma_C\alpha(\omega) \times \ln[(1-s^{**}(\lambda))Ak_\lambda^{**}]\}d\lambda \,\middle|\, k^{**}(\tau) \equiv k_\tau^{**}\right\}.$$

Let

$$\mu^{(t_0)i}(\tau; \tau, k_\tau^{**}) \equiv \mathbb{E}_\tau\left[\int_\tau^\infty e^{-\rho(\lambda-\tau)}\eta_i(\lambda)d\lambda \,\middle|\, k(\tau) = k_\tau^{**}\right] = \xi^{(t_0)i}(\tau, k_\tau^{**})$$

and

$$\mu^{(t_0)i}(\tau; t, k_t^{**}) \equiv \mathbb{E}_t\left[\int_t^\infty e^{-\rho(\lambda-\tau)}\eta_i(\lambda)d\lambda \,\middle|\, k(t) = k_t^{**}\right]$$

for $i \in \{C, G\}$ and $t \geq \tau \geq t_0$. Noting that

$$\mu^{(t_0)i}(\tau; t, k_t^{**}) \equiv e^{-\rho(t-\tau)}\mathbb{E}_t\left[\int_t^\infty e^{-\rho(\lambda-t)}\eta_i(\lambda)d\lambda \,\middle|\, k(t) = k_t^{**}\right] = e^{-\rho(t-\tau)}\xi^{(t_0)i}(t, k_t^{**}) = e^{-\rho(t-\tau)}\mu^{(t_0)i}(t; t, k_t^{**}) \tag{15}$$

for $i \in \{C, G\}$ and $k_t^{**} \in \Xi_t^{**}$, we have the following definition.

Definition 4.2 (Sub-game consistency). A solution imputation is said to meet the sub-game consistency if it satisfies condition (15).

That is, sub-game consistency requires that the extension of the solution policy to a situation with a later starting time and any feasible state brought about by prior optimal behaviors would remain optimal.

Definition 4.3 (Nash bargaining solution/Shapley value). For $\Gamma^{CM}(t_0, k_0)$ at time t_0, an allocation principle is called Nash bargaining solution/Shapley value if an imputation

$$\xi^{(t_0)i}(t_0, k_0) = J^{(t_0)i}(t_0, k_0) + \frac{1}{2}\left[J^{(t_0)CM}(t_0, k_0) - \sum_{j\in\{C,G\}} J^{(t_0)j}(t_0, k_0)\right],$$

is assigned to player i, for $i \in \{C, G\}$; and at time $\tau \in [t_0, \infty)$, an imputation

$$\xi^{(t_0)i}(\tau, k_\tau^{**}) = J^{(t_0)i}(\tau, k_\tau^{**}) + \frac{1}{2}\left[J^{(t_0)CM}(\tau, k_\tau^{**}) - \sum_{j\in\{C,G\}} J^{(t_0)j}(\tau, k_\tau^{**})\right],$$

is assigned to player i, for $i \in \{C, G\}$, $k_\tau^{**} \in \Xi_\tau^{**}$ and

$$J^{(t_0)CM}(\tau, k_\tau^{**}) \equiv \mathbb{E}_{t_0}\left\{\int_\tau^\infty e^{-\rho(\lambda - t_0)}\{\ln[\tau_k^{**}(\lambda)(A-\delta)k_\lambda^{**}] + \theta[As^{**}(\lambda) - (A-\delta)\tau_k^{**}(\lambda) - \delta] + (1+\phi)\sigma_C\alpha(\omega) \times \ln[(1-s^{**}(\lambda))Ak_\lambda^{**}]\}d\lambda \,\middle|\, k^{**}(\tau) \equiv k_\tau^{**}\right\}.$$

In the two-player game, Nash bargaining solution and Shapley value coincide with each other. Although Shapley value is commonly used, equal imputation of cooperative gains[j] may not be agreeable to some players especially when their size of noncooperative payoffs is asymmetric. For example, noncooperative payoffs of capitalist and politician may be significantly asymmetric in reality owing to *de facto* unequal social status as well as unequal opportunity. So, we also consider the following allocation principle in which players' shares of the gain from cooperation are proportional to the relative size of their expected noncooperative payoffs.[k]

Definition 4.4 (Proportional distribution). For $\Gamma^{CM}(t_0, k_0)$, an allocation principle is called proportional distribution if the imputation assigned to player i is

$$\xi^{(t_0)i}(t_0, k_0) = \frac{J^{(t_0)i}(t_0, k_0)}{\sum_{j\in\{C,G\}} J^{(t_0)j}(t_0, k_0)} J^{(t_0)CM}(t_0, k_0),$$

for $i \in \{C, G\}$; and in the sub-game $\Gamma^{CM}(\tau, k_\tau^{**})$ for $\tau \in [t_0, \infty)$, the imputation assigned to player i is

$$\xi^{(t_0)i}(\tau, k_\tau^{**}) = \frac{J^{(t_0)i}(\tau, k_\tau^{**})}{\sum_{j\in\{C,G\}} J^{(t_0)j}(\tau, k_\tau^{**})} J^{(t_0)CM}(\tau, k_\tau^{**}),$$

for $i \in \{C, G\}$ and $k_\tau^{**} \in \Xi_\tau^{**}$.

To be precise, the cooperative mechanism has two defining features. First, the production of the total payoff (or the "pie") available for distribution is based on

[j]Formally, here cooperative gains are defined as terms $J^{(t_0)CM}(t_0, k_0) - \sum_{j\in\{C,G\}} J^{(t_0)j}(t_0, k_0)$ and $J^{(t_0)CM}(\tau, k_\tau^{**}) - \sum_{j\in\{C,G\}} J^{(t_0)j}(\tau, k_\tau^{**})$ shown in Definition 4.3. These two terms are independent of the type of players, and hence both players receive equal imputation of cooperative gains under Shapley value, even though they may be asymmetric in noncooperative payoffs. One can easily tell the difference between *the cooperative imputation*, $\xi^{(t_0)i}(t_0, k_0)$, and *the imputation of cooperative gains*, $\frac{1}{2}[J^{(t_0)CM}(t_0, k_0) - \sum_{j\in\{C,G\}} J^{(t_0)j}(t_0, k_0)]$. That is, if the two sides have asymmetric non-cooperative payoffs, the Shapley value still assigns them equal cooperative gains, but their cooperative imputations are in general different from each other.

[k]In a general production economy, Roemer [2010] proves that the only Pareto-efficient allocation rule that can be Kantian-implemented is the proportional allocation rule. That is, such an allocation rule has a reasonable microfoundation to support it to achieve Pareto efficiency.

cooperation, namely government's tax policy and capitalist's savings strategy are jointly determined by maximizing the same objective function, as already shown in (13). Second, the pie is distributed between the politician and the capitalist via implementing one of the above allocation principles.

Now, we have the following main result.

Proposition 4.1 (Toward cooperative growth). *Suppose Assumptions* 2.1–2.3 *and* 3.1 *hold. Then,* $g^{**}(t) > g^{*}(t)$ *for any* $t \geq t_0$. *Meanwhile, the cooperative mechanism simultaneously meets group rationality, individual rationality, Pareto efficiency and subgame consistency, and neither the capitalist nor the politician will unilaterally deviate from cooperation.*

Group rationality confirms the basic legitimacy and tenability of cooperative mechanism. That is, compared to noncooperative mechanism, it produces a much bigger cake available for allocation. Also, it induces a higher equilibrium investment, a lower equilibrium rent-seeking level, and hence a faster speed of economic growth relative to the noncooperative mechanism.

Even so, the following arguments are worth emphasizing to avoid any misleading interpretations.

First, although the politician is allowed to be completely self-interested from the standpoint of human nature, cooperation does not suggest a path towards authoritarianism. Instead, its sustainability relies on democratic institutional arrangements.

On one hand, only when the economy is under democracy[1] (i.e., politicians face the risk of being replaced) can we reasonably expect politicians to have sufficient incentive/motive to promote the encompassing interest. Since all economic agents are under the democratic institutional constraint, politicians are game players other than rule designers (or dictators).

On the other hand, although it is possible for some dictators to provide good rules or policies, people generally do not desire dictatorships, especially under modern political civilization, and overwhelming numbers of dictators actually lead people to very poor economic outcomes. Therefore, cooperation is consistent with (and hence can be seen as a special realization of) democracy in the sense that well-intentioned politicians will do the right things, and more importantly, not so well-intentioned politicians are restricted or at least not induced to do the wrong things in the process of stimulating economic growth.

Second, the cooperative mechanism exhibits some socially beneficial properties. It encourages self-interested politicians to focus more on long-run benefits

[1]An infinitely lived politician may not be the best representation of democracy. For example, we cannot analyze the possible effects of short-termism and political cycles. However, we can relax this assumption by endogenizing the power endurance of a given politician. Even so, our major predication does not rely on the assumption of an infinitely lived politician.

of economic growth. That is, they will not be tempted to jeopardize sustainable growth for short-sighted benefits. It proposes a much healthier relationship between politicians and capitalists by allowing for rational bargaining between the power and citizens.[m]

Third, cooperative mechanism provides economic incentives with which politicians will not directly compete with capitalists. In China, one serious problem is that the government (represented by state-owned enterprises) directly competes with private enterprises in some economic fields, hence creating numerous rent-seeking opportunities and transferring a great amount of wealth from the people to the government (see, for instance, Coase and Wang [2012]). This also partly explains why the Chinese government is very rich while the per capita income level is still very low.

Fourth, since we ignore labor input in the production activity, we just use capitalists to represent households, and hence cooperative mechanism should not be misunderstood as crony capitalism. In other words, we stress the crucial role capital as well as the spirit of capitalism plays in promoting economic growth.

5. Concluding Remarks

The paper offers a model with closed-form solutions to comparatively study the growth performance of alternative institutional arrangements. We construct the microfoundation based on search and matching, against which economic growth prefers the cooperative relationship between capitalists and politicians. The key point is to provide effective incentives for politicians to internalize the negative externality of such distortions. Fortunately, the cooperative mechanism, which simultaneously respects individual rationality, group rationality, sub-game consistency and Pareto efficiency, generates an equilibrium arrangement that performs reasonably well in this point.

However, the issue regarding institutional transition between alternative states is left unexplored. Admittedly, top-down authority and rational cooperation just represent two special choices of institutional arrangement, meaning that there may be some states in between. In consequence, there exist different paths of institutional transition, e.g., not just a simple switch from the noncooperative mechanism to the cooperative mechanism, or vice versa (see, for example, Tian [2000, 2001]). One possible extension is hence to build a theory comparing and evaluating alternative paths of economic and political transitions from both short-run and long-run perspectives. In addition, given the observation of some real-world cases that

[m]In recent years, we actually observe that more and more politicians in China's local governments are trying to build up cooperation through rational bargaining with related citizens to resolve the dispute of compensation for expropriated land (also refer to the link: https://www.youtube.com/watch?v=XuRPFqhsBXQ&list=WL&index=11).

cooperation between politician and capitalist does not necessarily lead to economic growth but does lead to the increase of their payoffs, it is interesting to extend the current analysis by explicitly considering a possibility of corruption.[n] We, however, leave these possible extensions or applications to future research so that we are allowed to focus on the primary concern of the current study.

Appendix. A. Proofs

Proof of Proposition 3.1. To prove this proposition, we just prove these lemmas.

Lemma A.1. *For* $\Gamma^{NM}(t_0, k_0)$, $s^*(t) = 1 - \frac{\rho}{A}$, *and* $J^C(k(t))$ *can be explicitly derived.*

Lemma A.2. *For* $\Gamma^{NM}(t_0, k_0), \tau_k^*(t) = \frac{\rho}{(A-\delta)[\rho\theta+1+\phi\sigma_C\alpha(\omega)]}$, *and* $J^G(k(t))$ *can be explicitly derived.*

Lemma A.3. $\lim_{t\to\infty} e^{-\rho(t-t_0)} J^C(k(t)) = 0$ *a.s., i.e., the transversality condition is satisfied almost surely.*

Lemma A.4. $\lim_{t\to\infty} e^{-\rho(t-t_0)} J^G(k(t)) = 0$ *a.s., i.e., the transversality condition holds true almost surely.*

Proof. We omit it as it is quite similar to that of Lemma A.3. □

Proof of Lemma A.1. For the first Bellman equation in Definition 3.1, the FOC is

$$\sigma_C\alpha(\omega) = J_k^C(k(t))(1 - s(t))Ak(t). \tag{A.1}$$

Substituting this term into the Bellman equation produces

$$\begin{aligned}\rho J^C(k(t)) &- \frac{1}{2}\sigma^2 k^2(t) J_{kk}^C(k(t)) \\ &= \sigma_C\alpha(\omega)\{\ln[\sigma_C\alpha(\omega)] - \ln[J_k^C(k(t))]\} + J_k^C(k(t))k(t) \\ &\quad \times [(1 - \tau_k^*(t))(A - \delta) - n + \sigma^2] - \sigma_C\alpha(\omega).\end{aligned} \tag{A.2}$$

[n]We wish to thank a referee for pointing out this possible extension. The current framework has the potential to be extended along several dimensions to investigate more complicated circumstances. As a short paper, our ambition is not that big and we believe that the current content is informative enough in revealing the key message.

Based on the guess-and-verify approach, we try $J^C(k(t)) = C_1 + C_2 \ln k(t)$ for some parameters C_1 and C_2, to be determined.[o] Then, by (A.2) we get

$$C_2 = \frac{\sigma_C \alpha(\omega)}{\rho} \tag{A.3}$$

and

$$C_1 = \left[-\frac{\sigma^2}{2\rho^2} + \frac{\ln \rho}{\rho}\right] \sigma_C \alpha(\omega) + \frac{\sigma_C \alpha(\omega)}{\rho} \left\{\frac{1}{\rho}[(1 - \tau_k^*(t))(A - \delta) - n + \sigma^2] - 1\right\}.$$

Hence, by (A.1) and (A.3) we obtain $s^*(t) = 1 - \frac{\rho}{A}$, as required. □

Proof of Lemma A.2. For the second Bellman equation in Definition 3.1, the FOC is

$$(A - \delta)\tau_k(t) = \frac{1}{\theta + J_k^G(k(t))k(t)}. \tag{A.4}$$

Inserting this result into the Bellman equation reveals that

$$\begin{aligned}
&\rho J^G(k(t)) - \frac{1}{2}\sigma^2 k^2(t) J_{kk}^G(k(t)) \\
&\quad = \ln k(t) - \ln[\theta + J_k^G(k(t))k(t)] - \frac{J_k^G(k(t))k(t)}{\theta + J_k^G(k(t))k(t)} + \theta[As^*(t) - \delta] \\
&\qquad + J_k^G(k(t))k(t)[A - \delta - n + \sigma^2 - (1 - s^*(t))A] + \phi\sigma_C\alpha(\omega) \\
&\qquad \times \ln[(1 - s^*(t))Ak(t)] - \frac{\theta}{\theta + J_k^G(k(t))k(t)}.
\end{aligned} \tag{A.5}$$

If we put $J^G(k(t)) = C_3 + C_4 \ln k(t)$ for some parameters C_3 and C_4, to be determined, then using (A.5) produces

$$C_4 = \frac{1 + \phi\sigma_C\alpha(\omega)}{\rho} \tag{A.6}$$

and

$$\begin{aligned}
C_3 = &-\frac{\sigma^2}{2\rho^2}[1 + \phi\sigma_C\alpha(\omega)] - \frac{1}{\rho}\ln\left(\theta + \frac{1 + \phi\sigma_C\alpha(\omega)}{\rho}\right) + \frac{\phi}{\rho}[\sigma_C\alpha(\omega) \\
&\times \ln[(1 - s^*(t))A]] + \frac{\theta}{\rho}[As^*(t) - \delta] + \frac{1 + \phi\sigma_C\alpha(\omega)}{\rho^2} \\
&\times [A - \delta - n + \sigma^2 - (1 - s^*(t))A] - \frac{1}{\rho}.
\end{aligned}$$

So, (A.4) combines with (A.6) gives rise to the desired result. □

[o]As log utility is assumed, such a guess of the form of value function is very reasonable. In fact, this is the usually adopted guess under log preferences [e.g., Øksendal and Sulem, 2009]. The same reasoning applies to the guess of the following value functions.

Proof of Lemma A.3. It follows from Lemmas A.1 and A.2 that

$$dk(t) = \left(A - \delta - n + \sigma^2 - \rho - \frac{\rho}{\rho\theta + 1 + \phi\sigma_C\alpha(\omega)}\right) k(t)dt - \sigma k(t)dB(t).$$

By applying Itô formula,

$$\ln k(t) = \ln k_0 + \left(A - \delta - n + \frac{\sigma^2}{2} - \rho - \frac{\rho}{\rho\theta + 1 + \phi\sigma_C\alpha(\omega)}\right)(t - t_0) - \sigma[B(t) - B(t_0)].$$

Note that $e^{-\rho(t-t_0)}B(t) = \frac{t}{e^{\rho(t-t_0)}}\frac{B(t)}{t} \to 0$ almost surely as $t \to \infty$ by making use of the strong Law of Large Numbers for martingales, we have $\lim_{t\to\infty} e^{-\rho(t-t_0)} \ln k(t) = 0$ almost surely. Since C_1 and C_2 are finite constants conditional on $\tau_k^*(t)$ derived in Lemma A.2, the required result immediately follows. □

Proof of Corollary 3.1. Provided $s^*(t) = 1 - \frac{\rho}{A}$, it is immediate that:

$$\frac{\partial s^*(t)}{\partial \sigma_C} = \frac{\partial s^*(t)}{\partial \sigma_E} = \frac{\partial s^*(t)}{\partial \gamma} = \frac{\partial s^*(t)}{\partial \theta} = \frac{\partial s^*(t)}{\partial \phi} = 0, \frac{\partial s^*(t)}{\partial \rho} < 0.$$

For $\tau_k^*(t)$,

$$\frac{\partial \tau_k^*(t)}{\partial \theta} = \frac{-\rho^2}{(A-\delta)[\rho\theta + 1 + \phi\sigma_C\alpha(\omega)]^2} < 0,$$

$$\frac{\partial \tau_k^*(t)}{\partial \phi} = \frac{-\rho\sigma_C\alpha(\omega)}{(A-\delta)[\rho\theta + 1 + \phi\sigma_C\alpha(\omega)]^2} < 0$$

and

$$\frac{\partial \tau_k^*(t)}{\partial \sigma_C} = \frac{-\rho\phi[\alpha(\omega) - \alpha'(\omega)\omega]}{(A-\delta)[\rho\theta + 1 + \phi\sigma_C\alpha(\omega)]^2}. \tag{A.7}$$

For the function $f(x) \equiv \alpha(x) - \alpha'(x)x$, we have $f'(x) = -\alpha''(x)x > 0$ based on our specification, which hence implies that $f(x)$ is a strictly increasing function of x. Note that $f(0) = \alpha(0) - \alpha'(0)0 = 0$ and we just consider the case corresponding to $x > 0$, thus $f(x) \equiv \alpha(x) - \alpha'(x)x > 0$ for any $x > 0$. So, applying this result to (A.7) produces that $\frac{\partial \tau_k^*(t)}{\partial \sigma_C} < 0$. Moreover, we have

$$\frac{\partial \tau_k^*(t)}{\partial \sigma_E} = \frac{-\rho\phi\alpha'(\omega)(\frac{1}{\gamma} - 1)}{(A-\delta)[\rho\theta + 1 + \phi\sigma_C\alpha(\omega)]^2} < 0,$$

$$\frac{\partial \tau_k^*(t)}{\partial \gamma} = \frac{\rho\phi\alpha'(\omega)\sigma_E}{(A-\delta)[\rho\theta + 1 + \phi\sigma_C\alpha(\omega)]^2\gamma^2} > 0,$$

as well as

$$\frac{\partial \tau_k^*(t)}{\partial \rho} = \frac{1 + \phi\sigma_C\alpha(\omega)}{(A-\delta)[\rho\theta + 1 + \phi\sigma_C\alpha(\omega)]^2} > 0.$$

It follows from (12) that $g^*(t) = As^*(t) - (A-\delta)\tau_k^*(t) - \delta$. Thus, these required results are easily confirmed. □

Proof of Proposition 3.2. For the Bellman equation in Definition 3.2, the FOCs are

$$(A-\delta)\tau_k(t) = \frac{1}{\theta + J_k^{CM}(k(t))k(t)} \tag{A.8}$$

and

$$(1-s(t))A = \frac{(1+\phi)\sigma_C\alpha(\omega)}{\theta + J_k^{CM}(k(t))k(t)}. \tag{A.9}$$

Inserting (A.8) and (A.9) into the above Bellman equation produces

$$\begin{aligned}
&\rho J^{CM}(k(t)) - \frac{1}{2}\sigma^2 k^2(t) J_{kk}^{CM}(k(t)) \\
&\quad = \ln k(t) - \ln[\theta + J_k^{CM}(k(t))k(t)] + J_k^{CM}(k(t))k(t)(A-\delta-n+\sigma^2) \\
&\qquad + (1+\phi)\sigma_C\alpha(\omega)\{\ln[(1+\phi)\sigma_C\alpha(\omega)k(t)] - \ln[\theta + J_k^{CM}(k(t))k(t)]\} \\
&\qquad + \theta\left[A - \frac{(1+\phi)\sigma_C\alpha(\omega)+1}{\theta + J_k^{CM}(k(t))k(t)} - \delta\right] - \frac{J_k^{CM}(k(t))k(t)}{\theta + J_k^{CM}(k(t))k(t)} \\
&\qquad \times [1 + (1+\phi)\sigma_C\alpha(\omega)].
\end{aligned} \tag{A.10}$$

If we put $J^{CM}(k(t)) = C_5 + C_6 \ln k(t)$ for some parameters C_5 and C_6, remaining to be determined, then plugging it in (A.10) can pin down

$$C_6 = \frac{1 + (1+\phi)\sigma_C\alpha(\omega)}{\rho} \tag{A.11}$$

and

$$\begin{aligned}
C_5 = &-\frac{\sigma^2}{2\rho^2}[1 + (1+\phi)\sigma_C\alpha(\omega)] \\
&- \frac{1}{\rho}[1 + (1+\phi)\sigma_C\alpha(\omega)] \ln\left(\theta + \frac{1 + (1+\phi)\sigma_C\alpha(\omega)}{\rho}\right) \\
&+ \frac{(1+\phi)\sigma_C\alpha(\omega)}{\rho} \ln[(1+\phi)\sigma_C\alpha(\omega)] + \frac{\theta(A-\delta)}{\rho} \\
&+ \frac{1 + (1+\phi)\sigma_C\alpha(\omega)}{\rho}\left[\frac{1}{\rho}(A-\delta-n+\sigma^2) - 1\right].
\end{aligned}$$

We, by making use of (A.8), (A.9) and (A.10), obtain the desired results. And along the derived cooperative-equilibrium path, we have $\lim_{t\to\infty} e^{-\rho(t-t_0)} J^{CM}(k(t)) = 0$ almost surely, i.e., the transversality condition is fulfilled almost surely for (13). Since the proof is quite similar to that of Lemma A.3, we thus take it as omitted. □

Proof of Corollary 3.3. First, based on Proposition 3.2 we can get

$$\frac{\partial g^{**}(t)}{\partial \phi} = \frac{-\rho^2 \sigma_C \alpha(\omega)\theta}{[\rho\theta + 1 + (1+\phi)\sigma_C \alpha(\omega)]^2} < 0$$

and

$$\frac{\partial g^{**}(t)}{\partial \rho} = \frac{-[1 + (1+\phi)\sigma_C \alpha(\omega)]^2}{[\rho\theta + 1 + (1+\phi)\sigma_C \alpha(\omega)]^2} < 0.$$

Moreover, we can get

$$\frac{\partial g^{**}(t)}{\partial(\sigma_C \alpha(\omega))} = \frac{-\rho^2 (1+\phi)\theta}{[\rho\theta + 1 + (1+\phi)\sigma_C \alpha(\omega)]^2} < 0.$$

Also, we can show that

$$\frac{\partial(\sigma_C \alpha(\omega))}{\partial \sigma_C} = \alpha(\omega) - \alpha'(\omega)\omega > 0, \quad \frac{\partial(\sigma_C \alpha(\omega))}{\partial \sigma_E} = \alpha'(\omega)\left(\frac{1}{\gamma} - 1\right) > 0,$$

as well as

$$\frac{\partial(\sigma_C \alpha(\omega))}{\partial \gamma} = \alpha'(\omega)\sigma_E\left(-\frac{1}{\gamma^2}\right) < 0$$

by our assumption imposed on $\alpha(\cdot)$. Hence, by using the chain rule of calculus, we have

$$\frac{\partial g^{**}(t)}{\partial \sigma_C} = \underbrace{\frac{\partial g^{**}(t)}{\partial(\sigma_C \alpha(\omega))}}_{<0} \cdot \underbrace{\frac{\partial(\sigma_C \alpha(\omega))}{\partial \sigma_C}}_{>0} < 0.$$

Similarly, we can get $\frac{\partial g^{**}(t)}{\partial \sigma_E} < 0$ as well as $\frac{\partial g^{**}(t)}{\partial \gamma} > 0$. Finally, it is easy to verify that $\frac{\partial g^{**}(t)}{\partial \theta} > 0$ based on the formula of $g^{**}(t)$. □

Proof of Proposition 4.1. It follows from Propositions 3.1 and 3.2 that $s^{**}(t) > s^*(t)$ and $\tau_k^{**}(t) < \tau_k^*(t)$ for any $t \geq t_0$. As a consequence, the first part of the required assertion immediately follows by using (12). Then, we just need to prove the following lemmas. □

Lemma A.5 (Group rationality). *There exists at least one combination*[p] *of search intensity, matching technology, market composition as well as governance type such that* $J^{CM}(k(t)) > J^C(k(t)) + J^G(k(t))$ *along any given trajectory* $\{k(t)\}_{t=t_0}^{\infty}$ *with* $J^{CM}(k(t)), J^C(k(t))$ *and* $J^G(k(t))$ *established in Propositions* 3.2 *and* 3.1.

[p] As is clear soon, we just obtain this limited result because it is almost impossible to show that $\Psi > 1$ without resorting to additional assumptions or restrictions. Importantly, these additional restrictions on parameters are hardly to be economically interpretable, we hence just use numerical results to illustrate the existence of such combinations of parameters. We admit that we cannot provide a full mathematical proof, but we believe that there are sufficiently various combinations of these parameters such that the inequality holds true.

Lemma A.6 (Sub-game consistent solution). *An instantaneous payment at time $\tau \in [t_0, \infty)$ equaling*

$$\eta_i(\tau) = \rho\xi_\tau^{(t_0)i}(\tau, k_\tau^{**}) - \frac{1}{2}\sigma^2(k_\tau^{**})^2\xi_{k_\tau^{**}k_\tau^{**}}^{(t_0)i}(\tau, k_\tau^{**}) - \xi_{k_\tau^{**}}^{(t_0)i}(\tau, k_\tau^{**})k_\tau^{**}\left\{A - \delta - n + \sigma^2 - \frac{\rho[1 + (1+\phi)\sigma_C\alpha(\omega)]}{\rho\theta + 1 + (1+\phi)\sigma_C\alpha(\omega)}\right\},$$

for $i \in \{C, G\}$ and $k_\tau^{**} \in \Xi_\tau^{**}$, yields a sub-game consistent solution for $\Gamma^{CM}(\tau, k_\tau^{**})$.

Proof. It is quite similar to the proof of Theorem 5.8.3 in Yeung and Petrosyan [2006], so we take it as omitted to economize on the space of the paper. □

Lemma A.7. *The Nash bargaining solution/Shapley value is sub-game consistent, and it satisfies individual rationality. Moreover, neither the capitalist nor the politician will unilaterally deviate from cooperation.*

Lemma A.8. *The proportional-distribution imputation is sub-game consistent, and it satisfies individual rationality. Moreover, neither the capitalist nor the politician will unilaterally deviate from cooperation.*

Proof of Lemma A.5. We know that $J^C(k(t)) = C_1 + C_2 \ln k(t)$ with C_1 and C_2 given in the proof of Lemma A.1, $J^G(k(t)) = C_3 + C_4 \ln k(t)$ with C_3 and C_4 given in the proof of Lemma A.2, and $J^{CM}(k(t)) = C_5 + C_6 \ln k(t)$ with C_5 and C_6 given in the proof of Proposition 3.2. First, it follows from (A.3), (A.6) and (A.11) that $C_2 + C_4 = C_6$. To prove this lemma, we just need to verify that $C_5 > C_1 + C_3$ holds true. In fact, $C_5 - (C_1 + C_3) > 0$ is equivalent to

$$\Psi \equiv \exp\left\{\frac{\rho\theta[\rho\theta + 1 + \phi\sigma_C\alpha(\omega)] + \sigma_C\alpha(\omega)}{\rho\theta + 1 + \phi\sigma_C\alpha(\omega)}\right\} \times \left[\frac{\rho\theta + 1 + \phi\sigma_C\alpha(\omega)}{\rho\theta + 1 + (1+\phi)\sigma_C\alpha(\omega)}\right]\left(\frac{(1+\phi)\sigma_C\alpha(\omega)}{\rho\theta + 1 + (1+\phi)\sigma_C\alpha(\omega)}\right)^{(1+\phi)\sigma_C\alpha(\omega)} > 1.$$

If we set the following numerical values

$$\begin{cases} (1+\phi)\sigma_C\alpha(\omega) = \dfrac{1}{2} \\ \phi\sigma_C\alpha(\omega) = \dfrac{1}{6} \\ \rho\theta = \dfrac{5}{6}. \end{cases} \tag{A.12}$$

Then, $\Psi = \exp(1) \times (\frac{6}{7})(\frac{3}{14})^{\frac{1}{2}} > 1 \Leftrightarrow 1 > \ln(\frac{7}{6}) + \frac{1}{2}\ln(\frac{14}{3})$. Since $\ln(\frac{7}{6}) + \frac{1}{2}\ln(\frac{14}{3}) < 0.93 < 1$, the required assertion follows. Furthermore, we consider another numerical example as

$$\begin{cases} (1+\phi)\sigma_C\alpha(\omega) = \dfrac{1}{3} \\ \phi\sigma_C\alpha(\omega) = \dfrac{1}{6} \\ \rho\theta = \dfrac{5}{6}. \end{cases} \tag{A.13}$$

Substituting this into the formula of Ψ gives rise to $\Psi = \exp(\frac{11}{12}) \times (\frac{12}{13})(\frac{2}{13})^{\frac{1}{3}} > 1 \Leftrightarrow \frac{11}{12} > \ln(\frac{13}{12}) + \frac{1}{3}\ln(\frac{13}{2})$. Since $\frac{11}{12} \approx 0.916666, \ln(\frac{13}{12}) + \frac{1}{3}\ln(\frac{13}{2}) < 0.71 < 1$, we obtain the required assertion. To summarize, $\Psi > 1$ holds true for (A.12) and (A.13), i.e., $C_5 > C_1 + C_3$ is verified under both (A.12) and (A.13), which are two reasonable cases for the present model. As is obvious, even though it is mathematically difficult to obtain $C_5 > C_1+C_3$ for any given parameter combinations, there exist sufficiently many numerical examples such that $C_5 > C_1 + C_3$ holds true, and we leave more detailed computations and verifications to interested readers to economize on the space of paper. □

Proof of Lemma A.7. Note that the equilibrium feedback strategies in (10), (11) and (13) are Markovian in the sense that they just depend on current state and current time. Hence one can readily observe by comparing the Bellman equations in Definitions 3.1 and 3.2 for different values of $\tau \in [t_0, \infty)$ that

$$\begin{pmatrix} s^{*(t_0)}(t, k_t^*) \\ \tau_k^{*(t_0)}(t, k_t^*) \end{pmatrix} = \begin{pmatrix} s^{*(\tau)}(t, k_t^*) \\ \tau_k^{*(\tau)}(t, k_t^*) \end{pmatrix}$$

for $t_0 \le \tau \le t < \infty$ and $k_t^* \equiv k^*(t)$, the noncooperative equilibrium trajectory of capital per capita determined by Proposition 3.1 at time t, and similarly

$$\begin{pmatrix} s^{**(t_0)}(t, k_t^{**}) \\ \tau_k^{**(t_0)}(t, k_t^{**}) \end{pmatrix} = \begin{pmatrix} s^{**(\tau)}(t, k_t^{**}) \\ \tau_k^{**(\tau)}(t, k_t^{**}) \end{pmatrix}$$

for $t_0 \le \tau \le t < \infty$ and $k^{**}(t) \equiv k_t^{**} \in \Xi_t^{**}$, the cooperative-equilibrium trajectory of capital per capita determined by (14). Moreover, along the noncooperative trajectory, namely $\{k_t^*\}_{t=t_0}^{\infty}$, one can obtain

$$\begin{aligned} J^{(t_0)C}(\tau, k_\tau^*) &\equiv \mathbb{E}_{t_0}\left[\int_\tau^\infty e^{-\rho(\lambda - t_0)}\sigma_C\alpha(\omega)\ln((1 - s^{*(t_0)}(\lambda, k_\lambda^*))Ak_\lambda^*)d\lambda \,\middle|\, k^*(\tau) = k_\tau^*\right] \\ &= \mathbb{E}_{t_0}\left[\int_\tau^\infty e^{-\rho(\lambda - \tau)}\sigma_C\alpha(\omega)\ln((1 - s^{*(\tau)}(\lambda, k_\lambda^*))Ak_\lambda^*)d\lambda \,\middle|\, k^*(\tau) = k_\tau^*\right] \\ &\quad \times e^{-\rho(\tau - t_0)} \end{aligned}$$

$$= \mathbb{E}_\tau\left[\int_\tau^\infty e^{-\rho(\lambda-\tau)}\sigma_C\alpha(\omega)\ln((1-s^{*(\tau)}(\lambda,k_\lambda^*))Ak_\lambda^*)d\lambda \,\middle|\, k^*(\tau)=k_\tau^*\right]$$
$$\times e^{-\rho(\tau-t_0)}$$
$$\equiv e^{-\rho(\tau-t_0)}J^{(\tau)C}(\tau,k_\tau^*),$$

where $J^{(t_0)C}(\tau,k_\tau^*)$ measures the expected present value of the capitalist's payoff in the time interval $[\tau,\infty)$ when $k^*(\tau)=k_\tau^*$ and the game starts from time $t_0\le\tau$. For the politician, we can similarly obtain $J^{(t_0)G}(\tau,k_\tau^*)=e^{-\rho(\tau-t_0)}J^{(\tau)G}(\tau,k_\tau^*)$, in which $J^{(t_0)G}(\tau,k_\tau^*)$ measures the expected present value of the politician's payoff in the time interval $[\tau,\infty)$ when $k^*(\tau)=k_\tau^*$ and the game starts from time $t_0\le\tau$. Similarly, for the cooperative game $\Gamma^{CM}(t_0,k_0)$, we can obtain $J^{(t_0)CM}(\tau,k_\tau^{**})=e^{-\rho(\tau-t_0)}J^{(\tau)CM}(\tau,k_\tau^{**})$, where $J^{(t_0)CM}(\tau,k_\tau^{**})$ measures the expected present value of the cooperative payoff in the time interval $[\tau,\infty)$ when $k^{**}(\tau)=k_\tau^{**}$ and the game starts from time $t_0\le\tau$.

Now, we can establish the Nash bargaining solution/Shapley value along the cooperative-equilibrium trajectory $\{k_\tau^{**}\}_{\tau=t_0}^\infty$ as

$$\xi^{(t_0)i}(\tau,k_\tau^{**})$$
$$= J^{(t_0)i}(\tau,k_\tau^{**})+\frac{1}{2}\left[J^{(t_0)CM}(\tau,k_\tau^{**})-\sum_{j\in\{C,G\}}J^{(t_0)j}(\tau,k_\tau^{**})\right]$$
$$= e^{-\rho(\tau-t_0)}\left\{J^{(\tau)i}(\tau,k_\tau^{**})+\frac{1}{2}\left[J^{(\tau)CM}(\tau,k_\tau^{**})-\sum_{j\in\{C,G\}}J^{(\tau)j}(\tau,k_\tau^{**})\right]\right\}$$
$$= e^{-\rho(\tau-t_0)}\xi^{(\tau)i}(\tau,k_\tau^{**}),$$

for $i\in\{C,G\}, t_0\le\tau<\infty$ and $k_\tau^{**}\in\Xi_\tau^{**}$. Moreover, individual rationality immediately follows from the group rationality proved by Lemma A.5 and also Definitions 4.1 and 4.3.

At date $t\ge t_0$, if no one deviates from cooperation, the payoff allocation is

$$\xi^i(k^{**}(t)) = J^i(k^{**}(t))+\frac{1}{2}\left[J^{CM}(k^{**}(t))-\sum_{j\in\{C,G\}}J^j(k^{**}(t))\right],$$

for $i\in\{C,G\}$. It follows from Lemma A.5 that $\xi^i(k^{**}(t))>J^i(k^{**}(t))$ for $i\in\{C,G\}$. First, if the capitalist unilaterally deviates from cooperation, he gets payoff $J^C(\hat{k}(t))=C_1+C_2\ln\hat{k}(t)$ with C_1 and C_2 given in the proof of Lemma A.1, and $\hat{k}(t)$ is a solution of

$$d\hat{k}(t)=\left(A-\delta-n+\sigma^2-\frac{\rho[\rho\theta+2+(1+\phi)\sigma_C\alpha(\omega)]}{\rho\theta+1+(1+\phi)\sigma_C\alpha(\omega)}\right)\hat{k}(t)dt-\sigma\hat{k}(t)dB(t).$$

We know that $J^C(k^{**}(t))=C_1+C_2\ln k^{**}(t)$ with the same C_1 and C_2 except that $k^{**}(t)$ is given by (14). As it is easy to see that $k^{**}(t)>\hat{k}(t)$, we arrive at

$J^C(\hat{k}(t)) < J^C(k^{**}(t)) < \xi^C(k^{**}(t))$. Second, if the politician unilaterally deviates from cooperation, he will get payoff $J^G(\tilde{k}(t)) = C_3 + C_4 \ln \tilde{k}(t)$ with C_3 and C_4 given in the proof of Lemma A.2, and $\tilde{k}(t)$ is a solution of

$$d\tilde{k}(t) = \left(A - \delta - n + \sigma^2 - \frac{\rho\left[\frac{\rho\theta+1+(1+\phi)\sigma_C\alpha(\omega)}{\rho\theta+1+\phi\sigma_C\alpha(\omega)} + (1+\phi)\sigma_C\alpha(\omega)\right]}{\rho\theta + 1 + (1+\phi)\sigma_C\alpha(\omega)}\right) \times \tilde{k}(t)dt - \sigma\tilde{k}(t)dB(t).$$

Since $J^G(k^{**}(t)) = C_3 + C_4 \ln k^{**}(t)$ with the same C_3 and C_4 except that $k^{**}(t)$ is given by (14), we have $J^G(\tilde{k}(t)) < J^G(k^{**}(t)) < \xi^G(k^{**}(t))$ because $k^{**}(t) > \tilde{k}(t)$. To sum up, unilateral deviation always results in less payoff, hence neither the capitalist nor the politician will unilaterally deviate from cooperation.

Proof of Lemma A.8. In fact, the proof is quite similar to that of Lemma A.7. That is, given the Markovian property we can get the following equalities:

$$\begin{aligned} J^{(t_0)C}(\tau, k^{**}_\tau) &= e^{-\rho(\tau - t_0)} J^{(\tau)C}(\tau, k^{**}_\tau), \\ J^{(t_0)G}(\tau, k^{**}_\tau) &= e^{-\rho(\tau - t_0)} J^{(\tau)G}(\tau, k^{**}_\tau), \\ J^{(t_0)CM}(\tau, k^{**}_\tau) &= e^{-\rho(\tau - t_0)} J^{(\tau)CM}(\tau, k^{**}_\tau), \end{aligned}$$

for $t_0 \le \tau < \infty$ and $k^{**}_\tau \in \Xi^{**}_\tau$. Thus, we see that

$$\begin{aligned} \xi^{(t_0)i}(\tau, k^{**}_\tau) &= \frac{J^{(t_0)i}(\tau, k^{**}_\tau)}{\sum_{j\in\{C,G\}} J^{(t_0)j}(\tau, k^{**}_\tau)} J^{(t_0)CM}(\tau, k^{**}_\tau) \\ &= \frac{e^{-\rho(\tau-t_0)} J^{(\tau)i}(\tau, k^{**}_\tau)}{\sum_{j\in\{C,G\}} e^{-\rho(\tau-t_0)} J^{(\tau)j}(\tau, k^{**}_\tau)} e^{-\rho(\tau-t_0)} J^{(\tau)CM}(\tau, k^{**}_\tau) \\ &= e^{-\rho(\tau-t_0)} \left[\frac{J^{(\tau)i}(\tau, k^{**}_\tau)}{\sum_{j\in\{C,G\}} J^{(\tau)j}(\tau, k^{**}_\tau)} J^{(\tau)CM}(\tau, k^{**}_\tau)\right] \\ &= e^{-\rho(\tau-t_0)} \xi^{(\tau)i}(\tau, k^{**}_\tau), \end{aligned}$$

for $i \in \{C, G\}$, $t_0 \le \tau < \infty$ and $k^{**}_\tau \in \Xi^{**}_\tau$. Additionally, one can verify individual rationality by directly applying Lemma A.5, Definitions 4.1 and 4.4. □

Since by Lemma A.5 the payoff allocation under cooperation satisfies

$$\xi^i(k^{**}(t)) = \frac{J^i(k^{**}(t))}{\sum_{j\in\{C,G\}} J^j(k^{**}(t))} J^{CM}(k^{**}(t)) > J^i(k^{**}(t)),$$

for $i \in \{C, G\}$, no one will unilaterally deviate from cooperation following the same reason shown in the proof of Lemma A.7. □

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References

Acemoglu, D., Golosov, M. and Tsyvinski, A. [2008] Markets versus governments, *J. Monet. Econ.* **55**, 159–189.

Acemoglu, D., Johnson, S. and Robinson, J. A. [2005] Institutions as a fundamental cause of development, in *Handbook of Economic Growth*, eds. Philippe Aghion and Steven Durlauf, (Amsterdam, Holland, Elsevier), pp. 386–472.

Acemoglu, D. and Robinson, J. A. [2012] *Why Nations Fail*: *The Origins of Power*, *Prosperity*, *and Poverty* (Crown Business, New York).

Aidt, T. S. [2003] Economic analysis of corruption: A survey, *Econ. J.* **113**, F632–F652.

Barro, R. J. [1973] The control of politicians: An economic model, *Public Choice* **14**, 19–42.

Baumol, W. J., Litan, R. E. and Schramm, C. J. [2007] *Good Capitalism*, *Bad Capitalism*, *and the Economics of Growth and Prosperity* (Yale University Press).

Buchanan, J. M. and Tullock, G. [1962] *The Calculus of Consent* (University of Michigan Press, Michigan).

Coase, R. H. and Wang, N. [2012] *How China Became Capitalist* (Palgrave Macmillan, New York).

Ferejohn, J. [1986] Incumbent performance and electoral control, *Public Choice* **50**, 5–25.

Frye, T. and Shleifer, A. [1997] The invisible hand and the grabbing hand, *Am. Econ. Rev.* **87**, 354–358.

He, Z. and Krishnamurthy, A. [2012] A model of capital and crises, *Rev. Econ. Stud.* **79**, 735–777.

Huang, Y. [2008] *Capitalism with Chinese Characteristics*: *Entrepreneurship and the State* (Cambridge University Press, New York).

Hurwicz, L. [1996] Institutions as families of game-forms, *Jpn. Econ. Rev.* **47**, 13–132.

Kaitala, V. and Pohjola, M. [1990] Economic development and agreeable redistribution in capitalism: Efficient game equilibria in a two-class neoclassical growth model. *Int. Econ. Rev.* **31**, 421–438.

Lancaster, K. [1973] The dynamic inefficiency of capitalism, *J. Political Econ.* **81**, 1092–1109.

Leong, C. K. and Huang, W. [2010] A stochastic differential game of capitalism, *J. Math. Econ.* **46**, 552–561.

Mauro, P. [1995] Corruption and growth, *Q. J. Econ.* **110**, 681–712.

Merton, R. C. [1975] An asymptotic theory of growth under uncertainty, *Rev. Econ. Stud.* **42**, 375–393.

Montinola, G., Qian, Y. and Weingast, B. R. [1995] Federalism, Chinese style: The political basis for economic success in China, *World Politics*, **48**, 50–81.

Murphy, K. M., Shleifer, A. and Vishny, R. W. [1993] Why is rent-seeking so costly to growth? *Am. Econ. Rev.* **83**, 409–414.

North, D. C. [1990] *Institutions*, *Institutional Change and Economic Performance* (Cambridge University Press, New York).

Øksendal, B. and Sulem, A. [2009] *Applied Stochastic Control of Jump Diffusions*, 3rd edition (Springer).
Olson, M. C. [2000] *Power and Prosperity*: *Outgrowing Communist and Capitalist Dictatorships* (Basic Books, New York).
Petrongolo, B. and Pissarides, C. A. [2001] Looking into the black box: A survey of the matching function, *J. Econ. Lit.* **39**, 390–431.
Roemer, J. E. [2010] Kantian equilibrium, *Scandinavian J. Econ.* **112**, 1–24.
Seierstad, A. [1993] The dynamic efficiency of capitalism, *J. Econ. Dyn. Control* **17**, 877–886.
Tian, G. [2000] Property rights and the nature of Chinese collective enterprises, *J. Comp. Econ.* **28**, 247–268.
Tian, G. [2001] A theory of ownership arrangements and smooth transition to a free market economy, *J. Inst. Theor. Econ.* **157**, 380–412.
Turnovsky, S. J. [2000] Fiscal policy, elastic labor supply, and endogenous growth, *J. Monetary Econ.* **45**, 185–210.
Xu, C. [2011] The fundamental institutions of China's reforms and development, *J. Econ. Lit.* **49**, 1076–1151.
Yared, P. [2010] Politicians, taxes and debt, *Rev. Econ. Stud.* **77**, 806–840.
Yeung, D. W. K. and Petrosyan, L. [2006] *Cooperative Stochastic Differential Games* (Springer, New York).