Voting on Redistribution with Retaining Top Talent*

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March 27, 2023

Abstract

We consider majority voting over selfishly optimal nonlinear income tax schedules proposed by a continuum of individuals who differ in their privately observable skills. A tax schedule is feasible if it respects the participation constraint for the highest skill type ("top talent") and the usual participation, incentive compatibility, and government budget constraints. We provide the conditions under which the median skill type's proposal is a Condorcet winner, which favors the median voter at the expense of both rich and poor. Perhaps surprisingly, the redistributive motive of the median voter will, ceteris paribus, be reinforced by tightening the usual participation constraint; however, it can be weakened by tightening the participation constraint for top talent. Compared with the case that ignores these participation constraints, adding them results in marginal tax rates that are higher in the lower part of the distribution but are lower in the upper part, therefore being less progressive.

Keywords: Nonlinear taxation; income redistribution; selfishly optimal tax scheme; citizen-candidate model; majority voting; the median voter.

JEL classification codes: D70, D82, H20.

1 Introduction

Determining income redistribution via a tax-transfer system is essentially a political economy issue. Different income groups in a society exert political influence on the structure of tax and transfer systems through alternative channels. The Director's Law (Stigler 1970) suggests that under some plausible conditions, the middle-income classes represent the dominant group and therefore can lower their tax burdens at the expense of both low- and high-income groups. Recent empirical evidence shows that a strong middle class plays a decisive role in redistribution and that most democracies implement the redistribution preferred by the middle classes.[1] Moreover, with the deepening of economic globalization, we observe the rise of political extremes from both the right and the left, leading to deeply divided societies (Schwab and Vanham 2021).[2]

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*Declarations of interest: none.

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1See, e.g., Agranov and Palfrey (2015), Corneo and Neher (2015), Gründler and Köllner (2017), Jacobs, Jongen, and Zoutman (2017), and Bierbrauer, Boyer, and Peichl (2021).

2Typically, in the European Union, we see the rise of extreme right parties amid a backdrop of labor migrants and concerns about the implications for the sustainability of the welfare state. More extremely, the former UK Prime Minister Liz Truss proposed removing the top tier of income tax, effectively cutting taxes for citizens making more than $ 168,000 per year. In contrast, in the United States (US), there was a debate on the desirable

On the one hand, as the global competition for capital and talent grows, the upper classes can resort to their exit option of emigrating in the face of heavy tax burdens, leading to the flight of capital and wealth and an exodus of skilled professionals. The wealthy have high degrees of international mobility,[3] and so their political influence can be exerted through the threat of voting with their feet. Indeed, Kleven, Landais, and Saez (2013), Kleven, Landais, Saez, and Schultz (2014), and Akcigit, Baslandze, and Stantcheva (2016) estimate large migration elasticities (over 1) concerning the income tax rate for highly skilled individuals.[4]

On the other hand, income and wealth inequalities have increased in most countries, of which the typical examples are the US, China, and India.[5] Thus, the situation facing the working poor at the bottom of the global value chain worsens.[6] Workers often feel they have been treated unfairly,[7] and may exert their political influence by participating in riots and violent protests (Passarelli and Tabellini 2017, Campante and Chor 2012, Alesina and Perotti 1996, and Parvin 1973), such as the Arab Spring and the Occupy Wall Street movement initiated in 2010 and 2011, respectively.[8]

Given such observations, we address how the *usual* participation constraint[9] (the participation constraint for the lowest-income group) and an additional participation constraint for the highest-income group would affect income redistribution in a voting equilibrium. To this end, we consider pairwise majority voting over selfishly optimal nonlinear income tax schedules. Such schedules are proposed by a continuum of individuals who share the same quasilinear-in-consumption preferences for consumption and labor supply;[10] however, they differ in their

MTRs levied on top earners during the 2020 US presidential elections — the proposals of Democratic Party candidates, such as Senators Bernie Sanders and Elizabeth Warren. Additionally, the 2021 Biden tax plan would raise the top marginal income tax rate from 37 percent to 39.6 percent and increase the federal corporate income tax rate from 21 percent to 28 percent (see https://taxfoundation.org/biden-tax-plan-tracker/).

[3]See the evidence reported by Docquier and Marfouk (2006), Wildasin (2006), OECD (2008), Boeri, Brucker, Doquier, and Rapoport (2012), and Papademetriou and Sumption (2013).

[4]See also the recent review article, Kleven, Landais, Muñoz, and Stantcheva (2020), as well as the references cited therein. Additionally, recent empirical evidence reveals that a significant size of tax evasion or tax avoidance is performed by the wealthy, see Piketty, Saez, and Stantcheva (2014), Seim (2017), Alstadsæter, Johannesen, and Zucman (2019), and Brülhart, Gruber, Krapf, and Schmidheiny (2022).

[5]See Atkinson, Piketty, and Saez (2011), Stiglitz (2012), Piketty (2014), Xie and Zhou (2014), Chetty, Grusky, Hell, Hendren, Manduca, and Narang (2017), Chetty, Hendren, Jones, and Porter (2020), Fan, Yi, and Zhang (2021), and Blanchet, Chancel, and Gethin (2022). See also Schwab and Vanham (2021) and the related references cited therein.

[6]As noticed by Stiglitz (2018), the demand for unskilled labor in advanced countries declines with the opening of trade and with that, its wage, as workers' bargaining power is weakened as firms threaten to relocate if workers do not accept lower wages or worse working conditions. Many jobs held by workers with the lowest skills are at the bottom of the global value chain and are characterized as low-paid, insecure, and dangerous (Gereffi and Luo 2014).

[7]One should be noted that some subjective perceptions, or even irrational factors, may impose significant influence on individual preferences regarding redistribution, such as political ideologies, beliefs and representations about social mobility, and fairness concerns (e.g., Dixit and Londregan 1998; Benabou and Ok 2001; Alesina and Angeletos 2005; Benabou and Tirole 2006; Höchtl, Sausgruber, and Tyran 2012; Alesina, Miano, and Stantcheva 2018; and Alesina, Murard, and Rapoport 2021).

[8]Since the influential work of Karl Marx, it is well recognized that high levels of income inequality would lead to social-political instability and even political revolution that leads to radical redistribution, a concern highlighted recently by Stiglitz (2012), Piketty (2014), and Schwab and Vanham (2021).

[9]As a caveat, this term applies only to the case with type-independent participation constraints. Because individual indirect utility under quasilinear-in-consumption preferences is monotonically nondecreasing in skill type, the participation constraints for all skill types are trivially fulfilled as long as the participation constraint for the lowest skill type is satisfied. More importantly, doing so leads the formal analysis to a nontrivial technical level that is still mathematically manageable and gives rise to sharp predictions concerning income redistribution under plausible conditions. Considering type-dependent participation constraints with type-dependent reservation utilities for a continuum of skill types can make the formal analysis highly untractable.

[10]This kind of individual preferences is often used in the related literature, see, for instance, the seminal paper of Diamond (1998).

privately observable skills (Mirrlees 1971).[11] A tax schedule is *feasible* if it satisfies the usual participation, incentive compatibility, and government budget constraints and simultaneously respects the participation constraint for the highest-skilled individuals ("top talent"). The top talent's reservation utility can be interpreted as the highest possible utility they would attain if moving to another jurisdiction or country. The bottom skill type's reservation utility can be referred to as the highest possible utility they would realize if engaging in protests against the status quo. Assume that the top talent's reservation utility is greater than the bottom type's; therefore, both constraints play a nontrivial role. An individual of any given skill type can propose a feasible tax schedule for all skill types as if he were a dictator, thus to maximize the utility of his own type. All such tax schedules comprise a set of feasible candidates, over which the majority rule is used to select the winner.

We first derive and characterize the selfishly optimal income schedule proposed by any given skill type, and then we identify the conditions under which the proposal of the median skill type (the median voter) is the Condorcet winner. The Condorcet-winning tax schedule has four main features. (1) It is consistent with the Director's Law, aiming to redistribute resources from the rich and poor toward the middle. (2) The Condorcet-winning tax schedule coincides with the schedule obtained under a maxi-max social welfare function for low skills in the lower part of the skill distribution. In contrast, for high skills in the upper part of the distribution, it coincides with the schedule obtained under a maxi-min social welfare function. (3) To guarantee incentive compatibility, there must be an endogenously determined bunching region of intermediate skill types, including the median skill type. In this way, all skill types within this region receive the same income allocation, which builds a *bridge* connecting the maxi-max and maxi-min income schedules.[12] (4) These two participation constraints are always binding in the selfishly optimal allocation proposed by the median voter.

From an analytical perspective, the contribution of this article has been to identify a set of conditions under which individual preferences are single-peaked on the continuum of skill types; thus, Black (1948)'s median voter theorem can be applied to establish the existence of a Condorcet winner. To obtain the single-peakedness property, we must first provide two sets of comparative statics concerning the skill type of any given proposer. The first set concerns the Lagrange multipliers: a marginal increase in the proposer's skill type decreases the Lagrange multiplier associated with the participation constraint for top talent; however, an increase in the Lagrange multiplier is associated with the usual participation constraint. These two Lagrange multipliers measure the marginal utilities imposed on the proposer induced by a given reduction in the reservation utilities of the top talent and the bottom type, respectively. As the skill type of a proposer increases, it becomes further away from the bottom type and closer to the top type; thus, the participation threat from the top talent becomes relatively weaker, and participation threat from the bottom type becomes stronger. This separation occurs because the top type is now more likely to face the same income schedule as the proposer. The second set involves

[11] We provide two comments concerning the current economic environment. First, Höchtl, Sausgruber, and Tyran (2012) provide experimental evidence that redistribution outcomes could appear as if all voters were exclusively motivated by self-interest. Indeed, other empirical evidence also reveals that self-interest is an essential motivation behind the demand for redistribution, such as Fong (2001) and Sznycer, Seal, Sell, Lim, Porat, Shalvi, Halperin, Cosmides, and Tooby (2017). Second, it would be empirically reasonable to eliminate the income effect on taxable incomes (Gruber and Saez 2002); therefore, adopting quasilinear-in-consumption preferences is not only analytically tractable for establishing single-peakedness of individual preferences.

[12] Because the graph of the maxi-min income schedule lies below the corresponding graph showing the maxi-max income schedule, the sufficient condition for incentive compatibility is violated — namely, there is a downward jump discontinuity at the proposer's skill type in the resulting income allocation; therefore, the income allocation is no longer monotonically nondecreasing in skill level. Therefore, to yield a truth-telling allocation, we must technically iron the income schedule by building a bridge over a bunching region containing the proposer's skill type to connect the lower and upper parts of the selfishly optimal income schedule under consideration. These observations are valid regardless of whether these two participation constraints have been considered.

the two skill types determining the endogenous bunching region. A marginal increase in the proposer's skill type results in an increase in these two skill types; an increase in the proposer's skill type leads to a bridge with a higher income level in the proposer's selfishly optimal income schedule. This occurrence is intuitively appealing because the bunching region contains the proposer's skill type; therefore, those types bunched with the proposer must receive a higher income as the proposer's skill type increases.

Furthermore, these two sets of comparative statics rely on some additional conditions, mainly comprising four parts. We show that experimentally tightening the participation constraint for top talent will, ceteris paribus, benefit both low and high skills, increasing the pre-tax incomes they receive. The first part is that doing so must be (weakly) preferable to a specific high-skill type (the highest skill type bunched with the median type) than to a certain low-skill type (the lowest skill type bunched with the median type). The second part is somewhat analogous to the first part. We show that tightening the usual participation constraint will, ceteris paribus, hurt both low- and high-skill types, reducing the pre-tax incomes they receive; thus, the second part is that doing so must be (weakly) less harmful to this low-skill type than to this high-skill type. These two parts are intuitively appealing. Take the first part as an example. The selfishly optimal income schedule coincides with the maxi-max schedule for low skills and coincides with the maxi-min schedule for high skills; therefore, the countervailing incentive provided by the participation constraint for top talent must be more relevant to the incentive under a maxi-min objective than to the incentive under a maxi-max objective. So, tightening this constraint must be (weakly) more beneficial to high skills than to low skills, other things equal.

Additionally, we show that tightening the participation constraint for top talent will, ceteris paribus, increase the indirect utility of top talent (positive effect) and reduce the indirect utility of the bottom type (negative effect). The third part is that the positive effect on the top talent is sufficiently larger than the negative effect on the bottom type for this tightening of this constraint. The fourth part relates to an experimental tightening of the usual participation constraint. We show that tightening the usual participation constraint will, ceteris paribus, reduce the indirect utility of top talent (negative effect) and increase the indirect utility of bottom type (positive effect). The fourth part is that the positive effect on the bottom type is sufficiently larger than the corresponding negative effect on the top talent.

We further characterize the equilibrium tax schedule regarding MTRs from three dimensions.

First, compared with the case in which these two participation constraints are ignored, all else fixed, adding these constraints results in higher MTRs for low skills and lower MTRs for high skills, therefore being less progressive. Without these two constraints, the Condorcet-winning tax schedule yields MTRs that are negative for low skills, positive for high skills, and zero at the endpoints of skill distribution (Brett and Weymark 2017). However, the current tax schedule features a positive MTR at the bottom, a negative MTR at the top, a threshold skill level below the median skill type (such that types lower than this threshold face positive MTRs), and another threshold above the median skill type (such that types higher than this threshold face negative MTRs).

Second, ceteris paribus, if the usual participation constraint increases in stringency — namely, in the tax design problem for a proposer of any skill type, the shadow price associated with this constraint increases — then both low and high skills face higher MTRs. In contrast, they face lower MTRs as the participation constraint for top talent increases in stringency.

Third, if the shadow price of the usual participation constraint is sufficiently larger than that of the participation constraint for top talent, then a positive joint effect of (experimentally) tightening these two constraints on the MTRs results in both the lower and upper parts of the distribution. If the usual participation constraint is much more binding than the participation constraint for top talent, the effect of increasing MTRs induced by tightening the usual participation constraint outweighs the effect of reducing MTRs induced by tightening the participation constraint for top talent, generating a positive joint effect on the MTRs when tightening these two constraints simultaneously.

These new features of income redistribution are owing to incorporating these two participation constraints. As mentioned, incomes allocated to the upper part of the distribution are determined by a maxi-min objective, while incomes allocated to the lower part are determined by a maxi-max objective. The graph of the maxi-min income schedule lies below the corresponding graph of the maxi-max income schedule. Without such constraints, the motive for downward redistribution under the maxi-min objective yields positive tax rates for high skills, with the downward incentive-compatibility constraints satisfied. Moreover, the motive for upward redistribution under the maxi-max objective yields negative tax rates and wage subsidies for low skills, providing that the upward incentive-compatibility constraints can be fulfilled to prevent individuals with lower types from mimicking those with higher types.

Using these observations, we can identify the modifications caused by these two participation constraints from three dimensions.

First, imposing only the usual participation constraint will not (essentially) alter the incentive provided by the maxi-min objective in the upper part of the distribution; however, this additional constraint can produce a countervailing incentive to the incentive provided by the maxi-max objective in the lower part. As such, while the tax rates for high skills do not change, the (negative) tax rates facing low skills increase. Nevertheless, because the resulting maxi-max MTRs are still lower than the corresponding maxi-min MTRs; therefore, the graph of the maxi-max income schedule still lies above the graph of the maxi-min income schedule, and this countervailing incentive cannot completely overpower the desire for upward redistribution.

Second, if only the participation constraint for top talent is added, the incentive provided by the maxi-max objective in the lower part of the distribution does not change, but it produces a countervailing incentive to the incentive provided by the maxi-min objective in the upper part. This countervailing incentive can reduce the positive tax rates facing high skills. Again, this constraint cannot completely overpower the desire for downward redistribution because the resulting maxi-min MTRs are still higher than the corresponding maxi-max MTRs.

Lastly, under both participation constraints, the countervailing incentives mentioned above now work in tandem for shaping the MTRs in both the lower and upper parts of the distribution. While their impacts emphasized in the previous two points carry over to the present situation, the effect of adding the usual participation constraint on the MTRs facing low skills is weakened by the joint consideration of the participation constraint for top talent. Similarly, the effect of adding the participation constraint for top talent on the MTRs facing high skills is weakened by the joint consideration of the usual participation constraint. Departing from cases considered in isolation, imposing the usual participation constraint also increases the tax rates facing highly skilled workers, and imposing the participation constraint for top talent also reduces the tax rates facing low skills. The graphs of the resulting income schedules under maxi-min and maxi-max objectives, respectively, become closer than before, although the resulting maxi-max MTRs are still smaller than the corresponding maxi-min MTRs. Therefore, for a proposer of any given skill type, his desire to move resources away from other skill types toward his own type is constrained in magnitude yet essentially unaltered in the face of these two participation constraints.

The remainder of this article is organized as follows. The next section reviews the related literature, section 3 describes the model economy, and section 4 derives selfishly optimal tax schedules and establishes the existence of a Condorcet winner under majority voting. Section 5 addresses the impact of considering these two participation constraints on equilibrium income redistribution. Section 6 concludes. The proofs of our results are presented in Appendix A, whereas Appendix B offers an analysis of the reference case with only the participation constraint for top talent, together with the incentive compatibility and government budget constraints.

2 Literature Review

We now discuss the relationship of this article to the related literature; our paper mainly contributes to the literature studying income redistribution determined by majority voting over selfishly optimal nonlinear income tax schedules, such as Röell (2012), Bohn and Stuart (2013), Brett and Weymark (2016, 2017, 2020), Dai (2020), and Dai and Tian (2023). Among these articles, our model is closely related to that of Brett and Weymark (2016), in line with Röell (2012) and Bohn and Stuart (2013), which imposes a participation constraint for the lowest skill type, namely, only the usual participation constraint has been considered. Considering the novel participation constraint for top talent and the usual participation constraint, we enrich the analysis of Brett and Weymark (2016) in the following three aspects.

First, in terms of identifying plausible conditions under which individual preferences are single-peaked on the set of skill types, our formal analysis is technically more involved than the extant research. Second, our model represents a better approximation of reality. Given the stylized fact that the highest (rather than the lowest) skilled individuals are equipped with high degrees of geographic mobility, the top talent's reservation utility must be greater than the bottom. Therefore, our model can capture potentially different implications of these two sorts of participation constraints for income redistribution through a politically democratic process. Finally, several novel observations arise from considering these two constraints jointly. For instance, if only the usual participation constraint is considered, then only the tax schedule in the lower part of the distribution is affected. Conversely, if only the participation constraint for top talent is considered, then only the tax schedule in the upper part of the distribution is affected. However, if both constraints work in tandem, they impose nontrivial influences on the tax schedule over the entire distribution.

Our paper also contributes to the substantial literature that examines the impact of labor mobility and the interjurisdictional tax competition that it induces on income redistribution, such as Mirrlees (1982), Bhagwati and Hamada (1982), Epple and Romer (1991), Hindriks (2001), Hamilton and Pestieau (2005), Wilson (2009), Simula and Trannoy (2010, 2012), Gordon and Cullen (2012), Morelli, Yang, and Ye (2012), Piketty and Saez (2013), Bierbrauer, Brett, and Weymark (2013), Lehmann, Simula, and Trannoy (2014), Lipatov and Weichenrieder (2015), Blumkin, Sadka, and Shem-Tov (2015), Dai, Gao, and Tian (2020), and Dai and Tian (2023). Imposing these two participation constraints yields negative MTRs facing some very highly skilled individuals and positive MTRs facing some very lowly-skilled individuals. This finding is reminiscent of Bierbrauer, Brett, and Weymark (2013), who consider a different model with strategic nonlinear income tax competition induced by perfect labor mobility, showing the most highly-skilled individuals can potentially receive a net transfer funded by taxes on lower-skilled individuals in equilibrium. Moreover, instead of showing that large migration elasticities could decrease significantly the ability of a national government to tax its top income earners, as demonstrated by Piketty and Saez (2013), Lehmann, Simula, and Trannoy (2014), and Dai, Gao, and Tian (2020), we show that tightening the participation constraint for top talent decreases the MTRs facing *all* skill types in both the lower and upper parts of the distribution. A similar finding is obtained by Simula and Trannoy (2010), but they follow the welfarist approach and consider type-dependent participation constraints for all skill types. Therefore, using a different modeling approach, similar to the well-known argument of Stigler (1957), we conclude that allowing migration between regions or explicitly incorporating a participation constraint for top talent could lower the amount of redistribution that a government wishes to undertake.

Moreover, recall that tightening the usual participation constraint can increase the MTRs facing *all* skill types in both the lower and upper parts of the distribution, which is somewhat analogous to the finding that the presence of labor mobility might result in more, rather than less, redistribution than in the absence of labor mobility (Leite-Monteiro 1997, Hindriks 2001, and Dai and Tian 2023). For example, Hindriks (2001) shows that greater mobility of the poor can increase the amount of redistribution in a majority voting equilibrium. The economic environments and mechanisms that govern this prediction undoubtedly differ; therefore, our study can be seen as complementary to theirs.

Additionally, regarding the methodology adopted to address the issue of income redistribution, our modeling approach entails two advantages relative to the migration and tax competition literature. The first advantage is that our approach avoids specifying the proper set of voters when individuals are allowed to vote with their feet across countries. For instance, as Mirrlees (1982) and Cremer and Pestieau (2004) indicated for normative studies, measuring social welfare has always been controversial when labor is mobile because the set of agents whose welfare is to count depends on the income tax itself. This issue certainly applies well to the current political economy setting. The second advantage is that we are concerned about the strategic interaction issues between competing governments that arise from the fiscal externality induced by labor mobility; therefore, we will not be involved with the technical issues of choosing the right solution concept and establishing a game equilibrium. In this sense, our approach might be analytically more tractable, allowing us to obtain clear-cut theoretical results and sharper predictions.

Finally, regarding the political economy conceptual framework adopted, our model is essentially in line with the citizen-candidate model of political competition pioneered by Osborne and Slivinski (1996) and Besley and Coate (1997). The departure is that everybody in our model could be a candidate for public office; thus, to address the issues of tax design and income redistribution through a specific political process, our analysis complements other electoral competition approaches, such as the conventional Downsian model and its variants (Lindbeck and Weibull 1987; Roemer 1998, 1999; Laslier and Picard 2002; De Donder and Hindriks 2003; Carbonell-Nicolau and Ok 2007; Casamatta, Cremer, and De Donder 2010; Bierbrauer and Boyer 2013, 2016).

3 The Model

In the economy, individuals differ in skill level w, which is their private information, called *type*. There is a continuum of types represented by $[\underline{w}, \overline{w}]$, where $0 < \underline{w} < \overline{w} < \infty$. For each type w, a continuum of individuals could exist, and the measure of such individuals is normalized to 1. The cumulative distribution function $F(w)$ is continuously differentiable with support $[\underline{w}, \overline{w}]$, and the corresponding density $f(w) = F'(w) > 0$ is continuous for all w.

All types have the same quasilinear-in-consumption preference represented by the parameterized utility function

$$u(y, x; w) = x - h\left(\frac{y}{w}\right), \tag{1}$$

where $x \geq 0$ denotes consumption (or after-tax income) and $y \equiv wl \geq 0$ denotes pre-tax income under a constant-returns-to-scale production technology. $l \in [0, \bar{l}]$ denotes labor supply in a type-w perfectly competitive labor market, for an exogenous upper bound $\bar{l} \in (0, \infty)$, and the disutility function $h(\cdot)$ is increasing, strictly convex, and three-times continuously differentiable on $\mathbb{R}_+$. An individual's budget constraint is given by $x \leq y - T(y)$, for an anonymous and continuously differentiable tax function $T : \mathbb{R}_+ \to \mathbb{R}$, to be determined by majority voting. Taxes can be negative, and negative income tax refers to transfer payments. Thus, taking the tax policy as given, individual utility maximization subject to the budget constraint gives the MTR for type w:

$$T'(y) = 1 - h'\left(\frac{y}{w}\right)\frac{1}{w} \equiv \tau(w). \tag{2}$$

That is, at an interior individual optimum, the marginal rate of substitution between income and consumption, $h'\left(\frac{y}{w}\right)\frac{1}{w}$, equals the retention rate, $1 - T'(y)$, for all $w \in [\underline{w}, \overline{w}]$. Let $(y(w), x(w))$ represent the resulting optimal allocation for any given tax function $T(\cdot)$. Using (1), the indirect utility for type w is thus written as

$$u\left(y(w), x(w); w\right) = x(w) - h\left(\frac{y(w)}{w}\right) \equiv U(w), \ \forall w \in [\underline{w}, \overline{w}]. \tag{3}$$

To simplify the expression, in what follows, we use $U(w)$ rather than $u\left(y(w), x(w); w\right)$ to denote the indirect utility of type w.

We focus on allocations, $\{y(w), x(w)\}_{w\in[\underline{w},\overline{w}]}$, that are integrable and satisfy the following three sets of constraints. The first set is the incentive compatibility (truth-telling) constraints:

$$x(w) - h\left(\frac{y(w)}{w}\right) \geq x(w') - h\left(\frac{y(w')}{w}\right), \ \forall w, w' \in [\underline{w}, \overline{w}].$$

Applying the envelope theorem, we obtain the first-order incentive-compatibility condition (FOIC):

$$U'(w) = h'\left(\frac{y(w)}{w}\right)\frac{y(w)}{w^2}, \ \forall w \in [\underline{w}, \overline{w}], \tag{4}$$

and the second-order incentive-compatibility condition (SOIC):

$$y'(w) \geq 0, \ \forall w \in [\underline{w}, \overline{w}]. \tag{5}$$

Since the taxation principle (Hammond 1979, Guesnerie 1998, Bierbrauer 2011) applies, conditions (4) and (5) guarantee incentive feasibility.

Assuming that taxation is purely redistributive, the second constraint is the resource feasibility constraint (or government budget constraint):

$$\int_{\underline{w}}^{\overline{w}} [y(w) - x(w)] f(w) dw \geq 0. \tag{6}$$

That is, aggregate consumption must be no greater than aggregate income under Walras' law.

The third set of constraints comprises the participation constraints for the lowest type and the highest type:

$$U(\underline{w}) \geq \underline{u}, \tag{7}$$

$$U(\overline{w}) \geq \overline{u}, \tag{8}$$

where $\overline{u} > 0$ and $\underline{u} \in (0, \overline{u})$ are given reservation utilities for the highest type and the lowest type, respectively. Because the indirect utility function is monotonically nondecreasing in type w, constraint (7) is often adopted to guarantee individual rationality while maintaining analytical tractability. Since each type is only concerned with promoting their own interests, the resulting redistribution could harm the highest-skilled individuals.[13] Constraint (8) is particularly relevant with the option of emigrating in the era of economic globalization. Furthermore, $\overline{u} > \underline{u}$ makes constraint (8) — *the participation constraint for top talent* — a nontrivial addition to *the usual participation constraint* (7).

[13]One can interpret the participation constraint for the highest type by considering an open-economy context with two countries connected by migration. In the benchmark case with two countries, indexed by A and B, top-type individuals born in the country A have indirect utility $U_A(\overline{w})$ if staying in the home country, and have indirect utility $U_B(\overline{w})$ if moving to country B. Thus, from the standpoint of country A, the reservation utility level $\overline{u}$ can be interpreted as $\overline{u} = U_B(\overline{w}) - m(\overline{w})$, where $m(\overline{w}) > 0$ denotes migration-related costs. We often assume that individual preferences remain the same in both countries. Therefore, for a fixed income tax levied on top incomes in country B, $\overline{u} = U_B(\overline{w}) - m(\overline{w})$ is an exogenous parameter for the tax collector of country A. This logic can be safely applied to circumstances with more than two competing countries.

In summary, an income tax schedule, $T(\cdot)$, is *feasible* if the allocation that results from its adoption satisfies the incentive, resource, and participation constraints described above. We focus on *selfishly optimal* income tax schedules, in the sense that each type $k \in [\underline{w}, \overline{w}]$ proposes a feasible allocation that is utility maximal. We first follow the first-order approach by ignoring the SOIC (5); therefore, type-k's problem reads as

$$\max_{x(\cdot),y(\cdot)} U(k) \tag{9}$$

subject to (3), (4), (6), (7), and (8). After solving Problem (9), we examine whether the SOIC (5) is fulfilled.

4 Voting over Selfishly Optimal Income Tax Schedules

In this section, we first derive the set of selfishly optimal income tax schedules by solving mechanism design problems; we then apply the majority voting process to this set to establish a voting equilibrium regarding income redistribution.

4.1 Selfishly optimal income tax schedules

After some algebra, we verify that the optimal schedule of before-tax incomes for Problem (9) is obtained by solving

$$\begin{aligned} \max_{y(\cdot)} & \int_{\underline{w}}^{k} G^M(w,y(w))dw + \int_{k}^{\overline{w}} G^R(w,y(w))dw \\ \text{subject to} & \int_{\underline{w}}^{\overline{w}} G^M(w,y(w))dw \geq \overline{u} \\ \text{and} & \int_{\underline{w}}^{\overline{w}} G^R(w,y(w))dw \geq \underline{u} \end{aligned} \tag{10}$$

where

$$\begin{aligned} G^M(w,y(w)) &= \left[y(w) - h\left(\frac{y(w)}{w}\right)\right] f(w) + \frac{y(w)}{w^2} h'\left(\frac{y(w)}{w}\right) F(w), \\ G^R(w,y(w)) &= \left[y(w) - h\left(\frac{y(w)}{w}\right)\right] f(w) - \frac{y(w)}{w^2} h'\left(\frac{y(w)}{w}\right) [1-F(w)]. \end{aligned} \tag{11}$$

Without these two participation constraints, using the objective function of Problem (10) yields that before-tax incomes allocated to types smaller than k coincide with the maxi-max utility solution, denoted $y^M(w)$; before-tax incomes allocated to types greater than k coincide with the maxi-min utility solution, denoted $y^R(w)$. That is, we let superscripts ^{M} and ^{R} represent "maxi-max" and (Rawlsian) "maxi-min" social welfare functions, respectively.

The Lagrange function of Problem (10) is

$$\begin{aligned} \mathcal{L}^k &= \int_{\underline{w}}^{k} G^M(w,y(w))dw + \int_{k}^{\overline{w}} G^R(w,y(w))dw \\ &\quad + \lambda \left[\int_{\underline{w}}^{\overline{w}} G^M(w,y(w))dw - \overline{u}\right] + \xi \left[\int_{\underline{w}}^{\overline{w}} G^R(w,y(w))dw - \underline{u}\right] \\ &= \int_{\underline{w}}^{k} \left[(1+\lambda)G^M(w,y(w)) + \xi G^R(w,y(w))\right] dw \\ &\quad + \int_{k}^{\overline{w}} \left[(1+\xi)G^R(w,y(w)) + \lambda G^M(w,y(w))\right] dw - \lambda\overline{u} - \xi\underline{u}, \end{aligned} \tag{12}$$

where λ and ξ, which depend on k, are nonnegative Lagrange multipliers associated with the participation constraint for top talent and the usual participation constraint, respectively.

Therefore, simple differentiation of (12) with respect to $y(w)$ yields the first-order conditions (FOCs):

$$\begin{aligned} (1+\lambda)\theta^M(w,y(w)) + \xi\theta^R(w,y(w)) &= 0, \ \forall w \in [\underline{w}, k), \\ (1+\xi)\theta^R(w,y(w)) + \lambda\theta^M(w,y(w)) &= 0, \ \forall w \in (k, \overline{w}], \end{aligned} \tag{13}$$

where

$$\begin{aligned} \theta^M(w,y(w)) &= \frac{\partial G^M(w,y(w))}{\partial y(w)} = \left[1 - h'\left(\frac{y(w)}{w}\right)\frac{1}{w}\right] f(w) \\ &+ \left[\frac{y(w)}{w^3}h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2}h'\left(\frac{y(w)}{w}\right)\right] F(w), \\ \theta^R(w,y(w)) &= \frac{\partial G^R(w,y(w))}{\partial y(w)} = \left[1 - h'\left(\frac{y(w)}{w}\right)\frac{1}{w}\right] f(w) \\ &- \left[\frac{y(w)}{w^3}h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2}h'\left(\frac{y(w)}{w}\right)\right] [1 - F(w)]. \end{aligned} \tag{14}$$

To ensure that Lagrangian (12) is strictly concave in income, we impose the following technical restrictions:

Assumption 4.1 *$\theta^M_y(w,y(w)) < 0$, $\theta^R_y(w,y(w)) < 0$, $(1+\lambda)\theta^M_y(w,y(w)) + \xi\theta^R_y(w,y(w)) < 0$, and $(1+\xi)\theta^R_y(w,y(w)) + \lambda\theta^M_y(w,y(w)) < 0$, for $\forall (w,y) \in [\underline{w}, \overline{w}] \times \mathbb{R}_+$.*

Note that $\theta^M_y(w,y(w)) < 0$ and $\theta^R_y(w,y(w)) < 0$ are sufficient, but not necessary, for $(1+\lambda)\theta^M_y(w,y(w)) + \xi\theta^R_y(w,y(w)) < 0$ and $(1+\xi)\theta^R_y(w,y(w)) + \lambda\theta^M_y(w,y(w)) < 0$ to be valid, for nonnegative Lagrange multipliers λ and ξ. $G^M(w,y(w))$ and $G^R(w,y(w))$ can be interpreted as virtual surpluses under maxi-max and maxi-min objectives, respectively, in the absence of participation constraints; thus, the assumption of $\theta^M_y(w,y(w)) < 0$ and $\theta^R_y(w,y(w)) < 0$ means that the corresponding virtual surpluses must be strictly concave in pre-tax income. Similarly, $(1+\lambda)\theta^M_y(w,y(w)) + \xi\theta^R_y(w,y(w)) < 0$ and $(1+\xi)\theta^R_y(w,y(w)) + \lambda\theta^M_y(w,y(w)) < 0$ imply that the virtual surpluses in the presence of these two participation constraints must be strictly concave in pre-tax income.

Furthermore, the following lemma provides a set of sufficient conditions imposed on the disutility and distribution functions such that Assumption 4.1 follows.

Lemma 4.1 *Suppose $h'''(\cdot) = 0$. Then, for $\forall (w,y) \in [\underline{w}, \overline{w}] \times \mathbb{R}_+$, we have $\theta^R_y(w,y(w)) < 0$, and $\theta^M_y(w,y(w)) < 0$ when $F(w)/(wf(w)) < 1/2$. We also identify the conditions under which $F(w)/(wf(w)) < 1/2$ is satisfied by Pareto, Weibull, and lognormal skill distributions.*

Proof. In Appendix A. ■

Now, using (2), (13), and (14), the formulas for selfishly optimal MTRs are derived as

$$\begin{aligned} \tau^M(w) &= T'\left(y^M(w)\right) = -\frac{F(w)}{f(w)}\left[\frac{y(w)}{w^3}h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2}h'\left(\frac{y(w)}{w}\right)\right] \le 0, \ \forall w \in [\underline{w}, \overline{w}], \\ \tau^{\tilde{M}}(w) &= T'\left(y^{\tilde{M}}(w)\right) \\ &= \left[\frac{\xi}{1+\xi+\lambda} - F(w)\right]\frac{1}{f(w)}\left[\frac{y(w)}{w^3}h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2}h'\left(\frac{y(w)}{w}\right)\right], \ \forall w \in [\underline{w}, \overline{w}], \\ \tau^{\tilde{R}}(w) &= T'\left(y^{\tilde{R}}(w)\right) \\ &= \left[\frac{1+\xi}{1+\xi+\lambda} - F(w)\right]\frac{1}{f(w)}\left[\frac{y(w)}{w^3}h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2}h'\left(\frac{y(w)}{w}\right)\right], \ \forall w \in [\underline{w}, \overline{w}], \\ \tau^R(w) &= T'\left(y^R(w)\right) = \frac{1-F(w)}{f(w)}\left[\frac{y(w)}{w^3}h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2}h'\left(\frac{y(w)}{w}\right)\right] \ge 0, \ \forall w \in [\underline{w}, \overline{w}]. \end{aligned} \tag{15}$$

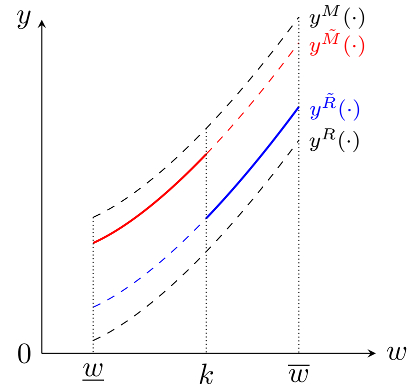

Figure 1: A downward jump discontinuity emerges at the proposer's skill type

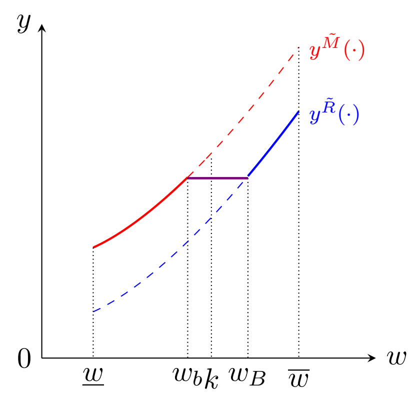

Figure 2: The incentive compatible income schedule with a bridge over a bunching region

We use superscripts $^{\tilde{R}}$ and $^{\tilde{M}}$ to highlight the maxi-min and maxi-max criteria with *binding* participation constraints,[14] namely, with $\lambda > 0$ and $\xi > 0$. Given (15), we have $\tau^M(w) < \tau^{\tilde{M}}(w) < \tau^{\tilde{R}}(w) < \tau^R(w)$ for all $w \in [\underline{w}, \overline{w}]$. Figure 1 shows a downward jump discontinuity in the resulting selfishly optimal income schedule. Because of this discontinuity, the solution to Problem (10) violates the SOIC (5); therefore, it is not incentive compatible. To deal with this issue, we must build a bridge connecting the maxi-max and maxi-min income schedules over a closed interval, denoted $[w_b, w_B]$, where the bridge endpoints $w_b, w_B \in [\underline{w}, \overline{w}]$ are to be endogenously determined. In other words, we must establish a bunching region such that those types of this region receive the same income allocation (see Figure 2).

The following proposition provides an incentive-compatible solution and the conditions determining the bridge endpoints.

Proposition 4.1 *The optimal schedule of before-tax incomes, $y^*(\cdot)$, for type k's problem is continuous on $[\underline{w}, \overline{w}]$, and is given by*

$$y^*(w) = \begin{cases} y^{\tilde{M}}(w) & \text{for } w \in [\underline{w}, w_b), \\ y^{\tilde{M}}(w_b) & \text{for } w \in [w_b, w_B] \text{ if } w_b > \underline{w}, \\ y^{\tilde{R}}(w_B) & \text{for } w \in [w_b, w_B] \text{ if } w_B < \overline{w}, \\ y^{\tilde{R}}(w) & \text{for } w \in (w_B, \overline{w}]. \end{cases} \tag{16}$$

[14] We subsequently prove that these two participation constraints are generally binding in a selfishly optimal proposal.

The optimal values of the Lagrange multipliers $\lambda, \xi \geq 0$, and the bridge endpoints w_b and w_B are determined by solving the binding participation constraints

$$\int_{\underline{w}}^{\overline{w}} G^M(w, y^*(w))dw = \overline{u} \quad \text{and} \quad \int_{\underline{w}}^{\overline{w}} G^R(w, y^*(w))dw = \underline{u},$$

together with either the first-order condition

$$(1+\lambda)\int_{w_b}^{k} \theta^M\left(w, y^{\tilde{M}}(w_b)\right)dw + \lambda \int_{k}^{w_B} \theta^M\left(w, y^{\tilde{M}}(w_b)\right)dw$$
$$+\ \xi \int_{w_b}^{k} \theta^R\left(w, y^{\tilde{M}}(w_b)\right)dw + (1+\xi)\int_{k}^{w_B} \theta^R\left(w, y^{\tilde{M}}(w_b)\right)dw = 0$$

if $w_b > \underline{w}$, or the first-order condition

$$(1+\lambda)\int_{w_b}^{k} \theta^M\left(w, y^{\tilde{R}}(w_B)\right)dw + \lambda \int_{k}^{w_B} \theta^M\left(w, y^{\tilde{R}}(w_B)\right)dw$$
$$+\ \xi \int_{w_b}^{k} \theta^R\left(w, y^{\tilde{R}}(w_B)\right)dw + (1+\xi)\int_{k}^{w_B} \theta^R\left(w, y^{\tilde{R}}(w_B)\right)dw = 0$$

if $w_B < \overline{w}$. Moreover, if $k \in (\underline{w}, \overline{w})$, then the two participation constraints must be binding at the optimum with $\lambda > 0$ and $\xi > 0$. If $k = \underline{w}$, then the participation constraint for top talent must be binding with $\lambda > 0$; and if $k = \overline{w}$, then the usual participation constraint must be binding with $\xi > 0$.

Proof. In Appendix A. ■

We provide three remarks on the incentive-compatible solution. First, the optimal schedule of before-tax incomes outside the bridge is still determined by solving the FOCs (13) of type k's reduced problem (9). Thus, considering the continuity of the selfishly optimal income schedule, the corresponding income allocations for all skill types can be determined accordingly as long as the two bridge endpoints w_b and w_B are determined. Second, the Lagrange multipliers must be determined jointly with the bridge endpoints because the participation constraints (7) and (8) take an integral form involving the income schedule over the entire skill distribution. Third, the integrands, $(1+\lambda)G^M(w, y(w)) + \xi G^R(w, y(w))$ and $(1+\xi)G^R(w, y(w)) + \lambda G^M(w, y(w))$, appearing in Lagrangian (12) can be given virtual surplus interpretations with accounting for these two participation constraints. Therefore, these bridging first-order conditions in Proposition 4.1 have the standard interpretation that the average of the marginal virtual surpluses on a bunching region must be zero. This interpretation implies that a marginal change in the level of income on the bridge does not affect the objective function of Problem (10).

Additionally, because of the countervailing incentives generated by these two participation constraints, compared to the case in which these two constraints are ignored, the graph of the income schedule under the maxi-max objective is pushed downward. Simultaneously, the graph of the income schedule under the maxi-min objective is pushed upward; therefore, the corresponding graphs of the income schedule under these two objectives become closer. Figure 1 shows that the red and blue lines lie between the two black dashed lines representing the income schedules without any participation constraint. Nevertheless, these two constraints do not overpower the desire for upward and downward redistribution toward the proposer in the sense that the graph of the maxi-max income schedule (denoted by the red line) is still above the graph of the maxi-min income schedule (denoted by the blue line).

4.2 The voting equilibrium

For any proposer of type $k \in [\underline{w}, \overline{w}]$, Proposition 4.1 has established his selfishly optimal proposal for income allocations (or, equivalently, tax schedule). To highlight the proposer's type, we let $(x^*(w,k), y^*(w,k))$ denote the allocation assigned to type-w individuals in the solution to the proposer k's problem. The resulting utility of type-w individuals is given by

$$U(w,k) = x^*(w,k) - h\left(\frac{y^*(w,k)}{w}\right). \tag{17}$$

Furthermore, it follows from type k's problem and the application of the implicit function theorem that the Lagrange multipliers, λ and ξ, and the bridge endpoints w_b and w_B are continuously differentiable functions of the skill parameter k. To establish the existence of a voting equilibrium by showing the single-peakedness of $U(w, \cdot)$ in the proposer's type, we must first provide the comparative statics of $\lambda(k), \xi(k), w_b(k)$, and $w_B(k)$ with respect to k. To this end, we must first impose the following:

Assumption 4.2 *The following three conditions hold:*

$$\text{(i)} \quad \frac{\partial y^{\tilde{M}}(w_b)}{\partial \lambda} \leq \frac{\partial y^{\tilde{R}}(w_B)}{\partial \lambda}, \quad \text{(ii)} \frac{\partial y^{\tilde{M}}(w_b)}{\partial \xi} \geq \frac{\partial y^{\tilde{R}}(w_B)}{\partial \xi}, \text{ and}$$

$$\text{(iii)} \quad -\frac{\partial \widetilde{U}(\overline{w})/\partial \lambda}{\partial \widetilde{U}(\underline{w})/\partial \lambda} > -\frac{\int_{w_b}^{w_B} \theta^M(w, y^{\tilde{M}}(w_b))dw}{\int_{w_b}^{w_B} \theta^R(w, y^{\tilde{M}}(w_b))dw} > -\frac{\partial \widetilde{U}(\overline{w})/\partial \xi}{\partial \widetilde{U}(\underline{w})/\partial \xi},$$

where

$$\widetilde{U}(\overline{w}) \equiv \int_{\underline{w}}^{\overline{w}} G^M(w, y^*(w))dw \quad \text{and} \quad \widetilde{U}(\underline{w}) \equiv \int_{\underline{w}}^{\overline{w}} G^R(w, y^*(w))dw.$$

In particular, the following lemma identifies specific requirements imposed on the underlying economic environment such that Assumption 4.2(i)-(ii) are satisfied with strict inequalities.

Lemma 4.2 *Suppose the disutility function of labor supply satisfies $-h'''(l)l/h''(l) \leq 2$. Then,*

$$(0 <)\frac{\partial y^{\tilde{M}}(w_b)}{\partial \lambda} < \frac{\partial y^{\tilde{R}}(w_B)}{\partial \lambda} \quad \text{if} \quad \frac{\theta^M(w_b, y^{\tilde{M}}(w_b))}{\theta^M(w_B, y^{\tilde{R}}(w_B))} \leq \frac{\theta_y^M(w_b, y^{\tilde{M}}(w_b))}{\theta_y^R(w_B, y^{\tilde{R}}(w_B))}$$

and

$$(0 >)\frac{\partial y^{\tilde{M}}(w_b)}{\partial \xi} > \frac{\partial y^{\tilde{R}}(w_B)}{\partial \xi} \quad \text{if}$$
$$\frac{\theta^R(w_b, y^{\tilde{M}}(w_b))}{\theta^R(w_B, y^{\tilde{R}}(w_B))} \geq \max\left\{\frac{\theta_y^M(w_b, y^{\tilde{M}}(w_b))}{\theta_y^M(w_B, y^{\tilde{R}}(w_B))}, \frac{\theta_y^R(w_b, y^{\tilde{M}}(w_b))}{\theta_y^R(w_B, y^{\tilde{R}}(w_B))}\right\}.$$

Moreover, suppose the disutility function of labor supply is given by $h(l) = l^2/2$, the distribution function satisfies

$$\frac{\mathrm{d}\left[\frac{1-F(w)}{wf(w)}\right]}{\mathrm{d}w} < 0, \quad \frac{\mathrm{d}\left[\frac{F(w)}{wf(w)}\right]}{\mathrm{d}w} > 0, \quad \text{and} \quad \frac{F(w)}{wf(w)} < \frac{1}{2},$$

and the slope of the selfishly optimal income schedule, $\frac{\mathrm{d}y^*(w)}{\mathrm{d}w}$, has type-dependent lower and upper bounds. Then, Assumption 4.2(i)-(ii) are fulfilled if the right bridge endpoint, w_B, is sufficiently larger than left bridge endpoint, w_b; namely, the bridge built into the selfishly optimal income schedule is long enough. And the requirements about skill distribution can be satisfied by such empirically plausible distributions as Pareto, Weibull, and lognormal under relatively mild restrictions.

Proof. In Appendix A. ■

Condition $-h'''(l)l/h''(l) \leq 2$ is satisfied by reasonable functional forms, and thus is a mild restriction imposed on the disutility function of labor supply. Given (14), we can interpret the sufficient conditions identified by the first part of Lemma 4.2 as further technical restrictions placed on the disutility function of labor supply and the distribution function of skill types. The second part of Lemma 4.2 provides more concrete conditions, enabling us to get certain intuition about the underlying meaning of Assumption 4.2(i)-(ii).

The intuition of Assumption 4.2(i)-(ii) is the following. Note that λ and ξ are shadow prices of the participation constraint for top talent and the usual participation constraint, respectively. Furthermore, the incomes allocated to high skills are determined by a maxi-min objective, whereas the incomes allocated to low skills are determined by a maxi-max objective. As λ increases, if the participation constraint for top talent becomes more binding in determining the selfishly optimal income schedule, then highly skilled individuals can benefit from this change because it imposes a countervailing effect on the maxi-min objective. Additionally, low-skilled individuals can benefit from this change because it somewhat strengthens the maxi-max objective. Nevertheless, the following analysis requires that this change must be preferable to a specific high-skill type (i.e., the highest skill type bunched with the median type) than to a certain low-skill type (i.e., the lowest skill type bunched with the median type): $0 < \partial y^{\tilde{M}}(w_b)/\partial\lambda < \partial y^{\tilde{R}}(w_B)/\partial\lambda$. The case corresponding to ξ can be interpreted analogously.

Additionally, we can interpret Assumption 4.2(iii) as follows. First, it is easy to understand that tightening the participation constraint for top talent (the usual participation constraint) contributes to the increased indirect utility of the very highest (lowest) skilled individuals, everything else being equal. Formally, we have $\partial\widetilde{U}(\overline{w})/\partial\lambda > 0$ and $\partial\widetilde{U}(\underline{w})/\partial\xi > 0$ (see the proof of Lemma 4.3). Second, we have $\partial\widetilde{U}(\overline{w})/\partial\xi < 0$ and $\partial\widetilde{U}(\underline{w})/\partial\lambda < 0$ (see the proof of Lemma 4.3). That is, tightening the usual participation constraint (the participation constraint for top talent) decreases the indirect utility of the very highest (lowest) skilled individuals.

We then divide Assumption 4.2(iii) into two parts. The first part is given by

$$\frac{\partial\widetilde{U}(\overline{w})/\partial\lambda}{-\partial\widetilde{U}(\underline{w})/\partial\lambda} > \frac{\int_{w_b}^{w_B}\theta^M(w, y^{\tilde{M}}(w_b))dw}{-\int_{w_b}^{w_B}\theta^R(w, y^{\tilde{M}}(w_b))dw} \quad \text{and} \quad \frac{\partial\widetilde{U}(\underline{w})/\partial\xi}{-\partial\widetilde{U}(\overline{w})/\partial\xi} > \frac{-\int_{w_b}^{w_B}\theta^R(w, y^{\tilde{M}}(w_b))dw}{\int_{w_b}^{w_B}\theta^M(w, y^{\tilde{M}}(w_b))dw} \tag{18}$$

in which the denominators and numerators are all positive terms. Take the case of tightening the participation constraint for top talent as an example. Ceteris paribus, if the magnitude of the resulting positive effect on the indirect utility of the highest-skilled individuals is sufficiently larger than that of the corresponding negative effect on the indirect utility of the lowest-skilled, then the first inequality of (18) follows. In particular, (18) provides a lower bound for the ratio of these two magnitudes.

The second part is given by

$$\underbrace{\frac{\partial\widetilde{U}(\overline{w})}{\partial\lambda}}_{+}\cdot\underbrace{\frac{\partial\widetilde{U}(\underline{w})}{\partial\xi}}_{+} > \underbrace{\left[-\frac{\partial\widetilde{U}(\overline{w})}{\partial\xi}\right]}_{+}\cdot\underbrace{\left[-\frac{\partial\widetilde{U}(\underline{w})}{\partial\lambda}\right]}_{+}. \tag{19}$$

As such, if tightening these two participation constraints simultaneously, inequality (19) means that the product of the resulting positive effects is greater than the product of the corresponding negative effects. Therefore, we arrive at the following observation; as long as the magnitude of the positive effect of tightening a participation constraint is sufficiently larger than the magnitude of the corresponding negative effect, then both (18) and (19) will be fulfilled.

We are now ready to establish the following lemma.

Lemma 4.3 *Suppose Assumptions 4.1 and 4.2 hold. Then, for all $k \in [\underline{w}, \overline{w})$, a marginal increase in k results in*

(1) a decrease in $\lambda(k)$ and an increase in $\xi(k)$;

(2) an increase in $w_b(k)$ and $w_B(k)$ if $\underline{w} < w_b(k) < w_B(k) < \overline{w}$;

(3) an increase in $w_b(k)$ if $\underline{w} < w_b(k) < w_B(k) = \overline{w}$;

(4) an increase in $w_B(k)$ if $\underline{w} = w_b(k) < w_B(k) < \overline{w}$.

Proof. In Appendix A. ■

The Lagrange multipliers measure the proposer's marginal disutility caused by increasing the reservation utility (threat point) of the lowest (or the highest) type. As the proposer's type increases, it becomes closer to the highest type and more distant to the lowest type; therefore, the participation threat from the highest type weakens, whereas the participation threat from the lowest type strengthens. Therefore, as the proposer's type increases, the proposer's marginal disutility results from increasing the highest (lowest) type's reservation utility decreases (increases).

Given (17), an individual of type w has a (weakly) single-peaked preference on the set of types if

$$U(w,w) \geq U(w,k_1) \geq U(w,k_2) \quad \text{for } w < k_1 < k_2 \tag{20}$$

and simultaneously

$$U(w,w) \geq U(w,k_1) \geq U(w,k_2) \quad \text{for } w > k_1 > k_2, \tag{21}$$

for $\forall w, k_1, k_2 \in [\underline{w}, \overline{w}]$. The first inequalities in (20) and (21) are automatically satisfied because each type proposes a feasible income schedule that is selfishly optimal for them; therefore, they must weakly prefer what they obtain with their schedule to whatever anybody else proposes for them. Consequently, given (20) and (21), the single-peakedness of individual preferences can be established by proving that $U(w,k)$ is increasing in k when $k < w$ and is decreasing in k when $k > w$.

With the comparative statics established in Lemma 4.3 and the selfishly optimal income schedule shown in Proposition 4.1, we can demonstrate the following:

Lemma 4.4 *Suppose Assumptions 4.1 and 4.2 hold. Then, individual preferences represented by (17) are single-peaked on the set of skill types.*

Proof. In Appendix A. ■

In light of Lemma 4.4 and the median voter theorem (Black 1948), we obtain the following:

Proposition 4.2 *Suppose Assumptions 4.1 and 4.2 hold. Then, the selfishly optimal income tax schedule for the median skill type is a Condorcet winner when majority voting is restricted to the income tax schedules represented by (16), which are selfishly optimal for some skill type.*

5 Identifying the Impact of Participation Constraint(s) on Redistribution

This section first analyzes how accounting for participation constraint(s) affects the sign of MTRs facing low and high skills. We then address how the equilibrium tax size is shaped by experimentally tightening the participation constraint(s). To this end, we denote by $\text{PC}_{\underline{w}}$ and $\text{PC}_{\overline{w}}$ the usual participation constraint and the participation constraint for top talent, respectively. Let w_m be the median skill type.

Brett and Weymark (2017) establish the voting equilibrium in the case without any participation constraint. We write the corresponding MTRs as follows:

$$\tau^0(w) = \begin{cases} -\frac{F(w)}{f(w)}\left[\frac{y(w)}{w^3}h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2}h'\left(\frac{y(w)}{w}\right)\right] & \text{for } w \in [\underline{w}, w_b^0(w_m)), \\ \frac{1-F(w)}{f(w)}\left[\frac{y(w)}{w^3}h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2}h'\left(\frac{y(w)}{w}\right)\right] & \text{for } w \in (w_B^0(w_m), \overline{w}], \end{cases} \tag{22}$$

where $w_b^0(w_m), w_B^0(w_m)$ denote endogenous bridge endpoints when the proposer's type is w_m. With reference to Brett and Weymark (2016), we write the MTRs under $\text{PC}_{\underline{w}}$ as follows:

$$\tau^{\text{PC}_{\underline{w}}}(w) = \begin{cases} \left[\frac{\xi^{\text{PC}_{\underline{w}}}}{1+\xi^{\text{PC}_{\underline{w}}}} - F(w)\right] \frac{1}{f(w)} \left[\frac{y(w)}{w^3} h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2} h'\left(\frac{y(w)}{w}\right)\right] & \text{for } w \in [\underline{w}, w_b^{\text{PC}_{\underline{w}}}(w_m)), \\ \frac{1-F(w)}{f(w)} \left[\frac{y(w)}{w^3} h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2} h'\left(\frac{y(w)}{w}\right)\right] & \text{for } w \in (w_B^{\text{PC}_{\underline{w}}}(w_m), \overline{w}], \end{cases} \tag{23}$$

where $w_b^{\text{PC}_{\underline{w}}}(w_m), w_B^{\text{PC}_{\underline{w}}}(w_m)$ denote endogenous bridge endpoints, and $\xi^{\text{PC}_{\underline{w}}} > 0$ is the Lagrange multiplier associated to $\text{PC}_{\underline{w}}$.

Similarly, using (132) in Appendix B yields the following:

$$\tau^{\text{PC}_{\overline{w}}}(w) = \begin{cases} -\frac{F(w)}{f(w)} \left[\frac{y(w)}{w^3} h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2} h'\left(\frac{y(w)}{w}\right)\right] & \text{for } w \in [\underline{w}, w_b^{\text{PC}_{\overline{w}}}(w_m)), \\ \left[\frac{1}{1+\lambda^{\text{PC}_{\overline{w}}}} - F(w)\right] \frac{1}{f(w)} \left[\frac{y(w)}{w^3} h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2} h'\left(\frac{y(w)}{w}\right)\right] & \text{for } w \in (w_B^{\text{PC}_{\overline{w}}}(w_m), \overline{w}], \end{cases} \tag{24}$$

where $w_b^{\text{PC}_{\overline{w}}}(w_m), w_B^{\text{PC}_{\overline{w}}}(w_m)$ denote endogenous bridge endpoints, and $\lambda^{\text{PC}_{\overline{w}}} > 0$ is the Lagrange multiplier associated to $\text{PC}_{\overline{w}}$. Using (15) yields the following:

$$\tau^1(w) = \begin{cases} \left[\frac{\xi^1}{1+\xi^1+\lambda^1} - F(w)\right] \frac{1}{f(w)} \left[\frac{y(w)}{w^3} h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2} h'\left(\frac{y(w)}{w}\right)\right] & \text{for } w \in [\underline{w}, w_b^1(w_m)), \\ \left[\frac{1+\xi^1}{1+\xi^1+\lambda^1} - F(w)\right] \frac{1}{f(w)} \left[\frac{y(w)}{w^3} h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2} h'\left(\frac{y(w)}{w}\right)\right] & \text{for } w \in (w_B^1(w_m), \overline{w}], \end{cases} \tag{25}$$

where $w_b^1(w_m), w_B^1(w_m)$ denote endogenous bridge endpoints, and $\lambda^1, \xi^1 > 0$ are the Lagrange multipliers associated to $\text{PC}_{\overline{w}}$ and $\text{PC}_{\underline{w}}$, respectively.

Given (22)-(25), we arrive at the following observations:

(1) Without a participation constraint (namely, w/o PC), the equilibrium tax schedule features MTRs that are positive for high skills on the maxi-min part of the schedule, negative for low skills on the maxi-max part of the schedule, and are zero for the bottom and top skill types.

(2) In the case with only $\text{PC}_{\underline{w}}$, the equilibrium tax schedule features MTRs that are positive for high skills on the maxi-min part of the schedule, zero at the top, and positive at the bottom. A threshold skill type exists, which belongs to the maxi-max part of the schedule, and thus is below the median skill level, such that types below this threshold face positive MTRs, whereas other low-skill types beyond this threshold face negative tax rates.

(3) In the case with only $\text{PC}_{\overline{w}}$, the equilibrium tax schedule features MTRs that are negative for low skills on the maxi-max part of the schedule, zero at the bottom, and negative at the top. A threshold skill type exists, which belongs to the maxi-min part of the schedule, and thus is above the median skill level, such that types beyond this threshold face negative MTRs, whereas other high-skill types below this threshold face positive tax rates.

(4) In the case with both $\text{PC}_{\underline{w}}$ and $\text{PC}_{\overline{w}}$, the equilibrium tax schedule features MTRs that are positive at the bottom and negative at the top. A threshold exists, which is below the median skill level, such that types below this threshold face positive MTRs, whereas other low-skill types beyond this threshold face negative tax rates. Moreover, another threshold exists, which is above the median skill level, such that types beyond this threshold face negative MTRs, whereas other high-skill types below this threshold face positive tax rates.

The threshold skill types mentioned in observations (2)-(4) generally differ. Table 1 summarizes how the sign of MTRs at the endpoints of the skill distribution varies across the four scenarios.

Table 1: Signing MTRs at the bottom and top of distribution

	w/o PC	only PC$_{\underline{w}}$	only PC$_{\overline{w}}$	both PC$_{\underline{w}}$ & PC$_{\overline{w}}$
$T'(y(\underline{w}))$	0	+	0	+
$T'(y(\overline{w}))$	0	0	−	−

If we regard (22) as the benchmark case, then comparing (23)-(25) with (22) reveals the following:

Proposition 5.1 *Relative to the benchmark case without accounting for any participation constraint, we have:*

(i) adding only PC$_{\underline{w}}$ makes low skills face strictly higher MTRs, leaving the MTRs facing high skills unaltered;

(ii) adding only PC$_{\overline{w}}$ makes high skills face strictly lower MTRs, leaving the MTRs facing low skills unaltered;

(iii) adding both PC$_{\underline{w}}$ and PC$_{\overline{w}}$ makes low skills face strictly higher MTRs, and simultaneously makes high skills face strictly lower MTRs.

Proposition 5.1 identifies the redistributive effects of adding participation constraint(s); see also Table 2. First, in the case with only the usual participation constraint, low types in the lower part of skill distribution are worse off. In contrast, high types in the upper part of distribution are not affected, roughly implying a strengthening of "the tyranny of the middle class." The increased tax revenue collected from low skills is transferred to the middle. Second, in the case of only the participation constraint for top talent, low types are not affected while high types are better off, yielding "a curse of the middle class." That is, tax revenue collected from high skills reduces; therefore, the resources received by the middle decrease. Third, low types are worse off in the case with both participation constraints, whereas high types are better off, leaving the welfare effect on the middle ambiguous.

The intuition of Proposition 5.1 is as follows. The income tax schedule in the voting equilibrium redistributes resources from low and high-skill types toward the middle because a maxi-max social welfare function determines incomes allocated to low skills, and a maxi-min social welfare function determines incomes allocated to high skills. This result holds regardless of whether there is a binding participation constraint. Without a participation constraint, the motive for downward redistribution under the maxi-min objective yields positive tax rates on high skills, whereas the motive for upward redistribution under the maxi-max objective provides a reason to offer subsidies to low skills, and so they face negative tax rates. In the case of only the usual participation constraint, the incentive provided by the maxi-min objective is essentially unaltered. However, this additional constraint produces a countervailing incentive to the incentive provided by the maxi-max objective, thereby leading to the prediction of Proposition 5.1(i). By the same logic, in the case with only the participation constraint for top talent, the incentive provided by the maxi-max objective does not change; however, it produces a countervailing incentive to the incentive provided by the maxi-min objective, as claimed by Proposition 5.1(ii). The interpretation of Proposition 5.1(iii) follows from the combination of Proposition 5.1(i) and Proposition 5.1(ii).

To study the effect of tightening the participation constraints through increasing the shadow price(s) of participation constraint(s) on the equilibrium tax size, we rewrite the formulas for MTRs in ABC form (Diamond 1998 and Saez 2001):

Table 2: The effects of adding the participation constraint(s)

$T'(y(w))$	only PC$_{\underline{w}}$	only PC$_{\overline{w}}$	both PC$_{\underline{w}}$ & PC$_{\overline{w}}$
low incomes: $w \in [\underline{w}, w_b)$	$\uparrow$	nil	$\uparrow$
high incomes: $w \in (w_B, \overline{w}]$	nil	$\downarrow$	$\downarrow$
implication			
the middle class: $w \in [w_b, w_B]$	better-off	worse-off	ambiguous

$$\frac{\tau^{\mathrm{PC}_{\underline{w}}}(w)}{1-\tau^{\mathrm{PC}_{\underline{w}}}(w)} = \begin{cases} \frac{\frac{\xi^{\mathrm{PC}_{\underline{w}}}}{1+\xi^{\mathrm{PC}_{\underline{w}}}}-F(w)}{wf(w)}\left[1+\varepsilon(w)\right] & \text{for } w \in [\underline{w}, w_b^{\mathrm{PC}_{\underline{w}}}(w_m)), \\ \frac{1-F(w)}{wf(w)}\left[1+\varepsilon(w)\right] & \text{for } w \in (w_B^{\mathrm{PC}_{\underline{w}}}(w_m), \overline{w}], \end{cases} \tag{26}$$

where the elasticity of labor supply is

$$\varepsilon(w) \equiv \frac{h''\left(\frac{y(w)}{w}\right)\frac{y(w)}{w}}{h'\left(\frac{y(w)}{w}\right)} > 0. \tag{27}$$

Similarly, given (24), (25), and (27), we obtain

$$\frac{\tau^{\mathrm{PC}_{\overline{w}}}(w)}{1-\tau^{\mathrm{PC}_{\overline{w}}}(w)} = \begin{cases} -\frac{F(w)}{wf(w)}\left[1+\varepsilon(w)\right] & \text{for } w \in [\underline{w}, w_b^{\mathrm{PC}_{\overline{w}}}(w_m)), \\ \frac{\frac{1}{1+\lambda^{\mathrm{PC}_{\overline{w}}}}-F(w)}{wf(w)}\left[1+\varepsilon(w)\right] & \text{for } w \in (w_B^{\mathrm{PC}_{\overline{w}}}(w_m), \overline{w}], \end{cases} \tag{28}$$

and

$$\frac{\tau^{1}(w)}{1-\tau^{1}(w)} = \begin{cases} \frac{\frac{\xi^1}{1+\xi^1+\lambda^1}-F(w)}{wf(w)}\left[1+\varepsilon(w)\right] & \text{for } w \in [\underline{w}, w_b^{1}(w_m)), \\ \frac{\frac{1+\xi^1}{1+\xi^1+\lambda^1}-F(w)}{wf(w)}\left[1+\varepsilon(w)\right] & \text{for } w \in (w_B^{1}(w_m), \overline{w}]. \end{cases} \tag{29}$$

Then, we are ready to establish the following:

Proposition 5.2 *The following statements are true under the current economic environment:*

(i) Ceteris paribus , tightening the usual participation constraint increases the MTRs facing low skills in the case with only PC$_{\underline{w}}$ and increases the MTRs facing both low and high skills in the case with both PC$_{\underline{w}}$ and PC$_{\overline{w}}$.

(ii) Ceteris paribus, tightening the participation constraint for top talent reduces the MTRs facing high skills in the case with only PC$_{\overline{w}}$ and reduces the MTRs facing both low and high skills in the case with both PC$_{\underline{w}}$ and PC$_{\overline{w}}$.

(iii) If $\xi^1 \geq 1+\lambda^1$, then we have

$$\frac{\partial^2 \tau^1(w)}{\partial \xi^1 \partial \lambda^1} > 0.$$

That is, in the case with both PC$_{\underline{w}}$ and PC$_{\overline{w}}$, as the participation constraint for top talent becomes more binding (namely, λ^1 increases), the positive effect of tightening the usual participation constraint on MTRs grows. In other words, as the usual participation constraint becomes more binding (namely, ξ^1 increases), the negative effect of tightening the participation constraint for top talent on MTRs decreases.

Proof. In Appendix A. ■

Proposition 5.2(i)-(ii) show that tightening the usual participation constraint tends to increase the equilibrium tax size, whereas tightening the participation constraint for top talent tends to reduce the equilibrium tax size. Proposition 5.2(iii) shows that if the shadow price associated with $\mathrm{PC}_{\underline{w}}$ is sufficiently larger than the shadow price associated with $\mathrm{PC}_{\overline{w}}$, then the combined effect of tightening these two participation constraints on the equilibrium tax size is positive.

6 Concluding Summary

This article addresses how an equilibrium marginal tax schedule would look like in a setting populated by individuals who differ in privately observable skills and are involved in a contest of proposing competing income tax schedules, over which the pairwise majority rule is adopted to select the winner.[15] The focus of our theoretical analysis is to establish the existence of a voting equilibrium and provide a complete characterization of the equilibrium tax profile over the entire skill distribution. Since a continuum of permissible tax schedules exists in our problem, the single-crossing condition used by Gans and Smart (1996) cannot prove the existence of a Condorcet winner. Thus, we assume that individuals share the same quasilinear-in-consumption preferences and restrict attention to selfishly optimal tax schedules as candidates. In so doing, we establish single-peakedness of individual preferences under a set of reasonable conditions and then confirm the existence of a voting equilibrium by using Black (1948)'s median voter theorem.

Relative to the circumstances without these two participation constraints, we find that the resulting equilibrium tax schedule features higher MTRs for low-skill types in the lower part of the distribution and lower MTRs for high skills in the upper part. Ceteris paribus, a more binding participation constraint at the bottom increases MTRs in lower and upper parts of distribution. Conversely, a more binding participation constraint for top talent decreases the corresponding MTRs. This result is primarily owing to the countervailing incentives provided by these two participation constraints to the incentives implied by selfishly optimal marginal tax schedules. Furthermore, jointly considering these two constraints indicates that some highly skilled individuals face negative MTRs, whereas some lowly-skilled individuals face positive MTRs. Nonetheless, the Condorcet-winning tax schedule is still the one proposed and preferred by the median voter — namely, the desire to redistribute resources from the rich and poor toward the middle has not been entirely overpowered by these countervailing incentives.

In sum, for the political economy under consideration, the positive theory of equilibrium income redistribution is in line with the Director's Law in the sense that the middle-income groups can form a successful political coalition to extract resources from both the low-income and the high-income groups, which cannot be overturned by adding these two participation constraints. Nonetheless, the redistributive impact of the politico-economic equilibrium that features "a tyranny of the middle class" can be strengthened by adding the usual participation constraint, but it will be weakened by adding the participation constraint for top talent. Indeed, if only the participation constraint for top talent is considered, "a curse of the middle class" emerges, that is, the tax burden generated by retaining the top talent will be totally imposed on the middle class.

[15] As argued by Roberts (1977), if political parties in a democratic system choose a particular income tax schedule to maximize their chances of being elected, it is somewhat reasonable to view the selected tax schedule as being indirectly determined by a majority voting process.

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Appendix A: Proofs

Proof of Lemma 4.1. Define the elasticity of labor supply as

$$\varepsilon(w) \equiv \frac{h''(l(w))\, l(w)}{h'(l(w))} = \frac{h''\left(\frac{y}{w}\right)\frac{y}{w}}{h'\left(\frac{y}{w}\right)} \equiv \tilde{\varepsilon}(w, y). \tag{30}$$

Using (30) to re-express (14) as

$$\begin{aligned} \theta^M(w, y) &= \left[1 - h'\left(\frac{y}{w}\right)\frac{1}{w}\right] f(w) + \frac{1}{w^2} h'\left(\frac{y}{w}\right)\left[1 + \tilde{\varepsilon}(w, y)\right] F(w), \\ \theta^R(w, y) &= \left[1 - h'\left(\frac{y}{w}\right)\frac{1}{w}\right] f(w) - \frac{1}{w^2} h'\left(\frac{y}{w}\right)\left[1 + \tilde{\varepsilon}(w, y)\right]\left[1 - F(w)\right]. \end{aligned} \tag{31}$$

Simple differentiation of (31) yields

$$\begin{aligned} \theta^M_y(w, y) &= h''\left(\frac{y}{w}\right)\frac{1}{w^2} f(w)\left[\frac{F(w)}{wf(w)} - 1\right] \\ &\quad + \frac{1}{w^2} h'\left(\frac{y}{w}\right) F(w)\left[\tilde{\varepsilon}_y(w, y) + \left(\frac{h''\left(\frac{y}{w}\right)\frac{1}{w}}{h'\left(\frac{y}{w}\right)}\right)\tilde{\varepsilon}(w, y)\right], \\ \theta^R_y(w, y) &= -\, h''\left(\frac{y}{w}\right)\frac{1}{w^2} f(w)\left[\frac{1 - F(w)}{wf(w)} + 1\right] \\ &\quad - \frac{1}{w^2} h'\left(\frac{y}{w}\right)\left[1 - F(w)\right]\left[\tilde{\varepsilon}_y(w, y) + \left(\frac{h''\left(\frac{y}{w}\right)\frac{1}{w}}{h'\left(\frac{y}{w}\right)}\right)\tilde{\varepsilon}(w, y)\right]. \end{aligned} \tag{32}$$

Using (30) gives the expression of $\tilde{\varepsilon}_y(w, y)$ as follows:

$$\tilde{\varepsilon}_y(w, y) = \frac{h'''\left(\frac{y}{w}\right)\frac{y}{w^2}}{h'\left(\frac{y}{w}\right)} + \frac{h''\left(\frac{y}{w}\right)\frac{1}{w}}{h'\left(\frac{y}{w}\right)}\left[1 - \tilde{\varepsilon}(w, y)\right]. \tag{33}$$

Suppose $h'''(\cdot) = 0$, then (33) becomes

$$\tilde{\varepsilon}_y(w, y) + \frac{h''\left(\frac{y}{w}\right)\frac{1}{w}}{h'\left(\frac{y}{w}\right)}\tilde{\varepsilon}(w, y) = \frac{h''\left(\frac{y}{w}\right)\frac{1}{w}}{h'\left(\frac{y}{w}\right)}. \tag{34}$$

Plugging (34) in (32) and simplifying the algebra, we obtain

$$\begin{aligned} \theta^M_y(w, y) &= h''\left(\frac{y}{w}\right)\frac{1}{w^2} f(w)\left[2\left(\frac{F(w)}{wf(w)}\right) - 1\right], \\ \theta^R_y(w, y) &= -\, h''\left(\frac{y}{w}\right)\frac{1}{w^2} f(w)\left[2\left(\frac{1 - F(w)}{wf(w)}\right) + 1\right]. \end{aligned} \tag{35}$$

Then, we always have $\theta^R_y(w, y) < 0$, whereas $\theta^M_y(w, y) < 0$ is true only under circumstances featuring $F(w)/(wf(w)) < 1/2$.

To further study the effect of the condition $F(w)/(wf(w)) < 1/2$ on the distribution function, we consider some empirically plausible distributions. With a Pareto distribution, we have

$$\frac{F(w)}{wf(w)} = \frac{w^a}{a(\underline{w})^a} - \frac{1}{a},$$

in which $a > 0$ is the Pareto index. Evidently, $F(w)/(wf(w))$ is monotonically increasing in w under the Pareto distribution. With a Weibull distribution, we have

$$\frac{F(w)}{wf(w)} = \frac{\chi^\zeta}{\zeta} \cdot \frac{1}{w^\zeta} \cdot \left[e^{(w/\chi)^\zeta} - 1\right],$$

in which $\zeta > 0$ is the shape parameter and $\chi > 0$ is the scale parameter. Simple differentiation yields

$$\frac{\mathrm{d}\left[\frac{F(w)}{wf(w)}\right]}{\mathrm{d}w} = \frac{\chi^{\zeta}}{w^{\zeta+1}} + \frac{1}{w}e^{(w/\chi)^{\zeta}}\left[1-\left(\frac{\chi}{w}\right)^{\zeta}\right] > 0 \ \text{ for } \forall w \text{ if } \underline{w} \geq \chi.$$

With a lognormal distribution, we have

$$\frac{F(w)}{wf(w)} = \sigma\left[\int_{\underline{w}}^{\frac{\ln w-\mu}{\sigma}} e^{-t^2/2}dt\right]\exp\left[\frac{(\ln w-\mu)^2}{2\sigma^2}\right],$$

in which μ is the mean and σ^2 is the variance. After some algebra, we have

$$\frac{\mathrm{d}\left[\frac{F(w)}{wf(w)}\right]}{\mathrm{d}w} > 0 \text{ iff } (\mu-\ln w)\exp\left[\frac{(\ln w-\mu)^2}{2\sigma^2}\right]\left[1+\mathrm{erf}\left(\frac{\ln w-\mu}{\sqrt{2}\sigma}\right)\right] < 0.8\sigma.$$

As such, we have established relatively mild conditions under which $F(w)/(wf(w))$ is monotonically increasing in w. Therefore, to have $F(w)/(wf(w)) < 1/2$ under these distributions, it suffices to impose an upper bound on $\overline{w}$. ■

Proof of Proposition 4.1. To propose incentive-compatible allocations, a type-k proposer must solve the following problem:

$$\begin{aligned} &\max_{y(\cdot)} \int_{\underline{w}}^{k} G^M(w,y(w))dw + \int_{k}^{\overline{w}} G^R(w,y(w))dw \\ &\text{subject to } \int_{\underline{w}}^{\overline{w}} G^M(w,y(w))dw \geq \overline{u}, \\ &\int_{\underline{w}}^{\overline{w}} G^R(w,y(w))dw \geq \underline{u}, \text{ and } y'(w) \geq 0 \text{ for } \forall w \in [\underline{w},\overline{w}]. \end{aligned} \tag{36}$$

Problem (36) can be solved in two steps. In the first step, for fixed values of Lagrange multipliers — namely, λ and ξ associated with these two participation constraints — and bridge endpoints w_b and w_B, optimal before-tax incomes outside of the bridge are still determined by the FOCs given by (13), re-expressed as

$$\begin{aligned} (1+\lambda)\theta^M(w,y(w)) + \xi\theta^R(w,y(w)) &= 0, \ \forall w \in [\underline{w}, w_b), \\ (1+\xi)\theta^R(w,y(w)) + \lambda\theta^M(w,y(w)) &= 0, \ \forall w \in (w_B, \overline{w}]. \end{aligned} \tag{37}$$

For fixed values of λ and ξ, we let $\tilde{y}(w,\lambda,\xi)$ denote the solution to (37). Note that $\tilde{y}(w,\lambda,\xi) = y^{\tilde{M}}(w)$ for $w < w_b$, and $\tilde{y}(w,\lambda,\xi) = y^{\tilde{R}}(w)$ for $w > w_B$, for which $y^{\tilde{M}}(w)$ and $y^{\tilde{R}}(w)$ depend on both λ and ξ. Let $\bar{y}$ denote the before-tax income allocated to those types on the bridge over the bunching region $[w_b, w_B]$. Following the Lagrange-multiplier approach, the Lagrange function for the second step is

$$\begin{aligned} &\int_{\underline{w}}^{w_b} G^M\left(w, y^{\tilde{M}}(w)\right)dw + \int_{w_b}^{k} G^M(w,\bar{y})\,dw + \int_{k}^{w_B} G^R(w,\bar{y})\,dw \\ &+ \int_{w_B}^{\overline{w}} G^R\left(w, y^{\tilde{R}}(w)\right)dw + \lambda\int_{\underline{w}}^{w_b} G^M\left(w, y^{\tilde{M}}(w)\right)dw \\ &+ \lambda\left[\int_{w_b}^{w_B} G^M(w,\bar{y})\,dw + \int_{w_B}^{\overline{w}} G^M\left(w, y^{\tilde{R}}(w)\right)dw - \overline{u}\right] \\ &+ \xi\left[\int_{\underline{w}}^{w_b} G^R\left(w, y^{\tilde{M}}(w)\right)dw + \int_{w_b}^{w_B} G^R(w,\bar{y})\,dw + \int_{w_B}^{\overline{w}} G^R\left(w, y^{\tilde{R}}(w)\right)dw - \underline{u}\right]. \end{aligned} \tag{38}$$

There are three cases to consider.

Case 1: $\underline{w} < w_b < w_B < \overline{w}$. The continuity of the resulting income schedule yields

$$y^{\tilde{M}}(w_b) = \tilde{y}(w_b, \lambda, \xi) = \bar{y} = \tilde{y}(w_B, \lambda, \xi) = y^{\tilde{R}}(w_B). \tag{39}$$

The choice of λ, ξ, w_b, and w_B is determined by simultaneously solving

$$y^{\tilde{M}}(w_b) - y^{\tilde{R}}(w_B) = 0,$$

$$\begin{aligned}\int_{\underline{w}}^{w_b} G^M\left(w, y^{\tilde{M}}(w)\right) dw \ &+ \int_{w_b}^{w_B} G^M\left(w, y^{\tilde{M}}(w_b)\right) dw \\ &+ \int_{w_B}^{\overline{w}} G^M\left(w, y^{\tilde{R}}(w)\right) dw - \overline{u} = 0,\end{aligned}$$

$$\begin{aligned}\int_{\underline{w}}^{w_b} G^R\left(w, y^{\tilde{M}}(w)\right) dw \ &+ \int_{w_b}^{w_B} G^R\left(w, y^{\tilde{M}}(w_b)\right) dw \\ &+ \int_{w_B}^{\overline{w}} G^R\left(w, y^{\tilde{R}}(w)\right) dw - \underline{u} = 0,\end{aligned}$$

and

$$\begin{aligned}(1+\lambda)\int_{w_b}^{k} \theta^M\left(w, y^{\tilde{M}}(w_b)\right) dw \ &+ \xi \int_{w_b}^{k} \theta^R\left(w, y^{\tilde{M}}(w_b)\right) dw \\ + \ (1+\xi)\int_{k}^{w_B} \theta^R\left(w, y^{\tilde{M}}(w_b)\right) dw &+ \lambda \int_{k}^{w_B} \theta^M\left(w, y^{\tilde{M}}(w_b)\right) dw = 0.\end{aligned} \tag{40}$$

Equation (40) is obtained by differentiating Lagrange function (38) with respect to $\bar{y}$ and then evaluating the resulting derivative at $\bar{y} = y^{\tilde{M}}(w_b)$ in light of (39).

Case 2: $\underline{w} < w_b < w_B = \overline{w}$. Using continuity of the selfishly optimal income schedule, the types in $[w_b, \overline{w}]$ receive $y^{\tilde{M}}(w_b)$. Thus, the three variables w_b, λ, and ξ are determined by solving

$$\int_{\underline{w}}^{w_b} G^M\left(w, y^{\tilde{M}}(w)\right) dw + \int_{w_b}^{\overline{w}} G^M\left(w, y^{\tilde{M}}(w_b)\right) dw - \overline{u} = 0,$$

$$\int_{\underline{w}}^{w_b} G^R\left(w, y^{\tilde{M}}(w)\right) dw + \int_{w_b}^{\overline{w}} G^R\left(w, y^{\tilde{M}}(w_b)\right) dw - \underline{u} = 0,$$

and

$$\begin{aligned}(1+\lambda)\int_{w_b}^{k} \theta^M\left(w, y^{\tilde{M}}(w_b)\right) dw \ &+ \xi \int_{w_b}^{k} \theta^R\left(w, y^{\tilde{M}}(w_b)\right) dw \\ + \ (1+\xi)\int_{k}^{\overline{w}} \theta^R\left(w, y^{\tilde{M}}(w_b)\right) dw &+ \lambda \int_{k}^{\overline{w}} \theta^M\left(w, y^{\tilde{M}}(w_b)\right) dw = 0.\end{aligned} \tag{41}$$

Comparing equation (41) with equation (40) shows that the FOC for the optimal income over the bunching region is essentially the same whenever $w_b > \underline{w}$, regardless of whether $w_B < \overline{w}$ or $w_B = \overline{w}$.

Case 3: $\underline{w} = w_b < w_B < \overline{w}$. Now, the types in $[\underline{w}, w_B]$ receive $y^{\tilde{R}}(w_B)$. Hence, w_B, λ, and ξ are determined by solving

$$\int_{\underline{w}}^{w_B} G^M\left(w, y^{\tilde{R}}(w_B)\right) dw + \int_{w_B}^{\overline{w}} G^M\left(w, y^{\tilde{R}}(w)\right) dw - \overline{u} = 0,$$

$$\int_{\underline{w}}^{w_B} G^R\left(w, y^{\tilde{R}}(w_B)\right) dw + \int_{w_B}^{\overline{w}} G^R\left(w, y^{\tilde{R}}(w)\right) dw - \underline{u} = 0,$$

and

$$\begin{aligned}(1+\lambda)\int_{\underline{w}}^{k} \theta^M\left(w, y^{\tilde{R}}(w_B)\right) dw \ &+ \xi \int_{\underline{w}}^{k} \theta^R\left(w, y^{\tilde{R}}(w_B)\right) dw \\ + (1+\xi)\int_{k}^{w_B} \theta^R\left(w, y^{\tilde{R}}(w_B)\right) dw &+ \lambda \int_{k}^{w_B} \theta^M\left(w, y^{\tilde{R}}(w_B)\right) dw = 0.\end{aligned}$$

Therefore, we have provided the complete set of necessary conditions to determine the Lagrange multipliers and bridge endpoints.

We next verify whether the participation constraints are binding — namely, the Lagrange multipliers satisfy $\lambda > 0$ and $\xi > 0$ — in selfishly optimal allocations. To this end, we treat type-w's optimal consumption choice, $x(w)$, as an implicit function of $U(w)$ and $y(w)$, and write it as $\psi(U(w), y(w))$. Then, (3) can be rewritten as

$$U(w) = \psi(U(w), y(w)) - h\left(\frac{y(w)}{w}\right), \quad \forall w \in [\underline{w}, \overline{w}].$$

Differentiating this equation with respect to $U(w)$ and $y(w)$, respectively, gives:

$$\frac{\partial \psi(U(w), y(w))}{\partial U(w)} = 1 \quad \text{and} \quad \frac{\partial \psi(U(w), y(w))}{\partial y(w)} = \frac{1}{w} h'\left(\frac{y(w)}{w}\right) \tag{42}$$

for $\forall w \in [\underline{w}, \overline{w}]$. We must consider three possible scenarios concerning the proposer's type k, namely, $k \in (\underline{w}, \overline{w})$, $k = \underline{w}$, or $k = \overline{w}$.

First, for $k \in (\underline{w}, \overline{w})$, the Lagrange function of Problem (9) with considering the bunching region $[w_b, w_B]$ is

$$\begin{aligned}&\mathcal{L}\left(\{U(w), y(w)\}_{w\in[\underline{w},\overline{w}]}; \mu, \lambda, \xi, \{\eta(w)\}_{w\in[\underline{w},\overline{w}]}\right) \\ =\ & U(k)f(k) + \lambda \cdot [U(\overline{w}) - \overline{u}]\, f(\overline{w}) + \xi \cdot [U(\underline{w}) - \underline{u}]\, f(\underline{w}) \\ &+ \mu \cdot \int_{\underline{w}}^{w_b} [y(w) - \psi(U(w), y(w))]\, f(w) dw + \mu \cdot \int_{w_b}^{w_B} [\bar{y} - \psi(U(w), \bar{y})]\, f(w) dw \\ &+ \mu \cdot \int_{w_B}^{\overline{w}} [y(w) - \psi(U(w), y(w))]\, f(w) dw + \int_{\underline{w}}^{w_b} \eta(w)\left[\frac{y(w)}{w^2} h'\left(\frac{y(w)}{w}\right) - U'(w)\right] dw \\ &+ \int_{w_b}^{w_B} \eta(w)\left[\frac{\bar{y}}{w^2} h'\left(\frac{\bar{y}}{w}\right) - U'(w)\right] dw + \int_{w_B}^{\overline{w}} \eta(w)\left[\frac{y(w)}{w^2} h'\left(\frac{y(w)}{w}\right) - U'(w)\right] dw\end{aligned} \tag{43}$$

where μ, λ, and ξ are nonnegative Lagrange multipliers associated with the government budget constraint, the participation constraint for top talent, and the usual participation constraint, respectively. $\eta(w)$ is the costate variable associated with the FOIC (4). Integrating by parts and re-grouping terms, (43) is rewritten as:

$$
\begin{aligned}
&\mathcal{L}\left(\{U(w),y(w)\}_{w\in[\underline{w},\overline{w}]};\mu,\lambda,\xi,\{\eta(w)\}_{w\in[\underline{w},\overline{w}]}\right)\\
&= U(k)f(k)+\lambda\cdot\left[U(\overline{w})-\overline{u}\right]f(\overline{w})+\xi\cdot\left[U(\underline{w})-\underline{u}\right]f(\underline{w})\\
&\quad+\eta(\underline{w})U(\underline{w})-\eta(\overline{w})U(\overline{w})+\int_{\underline{w}}^{\overline{w}}\eta'(w)U(w)dw\\
&\quad+\mu\cdot\int_{\underline{w}}^{w_b}\left[y(w)-\psi(U(w),y(w))\right]f(w)dw+\mu\cdot\int_{w_b}^{w_B}\left[\bar{y}-\psi(U(w),\bar{y})\right]f(w)dw\\
&\quad+\mu\cdot\int_{w_B}^{\overline{w}}\left[y(w)-\psi(U(w),y(w))\right]f(w)dw+\int_{\underline{w}}^{w_b}\eta(w)\frac{y(w)}{w^2}h'\left(\frac{y(w)}{w}\right)dw\\
&\quad+\int_{w_b}^{w_B}\eta(w)\frac{\bar{y}}{w^2}h'\left(\frac{\bar{y}}{w}\right)dw+\int_{w_B}^{\overline{w}}\eta(w)\frac{y(w)}{w^2}h'\left(\frac{y(w)}{w}\right)dw.
\end{aligned}
$$

For our purpose, we do not need to present the complete set of necessary conditions for optimality.

Case 1: $\underline{w}<w_b<w_B<\overline{w}$. Following Simula and Trannoy (2010, 2012) and assuming the existence of an interior solution, the necessary conditions with respect to the indirect utilities (state variables) are given by:[16]

$$
\begin{aligned}
\{U(w)\}_{w\in(\underline{w},\overline{w})}:\ & f(k)\cdot\mathbb{I}_{\{w=k\}}+\eta'(w)-\mu\cdot\frac{\partial\psi(U(w),\bar{y})}{\partial U(w)}f(w)\cdot\mathbb{I}_{\{w\in[w_b,w_B]\}}\\
&-\mu\cdot\frac{\partial\psi(U(w),y(w))}{\partial U(w)}f(w)\cdot\mathbb{I}_{\{w\in(\underline{w},w_b)\cup(w_B,\overline{w})\}}=0,
\end{aligned}
\tag{44}
$$

$$
U(\underline{w}):\ \xi f(\underline{w})+\eta(\underline{w})-\mu\cdot\left[\frac{\partial\psi(U(w),y(w))}{\partial U(w)}f(w)\right]\Bigg|_{w=\underline{w}}=0, \tag{45}
$$

$$
U(\overline{w}):\ \lambda f(\overline{w})-\eta(\overline{w})-\mu\cdot\left[\frac{\partial\psi(U(w),y(w))}{\partial U(w)}f(w)\right]\Bigg|_{w=\overline{w}}=0, \tag{46}
$$

in which $\mathbb{I}$ denotes indicator function, together with

$$
\eta(\underline{w})\le 0\ \text{ and }\ \eta(\overline{w})\ge 0, \tag{47}
$$

and the complementary-slackness conditions:

$$
\begin{aligned}
&\lambda\ge 0,\quad U(\overline{w})\ge\overline{u},\quad \lambda\cdot\left[U(\overline{w})-\overline{u}\right]=0,\\
&\xi\ge 0,\quad U(\underline{w})\ge\underline{u},\quad \xi\cdot\left[U(\underline{w})-\underline{u}\right]=0.
\end{aligned}
\tag{48}
$$

Applying (42) and (47) to (45) and (46) yields:

$$
\begin{aligned}
(\xi-\mu)f(\underline{w}) &= -\eta(\underline{w})\ge 0,\\
(\lambda-\mu)f(\overline{w}) &= \eta(\overline{w})\ge 0.
\end{aligned}
\tag{49}
$$

In view of (42) and the fundamental theorem of calculus, (44) implies

$$
\eta(w)=\eta(\overline{w})-\int_{w}^{\overline{w}}\left[\mu f(t)\cdot\mathbb{I}_{\{t\in(\underline{w},\overline{w})\}}-f(k)\cdot\mathbb{I}_{\{t=k\}}\right]dt. \tag{50}
$$

[16]Note that these conditions can equivalently be derived based on variational techniques using Pontryagin's maximum principle.

Evaluating (50) at $\underline{w}$ and then using (49) yields, after algebraic simplification,

$$\mu = \lambda f(\overline{w}) + \xi f(\underline{w}) + \int_{\underline{w}}^{\overline{w}} f(k) \cdot \mathbb{I}_{\{t=k\}} dt > 0,$$

which, when combined with (49), gives $\xi \geq \mu > 0$ and $\lambda \geq \mu > 0$. Thus, one gets by the complementary-slackness conditions (48) that these two participation constraints must be binding in the optimum. The analysis of the other two cases with $\underline{w} < w_b < w_B = \overline{w}$ or $\underline{w} = w_b < w_B < \overline{w}$ is essentially the same, except for minor revisions imposed on equation (44).

Second, for $k = \underline{w}$, because of the downward jump discontinuity of the selfishly optimal income schedule at $k = \underline{w}$, the bunching region must be $[\underline{w}, w_B]$, namely, we just need to consider the case with $\underline{w} = w_b < w_B < \overline{w}$. The Lagrange function is

$$\begin{aligned}
&\mathcal{L}\left(\{U(w), y(w)\}_{w\in[\underline{w},\overline{w}]}; \mu, \lambda, \xi, \{\eta(w)\}_{w\in[\underline{w},\overline{w}]}\right) \\
= &\, U(\underline{w})f(\underline{w}) + \lambda \cdot [U(\overline{w}) - \overline{u}]\, f(\overline{w}) + \xi \cdot [U(\underline{w}) - \underline{u}]\, f(\underline{w}) \\
&+ \eta(\underline{w})U(\underline{w}) - \eta(\overline{w})U(\overline{w}) + \int_{\underline{w}}^{\overline{w}} \eta'(w)U(w)dw \\
&+ \mu \cdot \int_{\underline{w}}^{w_B} [\bar{y} - \psi(U(w), \bar{y})]\, f(w)dw + \mu \cdot \int_{w_B}^{\overline{w}} [y(w) - \psi(U(w), y(w))]\, f(w)dw \\
&+ \int_{\underline{w}}^{w_B} \eta(w) \frac{\bar{y}}{w^2} h'\left(\frac{\bar{y}}{w}\right) dw + \int_{w_B}^{\overline{w}} \eta(w) \frac{y(w)}{w^2} h'\left(\frac{y(w)}{w}\right) dw.
\end{aligned}$$

The necessary conditions with respect to the indirect utilities are given by:

$$\begin{aligned}
\{U(w)\}_{w\in(\underline{w},\overline{w})} :\ \eta'(w) &- \mu \cdot \frac{\partial\psi(U(w), \bar{y})}{\partial U(w)} f(w) \cdot \mathbb{I}_{\{w\in(\underline{w},w_B]\}} \\
&- \mu \cdot \frac{\partial\psi(U(w), y(w))}{\partial U(w)} f(w) \cdot \mathbb{I}_{\{w\in(w_B,\overline{w})\}} = 0,
\end{aligned} \tag{51}$$

$$U(\underline{w}) :\ (1+\xi)f(\underline{w}) + \eta(\underline{w}) - \mu \cdot \left[\frac{\partial\psi(U(w), \bar{y})}{\partial U(w)} f(w)\right]\Bigg|_{w=\underline{w}} = 0, \tag{52}$$

and

$$U(\overline{w}) :\ \lambda f(\overline{w}) - \eta(\overline{w}) - \mu \cdot \left[\frac{\partial\psi(U(w), y(w))}{\partial U(w)} f(w)\right]\Bigg|_{w=\overline{w}} = 0, \tag{53}$$

together with conditions (47) and (48). As previously, applying (42) and (47) to (52) and (53) yields:

$$\begin{aligned}
(1+\xi-\mu)f(\underline{w}) &= -\eta(\underline{w}) \geq 0, \\
(\lambda-\mu)f(\overline{w}) &= \eta(\overline{w}) \geq 0.
\end{aligned} \tag{54}$$

By (42) and the fundamental theorem of calculus, (51) implies

$$\eta(w) = \eta(\overline{w}) - \int_{w}^{\overline{w}} \mu f(t) \cdot \mathbb{I}_{\{t\in(\underline{w},\overline{w})\}} dt. \tag{55}$$

Applying (54) to (55) and re-grouping terms, one gets

$$\mu = \lambda f(\overline{w}) + (1+\xi)f(\underline{w}) > 0,$$

which, when combined with (54), gives $1 + \xi \geq \mu > 0$ and $\lambda \geq \mu > 0$. Using again the complementary-slackness conditions (48) reveals that at least the participation constraint for top talent must be binding in the optimum.

Last, for $k = \overline{w}$, because of the downward jump discontinuity of the selfishly optimal income schedule at $k = \overline{w}$, the bunching region must be $[w_b, \overline{w}]$, namely, we just need to consider the case with $\underline{w} < w_b < w_B = \overline{w}$. The Lagrange function is thus

$$\begin{aligned}
&\mathcal{L}\left(\{U(w), y(w)\}_{w\in[\underline{w},\overline{w}]}; \mu, \lambda, \xi, \{\eta(w)\}_{w\in[\underline{w},\overline{w}]}\right) \\
=\ & U(\overline{w})f(\overline{w}) + \lambda \cdot [U(\overline{w}) - \overline{u}]\, f(\overline{w}) + \xi \cdot [U(\underline{w}) - \underline{u}]\, f(\underline{w}) \\
&+ \eta(\underline{w})U(\underline{w}) - \eta(\overline{w})U(\overline{w}) + \int_{\underline{w}}^{\overline{w}} \eta'(w)U(w)dw \\
&+ \mu \cdot \int_{\underline{w}}^{w_b} [y(w) - \psi(U(w), y(w))]\, f(w)dw + \mu \cdot \int_{w_b}^{\overline{w}} [\bar{y} - \psi(U(w), \bar{y})]\, f(w)dw \\
&+ \int_{\underline{w}}^{w_b} \eta(w)\frac{y(w)}{w^2} h'\left(\frac{y(w)}{w}\right) dw + \int_{w_b}^{\overline{w}} \eta(w)\frac{\bar{y}}{w^2} h'\left(\frac{\bar{y}}{w}\right) dw.
\end{aligned}$$

The necessary conditions with respect to the indirect utilities are given by:

$$\begin{aligned}
\{U(w)\}_{w\in(\underline{w},\overline{w})}:\ \eta'(w) \ &- \ \mu \cdot \frac{\partial\psi(U(w), \bar{y})}{\partial U(w)} f(w) \cdot \mathbb{I}_{\{w\in[w_b,\overline{w})\}} \\
&- \ \mu \cdot \frac{\partial\psi(U(w), y(w))}{\partial U(w)} f(w) \cdot \mathbb{I}_{\{w\in(\underline{w},w_b)\}} = 0,
\end{aligned} \tag{56}$$

$$U(\underline{w}):\ \xi f(\underline{w}) + \eta(\underline{w}) - \mu \cdot \left[\frac{\partial\psi(U(w), y(w))}{\partial U(w)} f(w)\right]\Bigg|_{w=\underline{w}} = 0, \tag{57}$$

and

$$U(\overline{w}):\ (1+\lambda)f(\overline{w}) - \eta(\overline{w}) - \mu \cdot \left[\frac{\partial\psi(U(w), \bar{y})}{\partial U(w)} f(w)\right]\Bigg|_{w=\overline{w}} = 0, \tag{58}$$

together with conditions (47) and (48). Applying (47) and (42) to (57) and (58) gives

$$\begin{aligned}
(\xi - \mu)f(\underline{w}) \ &= \ -\eta(\underline{w}) \geq 0, \\
(1 + \lambda - \mu)f(\overline{w}) \ &= \eta(\overline{w}) \geq 0.
\end{aligned} \tag{59}$$

Combining (56) and (59) and manipulating algebraically, one gets

$$\mu = (1+\lambda)f(\overline{w}) + \xi f(\underline{w}) > 0,$$

which, when applied to (59), gives $\xi \geq \mu > 0$ and $1 + \lambda \geq \mu > 0$. Consequently, at least the usual participation constraint must be binding in the optimum. ■

Proof of Lemma 4.2. Using Proposition 4.1 and (13) yields

$$\begin{aligned}
(1+\lambda)\theta^M(w, y^*(w,k)) + \xi\theta^R(w, y^*(w,k)) \ &= \ 0,\ \forall w \in [\underline{w}, w_b(k)), \\
(1+\xi)\theta^R(w, y^*(w,k)) + \lambda\theta^M(w, y^*(w,k)) \ &= \ 0,\ \forall w \in (w_B(k), \overline{w}].
\end{aligned} \tag{60}$$

In light of Assumption 4.1 and Figure 1, implicit differentiation of (60) with respect to λ and ξ gives

$$\begin{aligned}
\frac{\partial y^*(w,k)}{\partial \lambda} \ &= \ -\frac{\theta^M(w, y^*(w,k))}{(1+\lambda)\theta^M_y(w, y^*(w,k)) + \xi\theta^R_y(w, y^*(w,k))} > 0, \\
\frac{\partial y^*(w,k)}{\partial \xi} \ &= \ -\frac{\theta^R(w, y^*(w,k))}{(1+\lambda)\theta^M_y(w, y^*(w,k)) + \xi\theta^R_y(w, y^*(w,k))} < 0, \quad \forall w \in [\underline{w}, w_b(k)),
\end{aligned} \tag{61}$$

and

$$\begin{aligned}\frac{\partial y^*(w,k)}{\partial \lambda} &= -\frac{\theta^M(w,y^*(w,k))}{(1+\xi)\theta_y^R(w,y^*(w,k))+\lambda\theta_y^M(w,y^*(w,k))} > 0,\\ \frac{\partial y^*(w,k)}{\partial \xi} &= -\frac{\theta^R(w,y^*(w,k))}{(1+\xi)\theta_y^R(w,y^*(w,k))+\lambda\theta_y^M(w,y^*(w,k))} < 0, \quad \forall w \in (w_B(k),\overline{w}].\end{aligned} \tag{62}$$

To highlight the dependence of selfishly optimal pre-tax incomes, $y^*(\cdot)$, on the Lagrange multipliers λ and ξ, we put

$$y^*(w) = \begin{cases}\tilde{y}^{\tilde{M}}(w,\lambda,\xi) & \text{for } w \in [\underline{w}, w_b),\\ \tilde{y}^{\tilde{R}}(w,\lambda,\xi) & \text{for } w \in (w_B, \overline{w}].\end{cases} \tag{63}$$

Given the continuity of the selfishly optimal income schedule, the selfishly optimal income allocation on the bridge over the bunching region $[w_b, w_B]$ is written as $\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi) = \tilde{y}^{\tilde{R}}(w_B,\lambda,\xi) \equiv \overline{y}$. Hence, using (61)-(63), continuity of the income schedule, and Assumption 4.1 yields:

$$\begin{aligned}&\frac{\partial y^{\tilde{M}}(w_b)}{\partial \lambda} - \frac{\partial y^{\tilde{R}}(w_B)}{\partial \lambda}\\ =\ &\frac{\partial \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial \lambda} - \frac{\partial \tilde{y}^{\tilde{R}}(w_B,\lambda,\xi)}{\partial \lambda}\\ =\ &-\frac{\theta^M(w_b,\overline{y})}{(1+\lambda)\theta_y^M(w_b,\overline{y})+\xi\theta_y^R(w_b,\overline{y})} + \frac{\theta^M(w_B,\overline{y})}{(1+\xi)\theta_y^R(w_B,\overline{y})+\lambda\theta_y^M(w_B,\overline{y})}\\ =\ &\frac{\overbrace{\theta^M(w_B,\overline{y})[(1+\lambda)\theta_y^M(w_b,\overline{y})+\xi\theta_y^R(w_b,\overline{y})]}^{-} - \overbrace{\theta^M(w_b,\overline{y})[(1+\xi)\theta_y^R(w_B,\overline{y})+\lambda\theta_y^M(w_B,\overline{y})]}^{-}}{\underbrace{[(1+\lambda)\theta_y^M(w_b,\overline{y})+\xi\theta_y^R(w_b,\overline{y})][(1+\xi)\theta_y^R(w_B,\overline{y})+\lambda\theta_y^M(w_B,\overline{y})]}_{+}}.\end{aligned} \tag{64}$$

Noting that

$$\begin{aligned}&\theta^M(w_B,\overline{y})\left[(1+\lambda)\theta_y^M(w_b,\overline{y})+\xi\theta_y^R(w_b,\overline{y})\right]\\ &\quad - \theta^M(w_b,\overline{y})\left[(1+\xi)\theta_y^R(w_B,\overline{y})+\lambda\theta_y^M(w_B,\overline{y})\right]\\ =\ &\lambda\left[\theta^M(w_B,\overline{y})\theta_y^M(w_b,\overline{y}) - \theta^M(w_b,\overline{y})\theta_y^M(w_B,\overline{y})\right]\\ &\quad + \xi\left[\theta^M(w_B,\overline{y})\theta_y^R(w_b,\overline{y}) - \theta^M(w_b,\overline{y})\theta_y^R(w_B,\overline{y})\right]\\ &\quad + \theta^M(w_B,\overline{y})\theta_y^M(w_b,\overline{y}) - \theta^M(w_b,\overline{y})\theta_y^R(w_B,\overline{y}) \leq 0\end{aligned}$$

if the following conditions are fulfilled:

$$\begin{aligned}\theta^M(w_B,\overline{y})\theta_y^M(w_b,\overline{y}) &\leq \theta^M(w_b,\overline{y})\theta_y^M(w_B,\overline{y}),\\ \theta^M(w_B,\overline{y})\theta_y^R(w_b,\overline{y}) &\leq \theta^M(w_b,\overline{y})\theta_y^R(w_B,\overline{y}),\end{aligned}$$

and simultaneously

$$\theta^M(w_B,\overline{y})\theta_y^M(w_b,\overline{y}) \leq \theta^M(w_b,\overline{y})\theta_y^R(w_B,\overline{y}).$$

Thus, applying these observations, combined with Assumption 4.1, to (64) reveals that

$$\begin{aligned}&\frac{\partial y^{\tilde{M}}(w_b)}{\partial \lambda} < \frac{\partial y^{\tilde{R}}(w_B)}{\partial \lambda} \quad \text{if}\\ &\frac{\theta^M(w_b,\overline{y})}{\theta^M(w_B,\overline{y})} \leq \min\left\{\frac{\theta_y^M(w_b,\overline{y})}{\theta_y^M(w_B,\overline{y})}, \frac{\theta_y^R(w_b,\overline{y})}{\theta_y^R(w_B,\overline{y})}, \frac{\theta_y^M(w_b,\overline{y})}{\theta_y^R(w_B,\overline{y})}\right\}.\end{aligned} \tag{65}$$

In the meantime, using (14) yields

$$\theta_y^M(w,y)-\theta_y^R(w,y)=h''\left(\frac{y}{w}\right)\frac{1}{w^3}\left[2+\frac{h'''\left(\frac{y}{w}\right)\frac{y}{w}}{h''\left(\frac{y}{w}\right)}\right].$$

Consequently, if $-\frac{h'''(\frac{y}{w})\frac{y}{w}}{h''(\frac{y}{w})}\leq 2$, then we have

$$\frac{\theta_y^M(w_b,\overline{y})}{\theta_y^R(w_B,\overline{y})}\leq\min\left\{\frac{\theta_y^M(w_b,\overline{y})}{\theta_y^M(w_B,\overline{y})},\frac{\theta_y^R(w_b,\overline{y})}{\theta_y^R(w_B,\overline{y})}\right\}$$

which when applied to (65) gives rise to the desired assertion.

In order to get more concrete conditions under which Assumption 4.2(i)-(ii) are fulfilled, we provide the following analysis. Firstly, for $\forall w<w_b(k)$, applying (31) and (35) in the proof of Lemma 4.1 to (61) and collecting terms, we obtain:

$$\begin{aligned}\frac{\partial y^*}{\partial\lambda}&=\frac{\left[1-h'\left(\frac{y^*}{w}\right)\frac{1}{w}\right]f(w)+\frac{1}{w^2}h'\left(\frac{y^*}{w}\right)\left[1+\tilde{\varepsilon}(w,y^*)\right]F(w)}{\xi h''\left(\frac{y^*}{w}\right)\frac{1}{w^2}f(w)\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]-(1+\lambda)h''\left(\frac{y^*}{w}\right)\frac{1}{w^2}f(w)\left[2\left(\frac{F(w)}{wf(w)}\right)-1\right]}\\&=\frac{w^2-wh'\left(\frac{y^*}{w}\right)+wh'\left(\frac{y^*}{w}\right)\left[1+\tilde{\varepsilon}(w,y^*)\right]\frac{F(w)}{wf(w)}}{\xi h''\left(\frac{y^*}{w}\right)\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]-(1+\lambda)h''\left(\frac{y^*}{w}\right)\left[2\left(\frac{F(w)}{wf(w)}\right)-1\right]}\end{aligned}\tag{66}$$

and

$$\begin{aligned}\frac{\partial y^*}{\partial\xi}&=\frac{\left[1-h'\left(\frac{y^*}{w}\right)\frac{1}{w}\right]f(w)-\frac{1}{w^2}h'\left(\frac{y^*}{w}\right)\left[1+\tilde{\varepsilon}(w,y^*)\right]\left[1-F(w)\right]}{\xi h''\left(\frac{y^*}{w}\right)\frac{1}{w^2}f(w)\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]-(1+\lambda)h''\left(\frac{y^*}{w}\right)\frac{1}{w^2}f(w)\left[2\left(\frac{F(w)}{wf(w)}\right)-1\right]}\\&=\frac{w^2-wh'\left(\frac{y^*}{w}\right)-wh'\left(\frac{y^*}{w}\right)\left[1+\tilde{\varepsilon}(w,y^*)\right]\frac{1-F(w)}{wf(w)}}{\xi h''\left(\frac{y^*}{w}\right)\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]-(1+\lambda)h''\left(\frac{y^*}{w}\right)\left[2\left(\frac{F(w)}{wf(w)}\right)-1\right]}.\end{aligned}\tag{67}$$

Similarly, for $\forall w>w_B(k)$, applying (31) and (35) to (62) and rearranging the algebra, we obtain:

$$\frac{\partial y^*}{\partial\lambda}=\frac{w^2-wh'\left(\frac{y^*}{w}\right)+wh'\left(\frac{y^*}{w}\right)\left[1+\tilde{\varepsilon}(w,y^*)\right]\frac{F(w)}{wf(w)}}{(1+\xi)h''\left(\frac{y^*}{w}\right)\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]-\lambda h''\left(\frac{y^*}{w}\right)\left[2\left(\frac{F(w)}{wf(w)}\right)-1\right]}\tag{68}$$

and

$$\frac{\partial y^*}{\partial\xi}=\frac{w^2-wh'\left(\frac{y^*}{w}\right)-wh'\left(\frac{y^*}{w}\right)\left[1+\tilde{\varepsilon}(w,y^*)\right]\frac{1-F(w)}{wf(w)}}{(1+\xi)h''\left(\frac{y^*}{w}\right)\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]-\lambda h''\left(\frac{y^*}{w}\right)\left[2\left(\frac{F(w)}{wf(w)}\right)-1\right]}.\tag{69}$$

Assume that $h(l)=l^2/2$, then it follows from (30) that $\tilde{\varepsilon}(w,y^*)\equiv 1$, and hence (66)-(69) can be further simplified as follows:

$$\frac{\partial y^*}{\partial\lambda}=\frac{w^2+\left[2\left(\frac{F(w)}{wf(w)}\right)-1\right]y^*}{\xi\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]-(1+\lambda)\left[2\left(\frac{F(w)}{wf(w)}\right)-1\right]}>0\quad\forall w<w_b(k),\tag{70}$$

$$\frac{\partial y^*}{\partial\xi}=\frac{w^2-\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]y^*}{\xi\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]-(1+\lambda)\left[2\left(\frac{F(w)}{wf(w)}\right)-1\right]}<0\quad\forall w<w_b(k),\tag{71}$$

and

$$\frac{\partial y^*}{\partial \lambda} = \frac{w^2 + \left[2\left(\frac{F(w)}{wf(w)}\right) - 1\right] y^*}{(1+\xi)\left[2\left(\frac{1-F(w)}{wf(w)}\right) + 1\right] - \lambda\left[2\left(\frac{F(w)}{wf(w)}\right) - 1\right]} > 0 \quad \forall w > w_B(k), \tag{72}$$

$$\frac{\partial y^*}{\partial \xi} = \frac{w^2 - \left[2\left(\frac{1-F(w)}{wf(w)}\right) + 1\right] y^*}{(1+\xi)\left[2\left(\frac{1-F(w)}{wf(w)}\right) + 1\right] - \lambda\left[2\left(\frac{F(w)}{wf(w)}\right) - 1\right]} < 0 \quad \forall w > w_B(k). \tag{73}$$

Taking derivative of (70) with respect to w yields

$$\begin{aligned} \frac{\partial^2 y^*}{\partial \lambda \partial w} &= \frac{2w + 2y^*\left(\frac{\mathrm{d}\left[\frac{F(w)}{wf(w)}\right]}{\mathrm{d}w}\right) + \left[2\left(\frac{F(w)}{wf(w)}\right) - 1\right]\frac{\partial y^*}{\partial w}}{\xi\left[2\left(\frac{1-F(w)}{wf(w)}\right) + 1\right] - (1+\lambda)\left[2\left(\frac{F(w)}{wf(w)}\right) - 1\right]} \\ &\quad - \frac{w^2 + \left[2\left(\frac{F(w)}{wf(w)}\right) - 1\right] y^*}{\left\{\xi\left[2\left(\frac{1-F(w)}{wf(w)}\right) + 1\right] - (1+\lambda)\left[2\left(\frac{F(w)}{wf(w)}\right) - 1\right]\right\}^2} \\ &\quad \times \left\{2\xi\left(\frac{\mathrm{d}\left[\frac{1-F(w)}{wf(w)}\right]}{\mathrm{d}w}\right) - 2(1+\lambda)\left(\frac{\mathrm{d}\left[\frac{F(w)}{wf(w)}\right]}{\mathrm{d}w}\right)\right\} \quad \forall w < w_b(k), \end{aligned} \tag{74}$$

and so we arrive at

$$\frac{\partial^2 y^*}{\partial \lambda \partial w} > 0 \quad \text{for} \quad \forall w < w_b(k) \tag{75}$$

if the following conditions are satisfied:

$$\frac{\mathrm{d}\left[\frac{1-F(w)}{wf(w)}\right]}{\mathrm{d}w} < 0, \quad \frac{\mathrm{d}\left[\frac{F(w)}{wf(w)}\right]}{\mathrm{d}w} > 0, \quad \text{and} \quad \frac{\partial y^*}{\partial w} \leq \frac{w + y^*\left(\frac{\mathrm{d}\left[\frac{F(w)}{wf(w)}\right]}{\mathrm{d}w}\right)}{\frac{1}{2} - \frac{F(w)}{wf(w)}} \tag{76}$$

for $F(w)/(wf(w)) < 1/2$ and $\forall w < w_b(k)$.

Similar to (74), we use (72) to get

$$\begin{aligned} \frac{\partial^2 y^*}{\partial \lambda \partial w} &= \frac{2w + 2y^*\left(\frac{\mathrm{d}\left[\frac{F(w)}{wf(w)}\right]}{\mathrm{d}w}\right) + \left[2\left(\frac{F(w)}{wf(w)}\right) - 1\right]\frac{\partial y^*}{\partial w}}{(1+\xi)\left[2\left(\frac{1-F(w)}{wf(w)}\right) + 1\right] - \lambda\left[2\left(\frac{F(w)}{wf(w)}\right) - 1\right]} \\ &\quad - \frac{w^2 + \left[2\left(\frac{F(w)}{wf(w)}\right) - 1\right] y^*}{\left\{(1+\xi)\left[2\left(\frac{1-F(w)}{wf(w)}\right) + 1\right] - \lambda\left[2\left(\frac{F(w)}{wf(w)}\right) - 1\right]\right\}^2} \\ &\quad \times \left\{2(1+\xi)\left(\frac{\mathrm{d}\left[\frac{1-F(w)}{wf(w)}\right]}{\mathrm{d}w}\right) - 2\lambda\left(\frac{\mathrm{d}\left[\frac{F(w)}{wf(w)}\right]}{\mathrm{d}w}\right)\right\} \quad \forall w > w_B(k), \end{aligned}$$

and so we arrive at

$$\frac{\partial^2 y^*}{\partial \lambda \partial w} > 0 \quad \text{for} \quad \forall w > w_B(k) \tag{77}$$

if the conditions given by (76) are satisfied for $\forall w > w_B(k)$.

For any given $w \in [\underline{w}, \overline{w}]$, a comparison of (70) with (72) reveals that these two terms of cross-partial derivatives share the same numerator, but differ in the denominator. To see the difference more precisely, noting that

$$\begin{aligned}&\overbrace{\left\{\xi\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]-(1+\lambda)\left[2\left(\frac{F(w)}{wf(w)}\right)-1\right]\right\}}^{+}\\ -&\underbrace{\left\{(1+\xi)\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]-\lambda\left[2\left(\frac{F(w)}{wf(w)}\right)-1\right]\right\}}_{+}=-\frac{2}{wf(w)}<0,\end{aligned} \tag{78}$$

and hence the term given by (70) is larger than the term given by (72), for any given $w \in [\underline{w}, \overline{w}]$. Recall that Assumption 4.2(i) calls for

$$\frac{\partial y^{\tilde{M}}(w_b)}{\partial \lambda} \leq \frac{\partial y^{\tilde{R}}(w_B)}{\partial \lambda}.$$

Under the conditions given by (76), (75) yields that $\partial y^{\tilde{M}}(w)/\partial\lambda$ is monotonically increasing in w for all $w < w_b(k)$, and also (77) yields that $\partial y^{\tilde{R}}(w)/\partial\lambda$ is monotonically increasing in w for all $w > w_B(k)$. Meanwhile, (78) implies that $\partial y^{\tilde{M}}(w)/\partial\lambda > \partial y^{\tilde{R}}(w)/\partial\lambda$ for any given w. Therefore, combining all of these facts, to get Assumption 4.2(i) we need to impose that the right bridge endpoint w_B is sufficiently larger than the left bridge endpoint w_b. That is, the bunching region, which includes the proposer's skill type, must be sufficiently large; or, the bridge in the selfishly optimal income schedule must be sufficiently long.

By the same token, concerning (71) and (73), we have

$$\begin{aligned}\frac{\partial^2 y^*}{\partial\xi\partial w} &= \frac{2w-2y^*\left(\frac{\mathrm{d}\left[\frac{1-F(w)}{wf(w)}\right]}{\mathrm{d}w}\right)-\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]\frac{\partial y^*}{\partial w}}{\xi\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]-(1+\lambda)\left[2\left(\frac{F(w)}{wf(w)}\right)-1\right]}\\ &\quad-\frac{w^2-\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]y^*}{\left\{\xi\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]-(1+\lambda)\left[2\left(\frac{F(w)}{wf(w)}\right)-1\right]\right\}^2}\\ &\quad\times\left\{2\xi\left(\frac{\mathrm{d}\left[\frac{1-F(w)}{wf(w)}\right]}{\mathrm{d}w}\right)-2(1+\lambda)\left(\frac{\mathrm{d}\left[\frac{F(w)}{wf(w)}\right]}{\mathrm{d}w}\right)\right\}\quad\forall w<w_b(k),\end{aligned}$$

and

$$\begin{aligned}\frac{\partial^2 y^*}{\partial\xi\partial w} &= \frac{2w-2y^*\left(\frac{\mathrm{d}\left[\frac{1-F(w)}{wf(w)}\right]}{\mathrm{d}w}\right)-\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]\frac{\partial y^*}{\partial w}}{(1+\xi)\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]-\lambda\left[2\left(\frac{F(w)}{wf(w)}\right)-1\right]}\\ &\quad-\frac{w^2-\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]y^*}{\left\{(1+\xi)\left[2\left(\frac{1-F(w)}{wf(w)}\right)+1\right]-\lambda\left[2\left(\frac{F(w)}{wf(w)}\right)-1\right]\right\}^2}\\ &\quad\times\left\{2(1+\xi)\left(\frac{\mathrm{d}\left[\frac{1-F(w)}{wf(w)}\right]}{\mathrm{d}w}\right)-2\lambda\left(\frac{\mathrm{d}\left[\frac{F(w)}{wf(w)}\right]}{\mathrm{d}w}\right)\right\}\quad\forall w>w_B(k).\end{aligned}$$

Hence, we have

$$\frac{\partial^2 y^{\tilde{M}}(w)}{\partial\xi\partial w} < 0 \text{ for } \forall w < w_b(k), \quad \text{and} \quad \frac{\partial^2 y^{\tilde{R}}(w)}{\partial\xi\partial w} < 0 \text{ for } \forall w > w_B(k), \tag{79}$$

if the following conditions are satisfied:

$$\frac{\mathrm{d}\left[\frac{1-F(w)}{wf(w)}\right]}{\mathrm{d}w} < 0, \quad \frac{\mathrm{d}\left[\frac{F(w)}{wf(w)}\right]}{\mathrm{d}w} > 0, \quad \text{and} \quad \frac{\partial y^*}{\partial w} \geq \frac{w - y^*\left(\frac{\mathrm{d}\left[\frac{1-F(w)}{wf(w)}\right]}{\mathrm{d}w}\right)}{\frac{1}{2} + \frac{1-F(w)}{wf(w)}}. \tag{80}$$

Recall that Assumption 4.2(ii) calls for

$$\frac{\partial y^{\tilde{M}}(w_b)}{\partial\xi} \geq \frac{\partial y^{\tilde{R}}(w_B)}{\partial\xi}.$$

Under the conditions given by (80), (79) yields that $\partial y^{\tilde{M}}(w)/\partial\xi$ is monotonically decreasing in w for all $w < w_b(k)$, and $\partial y^{\tilde{R}}(w)/\partial\xi$ is monotonically decreasing in w for all $w > w_B(k)$. Meanwhile, (78) implies that $\partial y^{\tilde{M}}(w)/\partial\xi < \partial y^{\tilde{R}}(w)/\partial\xi$ for any given w. Therefore, combining all of these facts, to get Assumption 4.2(ii) we need to impose that the right bridge endpoint w_B is sufficiently larger than the left bridge endpoint w_b.

In summary, (80) together with (76) yields

$$\frac{\mathrm{d}\left[\frac{1-F(w)}{wf(w)}\right]}{\mathrm{d}w} < 0, \quad \frac{\mathrm{d}\left[\frac{F(w)}{wf(w)}\right]}{\mathrm{d}w} > 0, \tag{81}$$

and

$$\underbrace{\frac{w - y^*\left(\frac{\mathrm{d}\left[\frac{1-F(w)}{wf(w)}\right]}{\mathrm{d}w}\right)}{\frac{1}{2} + \frac{1-F(w)}{wf(w)}}}_{+} \leq \frac{\partial y^*}{\partial w} \leq \underbrace{\frac{w + y^*\left(\frac{\mathrm{d}\left[\frac{F(w)}{wf(w)}\right]}{\mathrm{d}w}\right)}{\frac{1}{2} - \frac{F(w)}{wf(w)}}}_{+} \tag{82}$$

for $F(w)/(wf(w)) < 1/2$. Naturally, we need to guarantee that in condition (82) the lower bound is indeed smaller than the upper bound. After some algebra, we have

$$\begin{aligned} &\frac{w + y^*\left(\frac{\mathrm{d}\left[\frac{F(w)}{wf(w)}\right]}{\mathrm{d}w}\right)}{\frac{1}{2} - \frac{F(w)}{wf(w)}} - \frac{w - y^*\left(\frac{\mathrm{d}\left[\frac{1-F(w)}{wf(w)}\right]}{\mathrm{d}w}\right)}{\frac{1}{2} + \frac{1-F(w)}{wf(w)}} \\ &= \frac{\frac{1}{wf(w)}\left\{w + y^*\left(\frac{\mathrm{d}\left[\frac{F(w)}{wf(w)}\right]}{\mathrm{d}w}\right)\right\} + \left(\frac{1}{2} - \frac{F(w)}{wf(w)}\right) y^*\left(\frac{\mathrm{d}\left[\frac{1}{wf(w)}\right]}{\mathrm{d}w}\right)}{\left(\frac{1}{2} - \frac{F(w)}{wf(w)}\right)\left(\frac{1}{2} + \frac{1-F(w)}{wf(w)}\right)}. \end{aligned}$$

Thus, if $F(w)/(wf(w))$ is monotonically nondecreasing in w and $F(w)/(wf(w)) < 1/2$, then a sufficient condition to guarantee that the lower bound be smaller than the upper bound is that $f'(w) < 0$ and $wf'(w)/f(w) \leq -1$. That is, the density $f(w)$ monotonically decreases and the elasticity of the density with respect to skill type is no greater than -1.

Note that the proof of Lemma 4.1 has identified the conditions under which $F(w)/(wf(w))$ is monotonically increasing in w and $F(w)/(wf(w)) < 1/2$, for Pareto, Weibull, and lognormal skill distributions. Also, Boadway and Jacquet (2008) have derived the conditions under which

$(1-F(w))/(wf(w))$ is monotonically decreasing in w for these distributions. Therefore, one could argue that the conditions given by (81) are not too restrictive for these empirically plausible distributions. ■

Proof of Lemma 4.3. As previously, we have three cases to consider.

Case 1: $\underline{w} < w_b(k) < w_B(k) < \overline{w}$. Using Proposition 4.1 and (63), the four equations that solve for the two Lagrange multipliers and the two bridge endpoints are expressed as follows:

$$\begin{aligned}
\Upsilon(w_b, w_B, \lambda, \xi, k) &\equiv \tilde{y}^{\tilde{M}}(w_b, \lambda, \xi) - \tilde{y}^{\tilde{R}}(w_B, \lambda, \xi) = 0, \\
\Psi(w_b, w_B, \lambda, \xi, k) &\equiv \int_{\underline{w}}^{w_b} G^M\left(w, \tilde{y}^{\tilde{M}}(w, \lambda, \xi)\right) dw + \int_{w_b}^{w_B} G^M\left(w, \tilde{y}^{\tilde{M}}(w_b, \lambda, \xi)\right) dw \\
&\quad + \int_{w_B}^{\overline{w}} G^M\left(w, \tilde{y}^{\tilde{R}}(w, \lambda, \xi)\right) dw - \overline{u} = 0, \\
\Gamma(w_b, w_B, \lambda, \xi, k) &\equiv \int_{\underline{w}}^{w_b} G^R\left(w, \tilde{y}^{\tilde{M}}(w, \lambda, \xi)\right) dw + \int_{w_b}^{w_B} G^R\left(w, \tilde{y}^{\tilde{M}}(w_b, \lambda, \xi)\right) dw \\
&\quad + \int_{w_B}^{\overline{w}} G^R\left(w, \tilde{y}^{\tilde{R}}(w, \lambda, \xi)\right) dw - \underline{u} = 0,
\end{aligned} \tag{83}$$

and

$$\begin{aligned}
\Phi(w_b, w_B, \lambda, \xi, k) &\equiv (1+\lambda)\int_{w_b}^{k} \theta^M\left(w, \tilde{y}^{\tilde{M}}(w_b, \lambda, \xi)\right) dw \\
&\quad + \lambda \int_{k}^{w_B} \theta^M\left(w, \tilde{y}^{\tilde{M}}(w_b, \lambda, \xi)\right) dw + \xi \int_{w_b}^{k} \theta^R\left(w, \tilde{y}^{\tilde{M}}(w_b, \lambda, \xi)\right) dw \\
&\quad + (1+\xi)\int_{k}^{w_B} \theta^R\left(w, \tilde{y}^{\tilde{M}}(w_b, \lambda, \xi)\right) dw = 0.
\end{aligned} \tag{84}$$

Using Assumption 4.2, strictly increasing monotonicity of the selfishly optimal income schedule outside of the bridge, and (83) yields:

$$\begin{aligned}
\Upsilon_{w_b} &= \frac{\partial \tilde{y}^{\tilde{M}}(w_b, \lambda, \xi)}{\partial w_b} > 0, \quad \Upsilon_{w_B} = -\frac{\partial \tilde{y}^{\tilde{R}}(w_B, \lambda, \xi)}{\partial w_B} < 0, \\
\Upsilon_{\lambda} &= \frac{\partial \tilde{y}^{\tilde{M}}(w_b, \lambda, \xi)}{\partial \lambda} - \frac{\partial \tilde{y}^{\tilde{R}}(w_B, \lambda, \xi)}{\partial \lambda} \le 0, \\
\Upsilon_{\xi} &= \frac{\partial \tilde{y}^{\tilde{M}}(w_b, \lambda, \xi)}{\partial \xi} - \frac{\partial \tilde{y}^{\tilde{R}}(w_B, \lambda, \xi)}{\partial \xi} \ge 0, \text{ and } \Upsilon_k = 0.
\end{aligned} \tag{85}$$

Using Assumption 4.1, continuity of the selfishly optimal income schedule, (61), (62), and (83) yields:

$$\begin{aligned}
\Psi_{w_b} &= \int_{w_b}^{w_B} \theta^M\left(w, \tilde{y}^{\tilde{M}}(w_b, \lambda, \xi)\right) dw \cdot \frac{\partial \tilde{y}^{\tilde{M}}(w_b, \lambda, \xi)}{\partial w_b} > 0, \\
\Psi_{w_B} &= G^M\left(w_B, \tilde{y}^{\tilde{M}}(w_b, \lambda, \xi)\right) - G^M\left(w_B, \tilde{y}^{\tilde{R}}(w_B, \lambda, \xi)\right) = 0, \text{ and } \Psi_k = 0;
\end{aligned} \tag{86}$$

$$\begin{aligned}
\Psi_{\lambda} &= \int_{\underline{w}}^{w_b} \theta^M\left(w, \tilde{y}^{\tilde{M}}(w, \lambda, \xi)\right) \cdot \frac{\partial \tilde{y}^{\tilde{M}}(w, \lambda, \xi)}{\partial \lambda} dw \\
&\quad + \int_{w_b}^{w_B} \theta^M\left(w, \tilde{y}^{\tilde{M}}(w_b, \lambda, \xi)\right) dw \cdot \frac{\partial \tilde{y}^{\tilde{M}}(w_b, \lambda, \xi)}{\partial \lambda} \\
&\quad + \int_{w_B}^{\overline{w}} \theta^M\left(w, \tilde{y}^{\tilde{R}}(w, \lambda, \xi)\right) \cdot \frac{\partial \tilde{y}^{\tilde{R}}(w, \lambda, \xi)}{\partial \lambda} dw = \frac{\partial \widetilde{U}(\overline{w})}{\partial \lambda} > 0,
\end{aligned} \tag{87}$$

$$\begin{aligned}\Psi_\xi &= \int_{\underline{w}}^{w_b} \theta^M\left(w, \tilde{y}^{\tilde{M}}(w,\lambda,\xi)\right)\cdot \frac{\partial \tilde{y}^{\tilde{M}}(w,\lambda,\xi)}{\partial \xi}dw \\ &+ \int_{w_b}^{w_B} \theta^M\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)dw\cdot \frac{\partial \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial \xi} \\ &+ \int_{w_B}^{\overline{w}} \theta^M\left(w, \tilde{y}^{\tilde{R}}(w,\lambda,\xi)\right)\cdot \frac{\partial \tilde{y}^{\tilde{R}}(w,\lambda,\xi)}{\partial \xi}dw = \frac{\partial \widetilde{U}(\overline{w})}{\partial \xi} < 0;\end{aligned} \tag{88}$$

and also,

$$\begin{aligned}\Gamma_{w_b} &= \int_{w_b}^{w_B} \theta^R\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)dw\cdot \frac{\partial \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial w_b} < 0, \\ \Gamma_{w_B} &= G^R\left(w_B, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) - G^R\left(w_B, \tilde{y}^{\tilde{R}}(w_B,\lambda,\xi)\right) = 0, \text{ and } \Gamma_k = 0;\end{aligned} \tag{89}$$

$$\begin{aligned}\Gamma_\lambda &= \int_{\underline{w}}^{w_b} \theta^R\left(w, \tilde{y}^{\tilde{M}}(w,\lambda,\xi)\right)\cdot \frac{\partial \tilde{y}^{\tilde{M}}(w,\lambda,\xi)}{\partial \lambda}dw \\ &+ \int_{w_b}^{w_B} \theta^R\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)dw\cdot \frac{\partial \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial \lambda} \\ &+ \int_{w_B}^{\overline{w}} \theta^R\left(w, \tilde{y}^{\tilde{R}}(w,\lambda,\xi)\right)\cdot \frac{\partial \tilde{y}^{\tilde{R}}(w,\lambda,\xi)}{\partial \lambda}dw = \frac{\partial \widetilde{U}(\underline{w})}{\partial \lambda} < 0,\end{aligned} \tag{90}$$

$$\begin{aligned}\Gamma_\xi &= \int_{\underline{w}}^{w_b} \theta^R\left(w, \tilde{y}^{\tilde{M}}(w,\lambda,\xi)\right)\cdot \frac{\partial \tilde{y}^{\tilde{M}}(w,\lambda,\xi)}{\partial \xi}dw \\ &+ \int_{w_b}^{w_B} \theta^R\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)dw\cdot \frac{\partial \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial \xi} \\ &+ \int_{w_B}^{\overline{w}} \theta^R\left(w, \tilde{y}^{\tilde{R}}(w,\lambda,\xi)\right)\cdot \frac{\partial \tilde{y}^{\tilde{R}}(w,\lambda,\xi)}{\partial \xi}dw = \frac{\partial \widetilde{U}(\underline{w})}{\partial \xi} > 0.\end{aligned} \tag{91}$$

Similarly, making use of Assumption 4.1, (60)-(62), (84), and continuity of the selfishly optimal income schedule, we arrive at

$$\begin{aligned}\Phi_{w_b} &\equiv -\overbrace{\left[(1+\lambda)\theta^M\left(w_b, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)dw + \xi\theta^R\left(w_b, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)\right]}^{0} \\ &+ \int_{w_b}^{k}\left[\xi\theta^R_y\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) + (1+\lambda)\theta^M_y\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)\right]\frac{\partial \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial w_b}dw \\ &+ \int_{k}^{w_B}\left[(1+\xi)\theta^R_y\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) + \lambda\theta^M_y\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)\right]\frac{\partial \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial w_b}dw \\ &< 0, \\ \Phi_{w_B} &\equiv \lambda\theta^M\left(w_B, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) + (1+\xi)\theta^R\left(w_B, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) = 0, \\ \Phi_k &\equiv \theta^M\left(k, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) - \theta^R\left(k, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) > 0;\end{aligned} \tag{92}$$

$$\begin{aligned}\Phi_\lambda \;&\equiv\; \int_{w_b}^{w_B} \theta^M\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)dw \\ &+\int_{w_b}^{k}\left[\xi\theta_y^R\left(w,\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)+(1+\lambda)\theta_y^M\left(w,\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)\right]\frac{\partial\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial\lambda}dw \\ &+\int_{k}^{w_B}\left[(1+\xi)\theta_y^R\left(w,\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)+\lambda\theta_y^M\left(w,\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)\right]\frac{\partial\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial\lambda}dw \\ &= \int_{w_b}^{w_B} \theta^M\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)dw \\ &-\int_{w_b}^{k} \theta^M\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)dw-\int_{k}^{w_B} \theta^M\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)dw=0;\end{aligned} \tag{93}$$

and by the same token,

$$\begin{aligned}\Phi_\xi \;&\equiv\; \int_{w_b}^{w_B} \theta^R\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)dw \\ &+\int_{w_b}^{k}\left[\xi\theta_y^R\left(w,\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)+(1+\lambda)\theta_y^M\left(w,\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)\right]\frac{\partial\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial\xi}dw \\ &+\int_{k}^{w_B}\left[(1+\xi)\theta_y^R\left(w,\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)+\lambda\theta_y^M\left(w,\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)\right]\frac{\partial\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial\xi}dw \\ &= 0.\end{aligned} \tag{94}$$

Now, expressed in matrix form, the relevant comparative statics are as follows:

$$\begin{bmatrix}\Upsilon_{w_b} & \Upsilon_{w_B} & \Upsilon_\lambda & \Upsilon_\xi \\ \Psi_{w_b} & \Psi_{w_B} & \Psi_\lambda & \Psi_\xi \\ \Gamma_{w_b} & \Gamma_{w_B} & \Gamma_\lambda & \Gamma_\xi \\ \Phi_{w_b} & \Phi_{w_B} & \Phi_\lambda & \Phi_\xi\end{bmatrix}\begin{bmatrix}\mathrm{d}w_b \\ \mathrm{d}w_B \\ \mathrm{d}\lambda \\ \mathrm{d}\xi\end{bmatrix}=\begin{bmatrix}-\Upsilon_k \\ -\Psi_k \\ -\Gamma_k \\ -\Phi_k\end{bmatrix}\mathrm{d}k. \tag{95}$$

Substituting the related terms of (85)-(94) into (95) yields:

$$\underbrace{\begin{bmatrix}\Upsilon_{w_b} & \Upsilon_{w_B} & \Upsilon_\lambda & \Upsilon_\xi \\ \Psi_{w_b} & 0 & \Psi_\lambda & \Psi_\xi \\ \Gamma_{w_b} & 0 & \Gamma_\lambda & \Gamma_\xi \\ \Phi_{w_b} & 0 & 0 & 0\end{bmatrix}}_{\mathcal{H}}\begin{bmatrix}\mathrm{d}w_b \\ \mathrm{d}w_B \\ \mathrm{d}\lambda \\ \mathrm{d}\xi\end{bmatrix}=\begin{bmatrix}0 \\ 0 \\ 0 \\ -\Phi_k\end{bmatrix}\mathrm{d}k. \tag{96}$$

Thus, the determinant of $\mathcal{H}$ is derived as

$$\det(\mathcal{H}) \;=\; -\underbrace{\Phi_{w_b}}_{-}\cdot\underbrace{\Upsilon_{w_B}}_{-}\cdot\left(\underbrace{\Psi_\lambda}_{+}\cdot\underbrace{\Gamma_\xi}_{+}-\underbrace{\Gamma_\lambda}_{-}\cdot\underbrace{\Psi_\xi}_{-}\right) \tag{97}$$

where we have used (85), (92), (87)-(88), and (90)-(91). To sign $\det(\mathcal{H})$, we use (87)-(88), (90)-(91), and Assumption 4.2, and find that

$$\begin{aligned}
&\underbrace{\Psi_\lambda}_{+} \cdot \underbrace{\Gamma_\xi}_{+} - \underbrace{\Gamma_\lambda}_{-} \cdot \underbrace{\Psi_\xi}_{-} > 0 \\
\iff &\underbrace{\frac{\partial \widetilde{U}(\overline{w})}{\partial \lambda}}_{+} \cdot \underbrace{\frac{\partial \widetilde{U}(\underline{w})}{\partial \xi}}_{+} > \underbrace{\frac{\partial \widetilde{U}(\overline{w})}{\partial \xi}}_{-} \cdot \underbrace{\frac{\partial \widetilde{U}(\underline{w})}{\partial \lambda}}_{-} \\
\iff &\underbrace{-\frac{\partial \widetilde{U}(\overline{w})/\partial \lambda}{\partial \widetilde{U}(\underline{w})/\partial \lambda}}_{+} > \underbrace{-\frac{\partial \widetilde{U}(\overline{w})/\partial \xi}{\partial \widetilde{U}(\underline{w})/\partial \xi}}_{+}.
\end{aligned} \tag{98}$$

Hence, applying (98) to (97) gives rise to

$$\det(\mathcal{H}) < 0 \tag{99}$$

under Assumption 4.2.

In view of (99), (96), (97) and (92), we can apply Cramer's rule and get the following derivatives:

$$\frac{\mathrm{d}w_b(k)}{\mathrm{d}k} = \frac{1}{\det(\mathcal{H})} \begin{vmatrix} 0 & \Upsilon_{w_B} & \Upsilon_\lambda & \Upsilon_\xi \\ 0 & 0 & \Psi_\lambda & \Psi_\xi \\ 0 & 0 & \Gamma_\lambda & \Gamma_\xi \\ -\Phi_k & 0 & 0 & 0 \end{vmatrix} = -\frac{\Phi_k}{\Phi_{w_b}} > 0, \tag{100}$$

$$\begin{aligned}
\frac{\mathrm{d}w_B(k)}{\mathrm{d}k} &= \frac{1}{\det(\mathcal{H})} \begin{vmatrix} \Upsilon_{w_b} & 0 & \Upsilon_\lambda & \Upsilon_\xi \\ \Psi_{w_b} & 0 & \Psi_\lambda & \Psi_\xi \\ \Gamma_{w_b} & 0 & \Gamma_\lambda & \Gamma_\xi \\ \Phi_{w_b} & -\Phi_k & 0 & 0 \end{vmatrix} \\
&= \frac{\Phi_k \left[\Upsilon_{w_b}\left(\Psi_\lambda\Gamma_\xi - \Psi_\xi\Gamma_\lambda\right) - \Upsilon_\lambda\left(\Psi_{w_b}\Gamma_\xi - \Psi_\xi\Gamma_{w_b}\right) + \Upsilon_\xi\left(\Psi_{w_b}\Gamma_\lambda - \Psi_\lambda\Gamma_{w_b}\right)\right]}{\Phi_{w_b}\Upsilon_{w_B}\left(\Psi_\lambda\Gamma_\xi - \Psi_\xi\Gamma_\lambda\right)},
\end{aligned} \tag{101}$$

$$\frac{\mathrm{d}\lambda(k)}{\mathrm{d}k} = \frac{1}{\det(\mathcal{H})} \begin{vmatrix} \Upsilon_{w_b} & \Upsilon_{w_B} & 0 & \Upsilon_\xi \\ \Psi_{w_b} & 0 & 0 & \Psi_\xi \\ \Gamma_{w_b} & 0 & 0 & \Gamma_\xi \\ \Phi_{w_b} & 0 & -\Phi_k & 0 \end{vmatrix} = -\frac{\Upsilon_{w_B}\Phi_k\left(\Psi_{w_b}\Gamma_\xi - \Psi_\xi\Gamma_{w_b}\right)}{\det(\mathcal{H})}, \tag{102}$$

and

$$\frac{\mathrm{d}\xi(k)}{\mathrm{d}k} = \frac{1}{\det(\mathcal{H})} \begin{vmatrix} \Upsilon_{w_b} & \Upsilon_{w_B} & \Upsilon_\lambda & 0 \\ \Psi_{w_b} & 0 & \Psi_\lambda & 0 \\ \Gamma_{w_b} & 0 & \Gamma_\lambda & 0 \\ \Phi_{w_b} & 0 & 0 & -\Phi_k \end{vmatrix} = \frac{\Upsilon_{w_B}\Phi_k\left(\Psi_{w_b}\Gamma_\lambda - \Psi_\lambda\Gamma_{w_b}\right)}{\det(\mathcal{H})}. \tag{103}$$

To sign these derivatives and establish the related comparative statics, we need to appeal to Assumption 4.2. Specifically, using (86)-(87) and (89)-(90) gives rise to

$$\begin{aligned}
\Psi_{w_b}\Gamma_\lambda - \Psi_\lambda\Gamma_{w_b} = &\int_{w_b}^{w_B} \theta^M\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) dw \cdot \frac{\partial \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial w_b} \frac{\partial \widetilde{U}(\underline{w})}{\partial \lambda} \\
&- \int_{w_b}^{w_B} \theta^R\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) dw \cdot \frac{\partial \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial w_b} \frac{\partial \widetilde{U}(\overline{w})}{\partial \lambda} > 0 \\
\iff \quad &-\frac{\partial \widetilde{U}(\overline{w})/\partial \lambda}{\partial \widetilde{U}(\underline{w})/\partial \lambda} > -\frac{\int_{w_b}^{w_B} \theta^M\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) dw}{\int_{w_b}^{w_B} \theta^R\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) dw}
\end{aligned} \tag{104}$$

as assumed in Assumption 4.2. Hence, applying (104), (99), (85), and (92) to (103) reveals that

$$\frac{\mathrm{d}\xi(k)}{\mathrm{d}k} = \frac{\overbrace{\Upsilon_{w_B}\Phi_k}^{-} \cdot \overbrace{(\Psi_{w_b}\Gamma_\lambda - \Psi_\lambda\Gamma_{w_b})}^{+}}{\underbrace{\det(\mathcal{H})}_{-}} > 0$$

holds true under Assumption 4.2. Similarly, using (86), (88), (89), and (91) gives

$$\begin{aligned}
\Psi_{w_b}\Gamma_\xi - \Psi_\xi\Gamma_{w_b} = & \int_{w_b}^{w_B} \theta^M\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) dw \cdot \frac{\partial \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial w_b}\frac{\partial \widetilde{U}(\underline{w})}{\partial \xi} \\
& - \int_{w_b}^{w_B} \theta^R\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) dw \cdot \frac{\partial \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial w_b}\frac{\partial \widetilde{U}(\overline{w})}{\partial \xi} > 0 \qquad (105) \\
\iff \quad & -\frac{\partial \widetilde{U}(\overline{w})/\partial\xi}{\partial \widetilde{U}(\underline{w})/\partial\xi} < -\frac{\int_{w_b}^{w_B} \theta^M\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) dw}{\int_{w_b}^{w_B} \theta^R\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) dw}.
\end{aligned}$$

Applying Assumption 4.2, (105), (99), (85), and (92) to (102) yields

$$\frac{\mathrm{d}\lambda(k)}{\mathrm{d}k} = -\frac{\overbrace{\Upsilon_{w_B}\Phi_k}^{-} \cdot \overbrace{(\Psi_{w_b}\Gamma_\xi - \Psi_\xi\Gamma_{w_b})}^{+}}{\underbrace{\det(\mathcal{H})}_{-}} < 0.$$

Finally, applying (104)-(105), (98), (92), and (85) to (101) reveals that

$$\begin{aligned}
\frac{\mathrm{d}w_B(k)}{\mathrm{d}k} = & \frac{\overbrace{\Phi_k}^{+} \cdot \overbrace{\Upsilon_{w_b}(\Psi_\lambda\Gamma_\xi - \Psi_\xi\Gamma_\lambda)}^{+}}{\underbrace{\Phi_{w_b}\Upsilon_{w_B}}_{+} \cdot \underbrace{(\Psi_\lambda\Gamma_\xi - \Psi_\xi\Gamma_\lambda)}_{+}} \\
& - \frac{\overbrace{\Phi_k}^{+} \cdot \left[\overbrace{\Upsilon_\lambda(\Psi_{w_b}\Gamma_\xi - \Psi_\xi\Gamma_{w_b})}^{-} - \overbrace{\Upsilon_\xi(\Psi_{w_b}\Gamma_\lambda - \Psi_\lambda\Gamma_{w_b})}^{+}\right]}{\underbrace{\Phi_{w_b}\Upsilon_{w_B}}_{+} \cdot \underbrace{(\Psi_\lambda\Gamma_\xi - \Psi_\xi\Gamma_\lambda)}_{+}} > 0
\end{aligned}$$

holds true under Assumption 4.2.

Case 2: $\underline{w} < w_b(k) < w_B(k) = \overline{w}$. Only minor modifications are needed to the proof of Case 1. The terms in which $w > w_B(k)$ drop out as they are no longer relevant. In other words, the two Lagrange multipliers and w_b are determined by simultaneously solving

$$\begin{aligned}
\hat{\Psi}(w_b,\lambda,\xi,k) &\equiv \int_{\underline{w}}^{w_b} G^M\left(w, \tilde{y}^{\tilde{M}}(w,\lambda,\xi)\right) dw + \int_{w_b}^{\overline{w}} G^M\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) dw - \overline{u} = 0, \\
\hat{\Gamma}(w_b,\lambda,\xi,k) &\equiv \int_{\underline{w}}^{w_b} G^R\left(w, \tilde{y}^{\tilde{M}}(w,\lambda,\xi)\right) dw + \int_{w_b}^{\overline{w}} G^R\left(w, \tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right) dw - \underline{u} = 0,
\end{aligned}$$

and

$$\begin{aligned}\hat{\Phi}(w_b,\lambda,\xi,k) \;&\equiv (1+\lambda)\int_{w_b}^{k}\theta^M\left(w,\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)dw\\&+\lambda\int_{k}^{\overline{w}}\theta^M\left(w,\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)dw+\xi\int_{w_b}^{k}\theta^R\left(w,\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)dw\\&+(1+\xi)\int_{k}^{\overline{w}}\theta^R\left(w,\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)dw=0.\end{aligned}$$

Expressed in matrix form, implicit differentiation of the three equations yields:

$$\begin{bmatrix}\hat{\Psi}_{w_b} & \hat{\Psi}_{\lambda} & \hat{\Psi}_{\xi}\\ \hat{\Gamma}_{w_b} & \hat{\Gamma}_{\lambda} & \hat{\Gamma}_{\xi}\\ \hat{\Phi}_{w_b} & \hat{\Phi}_{\lambda} & \hat{\Phi}_{\xi}\end{bmatrix}\begin{bmatrix}\mathrm{d}w_b\\ \mathrm{d}\lambda\\ \mathrm{d}\xi\end{bmatrix}=\begin{bmatrix}-\hat{\Psi}_k\\ -\hat{\Gamma}_k\\ -\hat{\Phi}_k\end{bmatrix}\mathrm{d}k,$$

where the entries are given by

$$\begin{aligned}\hat{\Psi}_{w_b} \;&= \int_{w_b}^{\overline{w}}\theta^M\left(w,\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)dw\cdot\frac{\partial\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial w_b}>0,\\ \hat{\Psi}_k \;&= 0,\quad \hat{\Psi}_\lambda=\frac{\partial\widetilde{U}(\overline{w})}{\partial\lambda}>0,\quad \hat{\Psi}_\xi=\frac{\partial\widetilde{U}(\overline{w})}{\partial\xi}<0;\end{aligned}$$

and

$$\begin{aligned}\hat{\Gamma}_{w_b} \;&= \int_{w_b}^{\overline{w}}\theta^R\left(w,\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)\right)dw\cdot\frac{\partial\tilde{y}^{\tilde{M}}(w_b,\lambda,\xi)}{\partial w_b}<0,\\ \hat{\Gamma}_k \;&= 0,\quad \hat{\Gamma}_\lambda=\frac{\partial\widetilde{U}(\underline{w})}{\partial\lambda}<0,\quad \hat{\Gamma}_\xi=\frac{\partial\widetilde{U}(\underline{w})}{\partial\xi}>0,\end{aligned}$$

together with $\hat{\Phi}_{w_b}<0, \hat{\Phi}_\lambda=\hat{\Phi}_\xi=0$, and $\hat{\Phi}_k>0$. The remaining proof is analogous to that of Case 1, and hence is omitted to economize on the space.

Case 3: $\underline{w}=w_b(k)<w_B(k)<\overline{w}$. Exploiting Proposition 4.1 reveals that the two Lagrange multipliers and w_B are determined by simultaneously solving

$$\begin{aligned}\check{\Psi}(w_B,\lambda,\xi,k) \;&\equiv \int_{\underline{w}}^{w_B}G^M\left(w,\tilde{y}^{\tilde{R}}(w_B,\lambda,\xi)\right)dw+\int_{w_B}^{\overline{w}}G^M\left(w,\tilde{y}^{\tilde{R}}(w,\lambda,\xi)\right)dw-\overline{u}=0,\\ \check{\Gamma}(w_B,\lambda,\xi,k) \;&\equiv \int_{\underline{w}}^{w_B}G^R\left(w,\tilde{y}^{\tilde{R}}(w_B,\lambda,\xi)\right)dw+\int_{w_B}^{\overline{w}}G^R\left(w,\tilde{y}^{\tilde{R}}(w,\lambda,\xi)\right)dw-\underline{u}=0,\end{aligned}$$

and

$$\begin{aligned}\check{\Phi}(w_B,\lambda,\xi,k) \;&\equiv (1+\lambda)\int_{\underline{w}}^{k}\theta^M\left(w,\tilde{y}^{\tilde{R}}(w_B,\lambda,\xi)\right)dw\\&+\lambda\int_{k}^{w_B}\theta^M\left(w,\tilde{y}^{\tilde{R}}(w_B,\lambda,\xi)\right)dw+\xi\int_{\underline{w}}^{k}\theta^R\left(w,\tilde{y}^{\tilde{R}}(w_B,\lambda,\xi)\right)dw\\&+(1+\xi)\int_{k}^{w_B}\theta^R\left(w,\tilde{y}^{\tilde{R}}(w_B,\lambda,\xi)\right)dw=0.\end{aligned}$$

The proof that remains is quite analogous to that of Case 2, and therefore is available only upon request. ■

Proof of Lemma 4.4. Similar to (166), we make use of the binding participation constraint for the lowest type, and then get the derivative of individuals' gross utility with respect to proposer's type k as follows:

$$\begin{aligned}\frac{\partial U(w,k)}{\partial k} &= \int_{\underline{w}}^{w}\left[h''\left(\frac{y^*(t,k)}{t}\right)\frac{y^*(t,k)}{t^3}+h'\left(\frac{y^*(t,k)}{t}\right)\frac{1}{t^2}\right]\frac{\partial y^*(t,k)}{\partial k}dt \\ &= \int_{\underline{w}}^{w}\left[\theta^M(t,y^*(t,k))-\theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt.\end{aligned} \tag{106}$$

Obviously, we need to consider three cases:

Case 1: $\underline{w} < w_b(k) < w_B(k) < \overline{w}$. Making use of Proposition 4.1, Lemma 4.3, (61)-(62), and continuity of the selfishly optimal income schedule, we give the following:

$$\begin{aligned}\frac{\partial y^*(w,k)}{\partial k} &= \frac{\partial \tilde{y}^{\tilde{M}}(w,\lambda(k),\xi(k))}{\partial k} = \frac{\partial \tilde{y}^{\tilde{M}}}{\partial \lambda}\frac{\mathrm{d}\lambda(k)}{\mathrm{d}k}+\frac{\partial \tilde{y}^{\tilde{M}}}{\partial \xi}\frac{\mathrm{d}\xi(k)}{\mathrm{d}k} < 0, \quad \forall w < w_b(k), \\ \frac{\partial y^*(w,k)}{\partial k} &= \frac{\partial \tilde{y}^{\tilde{R}}(w,\lambda(k),\xi(k))}{\partial k} = \frac{\partial \tilde{y}^{\tilde{R}}}{\partial \lambda}\frac{\mathrm{d}\lambda(k)}{\mathrm{d}k}+\frac{\partial \tilde{y}^{\tilde{R}}}{\partial \xi}\frac{\mathrm{d}\xi(k)}{\mathrm{d}k} < 0, \quad \forall w > w_B(k),\end{aligned} \tag{107}$$

and

$$\begin{aligned}\frac{\partial y^*(w,k)}{\partial k} &= \frac{\partial \tilde{y}^{\tilde{M}}(w_b(k),\lambda(k),\xi(k))}{\partial k} \\ &= \underbrace{\frac{\partial \tilde{y}^{\tilde{M}}}{\partial w_b}\frac{\mathrm{d}w_b(k)}{\mathrm{d}k}}_{+}+\underbrace{\frac{\partial \tilde{y}^{\tilde{M}}}{\partial \lambda}\frac{\mathrm{d}\lambda(k)}{\mathrm{d}k}+\frac{\partial \tilde{y}^{\tilde{M}}}{\partial \xi}\frac{\mathrm{d}\xi(k)}{\mathrm{d}k}}_{-}, \ \forall w \in (w_b(k),w_B(k)).\end{aligned} \tag{108}$$

For the income allocation under consideration, the binding participation constraints amount to

$$\begin{aligned}&\int_{\underline{w}}^{w_b(k)} G^M(w,y^*(w,k))\,dw + \int_{w_b(k)}^{w_B(k)} G^M(w,y^*(w,k))\,dw \\ &\qquad + \int_{w_B(k)}^{\overline{w}} G^M(w,y^*(w,k))\,dw = \overline{u}, \\ &\int_{\underline{w}}^{w_b(k)} G^R(w,y^*(w,k))\,dw + \int_{w_b(k)}^{w_B(k)} G^R(w,y^*(w,k))\,dw \\ &\qquad + \int_{w_B(k)}^{\overline{w}} G^R(w,y^*(w,k))\,dw = \underline{u}.\end{aligned} \tag{109}$$

Differentiating both sides of (109) with respect to k yields:

$$\begin{aligned}&\overbrace{\int_{\underline{w}}^{w_b(k)} \theta^M(w,y^*(w,k))\frac{\partial y^*(w,k)}{\partial k}dw}^{-} + \int_{w_b(k)}^{w_B(k)} \overbrace{\theta^M(w,y^*(w,k))}^{+}\frac{\partial y^*(w,k)}{\partial k}dw \\ &\qquad \underbrace{+\int_{w_B(k)}^{\overline{w}} \theta^M(w,y^*(w,k))\frac{\partial y^*(w,k)}{\partial k}dw}_{-} = 0, \\ &\overbrace{\int_{\underline{w}}^{w_b(k)} \theta^R(w,y^*(w,k))\frac{\partial y^*(w,k)}{\partial k}dw}^{+} + \int_{w_b(k)}^{w_B(k)} \overbrace{\theta^R(w,y^*(w,k))}^{-}\frac{\partial y^*(w,k)}{\partial k}dw \\ &\qquad \underbrace{+\int_{w_B(k)}^{\overline{w}} \theta^R(w,y^*(w,k))\frac{\partial y^*(w,k)}{\partial k}dw}_{+} = 0,\end{aligned} \tag{110}$$

where we have used (107) and Assumption 4.1. Applying (110) to (108) reveals that

$$\frac{\partial y^*(w,k)}{\partial k} = \frac{\partial \tilde{y}^{\tilde{M}}\left(w_b(k), \lambda(k), \xi(k)\right)}{\partial k} > 0,\ \forall w \in (w_b(k), w_B(k)). \tag{111}$$

(a) Suppose $w < k$, then single-peakedness requires that $\partial U(w,k)/\partial k < 0$. If $w \leq w_b(k)$, then using (106) and (107) yields:

$$\frac{\partial U(w,k)}{\partial k} = \int_{\underline{w}}^{w} \underbrace{\left[\theta^M(t, y^*(t,k)) - \theta^R(t, y^*(t,k))\right]}_{+} \underbrace{\frac{\partial y^*(t,k)}{\partial k}}_{-} dt < 0,$$

as desired.

If, however, $w > w_b(k)$, then we need to make some preparations. Firstly, using the second equality of (110) and (60) gives

$$\begin{aligned}\int_{w_b(k)}^{w_B(k)} \theta^R\left(w, y^*(w,k)\right) \frac{\partial y^*(w,k)}{\partial k} dw = &\left(\frac{1+\lambda}{\xi}\right) \int_{\underline{w}}^{w_b(k)} \theta^M\left(w, y^*(w,k)\right) \frac{\partial y^*(w,k)}{\partial k} dw \\ &+ \left(\frac{\lambda}{1+\xi}\right) \int_{w_B(k)}^{\overline{w}} \theta^M\left(w, y^*(w,k)\right) \frac{\partial y^*(w,k)}{\partial k} dw.\end{aligned} \tag{112}$$

It follows from (84) that

$$\begin{aligned}&(1+\lambda)\int_{w_b}^{k} \theta^M\left(w, y^*(w,k)\right) dw + \lambda \int_{k}^{w_B} \theta^M\left(w, y^*(w,k)\right) dw \\ &+ \xi \int_{w_b}^{w_B} \theta^R\left(w, y^*(w,k)\right) dw + \int_{k}^{w_B} \theta^R\left(w, y^*(w,k)\right) dw = 0.\end{aligned} \tag{113}$$

Making use of (111) and (112), we get from (113) that

$$\begin{aligned}&\int_{k}^{w_B} \theta^R\left(w, y^*(w,k)\right) \frac{\partial y^*(w,k)}{\partial k} dw \\ =\ & -(1+\lambda)\int_{w_b}^{k} \theta^M\left(w, y^*(w,k)\right) \frac{\partial y^*(w,k)}{\partial k} dw - \lambda \int_{k}^{w_B} \theta^M\left(w, y^*(w,k)\right) \frac{\partial y^*(w,k)}{\partial k} dw \\ & -(1+\lambda)\int_{\underline{w}}^{w_b(k)} \theta^M\left(w, y^*(w,k)\right) \frac{\partial y^*(w,k)}{\partial k} dw \\ & -\left(\frac{\lambda\xi}{1+\xi}\right)\int_{w_B(k)}^{\overline{w}} \theta^M\left(w, y^*(w,k)\right) \frac{\partial y^*(w,k)}{\partial k} dw.\end{aligned} \tag{114}$$

Note that (106) can be rewritten as

$$\begin{aligned}\frac{\partial U(w,k)}{\partial k} =\ & \int_{\underline{w}}^{w_b(k)} \left[\theta^M(t, y^*(t,k)) - \theta^R(t, y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt \\ & + \int_{w_b(k)}^{k} \left[\theta^M(t, y^*(t,k)) - \theta^R(t, y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt \\ & - \int_{w}^{k} \left[\theta^M(t, y^*(t,k)) - \theta^R(t, y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt,\end{aligned}$$

and hence using (60) yields:

$$\begin{aligned}\frac{\partial U(w,k)}{\partial k} &= \left(\frac{1+\lambda+\xi}{\xi}\right)\int_{\underline{w}}^{w_b(k)} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt \\ &+ \int_{w_b(k)}^{k} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt - \int_{w_b(k)}^{w_B(k)} \theta^R(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt \\ &+ \int_{k}^{w_B(k)} \theta^R(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt \\ &- \int_{w}^{k} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt.\end{aligned} \tag{115}$$

Plugging (112) and (114) in (115) and rearranging the algebra, we obtain

$$\begin{aligned}\frac{\partial U(w,k)}{\partial k} &= \lambda\int_{\underline{w}}^{\overline{w}} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt \\ &\quad - \int_{w}^{k} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt \\ &= -\int_{w}^{k} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt < 0,\end{aligned}$$

where we have used (110) and (111), and hence the desired assertion follows.

(b) Suppose $w > k$, then single-peakedness requires that $\partial U(w,k)/\partial k > 0$. If $w > w_B(k)$, then (106) amounts to

$$\begin{aligned}\frac{\partial U(w,k)}{\partial k} &= \int_{\underline{w}}^{w_b(k)} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt \\ &\quad + \int_{w_b(k)}^{w_B(k)} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt \\ &\quad + \int_{w_B(k)}^{w} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt.\end{aligned}$$

Thus, making use of (60) gives

$$\begin{aligned}\frac{\partial U(w,k)}{\partial k} &= \left(\frac{1+\lambda+\xi}{\xi}\right)\int_{\underline{w}}^{w_b(k)} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt \\ &+ \int_{w_b(k)}^{w_B(k)} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt - \int_{w_b(k)}^{w_B(k)} \theta^R(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt \\ &+ \left(\frac{1+\lambda+\xi}{1+\xi}\right)\int_{w_B(k)}^{w} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt.\end{aligned} \tag{116}$$

Substituting (112) into (116) and rearranging the algebra, we arrive at

$$\begin{aligned}\frac{\partial U(w,k)}{\partial k} &= \int_{\underline{w}}^{w_b(k)} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt + \int_{w_b(k)}^{w_B(k)} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt \\ &- \left(\frac{\lambda}{1+\xi}\right)\int_{w_B(k)}^{\overline{w}} \theta^M\left(w,y^*(w,k)\right)\frac{\partial y^*(w,k)}{\partial k}dw \\ &+ \left(\frac{1+\lambda+\xi}{1+\xi}\right)\int_{w_B(k)}^{w} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt.\end{aligned} \tag{117}$$

To simplify (117) with (110), we do the following algebraic manipulation:

$$
\begin{aligned}
\frac{\partial U(w,k)}{\partial k} &= \int_{\underline{w}}^{w_b(k)} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt + \int_{w_b(k)}^{w_B(k)} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt \\
&+ \int_{w_B(k)}^{\overline{w}} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt - \int_{w_B(k)}^{\overline{w}} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt \\
&- \left(\frac{\lambda}{1+\xi}\right)\int_{w_B(k)}^{\overline{w}} \theta^M\left(w,y^*(w,k)\right)\frac{\partial y^*(w,k)}{\partial k}dw \\
&+ \left(\frac{1+\lambda+\xi}{1+\xi}\right)\int_{w_B(k)}^{w} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt.
\end{aligned}
\tag{118}
$$

Hence, (118) becomes, after collecting terms as well as applying (107),

$$
\frac{\partial U(w,k)}{\partial k} = \underbrace{-\left(\frac{1+\lambda+\xi}{1+\xi}\right)}_{-}\int_{w}^{\overline{w}} \underbrace{\theta^M(t,y^*(t,k))}_{+}\cdot\underbrace{\frac{\partial y^*(t,k)}{\partial k}}_{-}dt > 0
$$

as desired.

If, instead, we have $w \le w_B(k)$, then (106) amounts to

$$
\begin{aligned}
\frac{\partial U(w,k)}{\partial k} &= \int_{\underline{w}}^{w_b(k)} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt \\
&+ \int_{w_b(k)}^{k} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt \\
&+ \int_{k}^{w} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt.
\end{aligned}
$$

Consequently, in view of (60), we have

$$
\begin{aligned}
\frac{\partial U(w,k)}{\partial k} &= \left(\frac{1+\lambda+\xi}{\xi}\right)\int_{\underline{w}}^{w_b(k)} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt \\
&+ \int_{w_b(k)}^{k} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt - \int_{w_b(k)}^{w_B(k)} \theta^R(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt \\
&+ \int_{k}^{w_B(k)} \theta^R(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt \\
&+ \int_{k}^{w} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt.
\end{aligned}
\tag{119}
$$

Plugging (112) and (114) in (119) and simplifying, we obtain

$$
\frac{\partial U(w,k)}{\partial k} = \int_{k}^{w} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt > 0
$$

as desired.

Case 2: $\underline{w} < w_b(k) < w_B(k) = \overline{w}$. Only minor modifications are needed to the proof of Case 1. The terms in which $w > w_B(k)$ drop out as they are no longer relevant.

Case 3: $\underline{w} = w_b(k) < w_B(k) < \overline{w}$. One can verify that the proof is quite similar to those of the previous two cases, and hence we omit it to save space. ■

Proof of Proposition 5.2. In light of formulas (26), (28) and (29), Proposition 5.2(i)-(ii) are straightforward. Hence, we will just show the proof of Proposition 5.2(iii). Simple differentiation of (29) yields:

$$\begin{aligned} \frac{\partial \frac{\tau^1(w)}{1-\tau^1(w)}}{\partial \xi^1} &= \frac{1+\varepsilon(w)}{wf(w)} \cdot \frac{1+\lambda^1}{(1+\xi^1+\lambda^1)^2} > 0, \\ \frac{\partial \frac{\tau^1(w)}{1-\tau^1(w)}}{\partial \lambda^1} &= -\frac{1+\varepsilon(w)}{wf(w)} \cdot \frac{\xi^1}{(1+\xi^1+\lambda^1)^2} < 0, \ \forall w < w_b^1(w_m); \end{aligned} \tag{120}$$

and

$$\begin{aligned} \frac{\partial \frac{\tau^1(w)}{1-\tau^1(w)}}{\partial \xi^1} &= \frac{1+\varepsilon(w)}{wf(w)} \cdot \frac{\lambda^1}{(1+\xi^1+\lambda^1)^2} > 0, \\ \frac{\partial \frac{\tau^1(w)}{1-\tau^1(w)}}{\partial \lambda^1} &= -\frac{1+\varepsilon(w)}{wf(w)} \cdot \frac{1+\xi^1}{(1+\xi^1+\lambda^1)^2} < 0, \ \forall w > w_B^1(w_m). \end{aligned} \tag{121}$$

Making use of chain rule gives rise to

$$\begin{aligned} \frac{\partial \frac{\tau^1(w)}{1-\tau^1(w)}}{\partial \xi^1} &= \left(1-\tau^1(w)\right)^{-2} \cdot \frac{\partial \tau^1(w)}{\partial \xi^1}, \\ \frac{\partial \frac{\tau^1(w)}{1-\tau^1(w)}}{\partial \lambda^1} &= \left(1-\tau^1(w)\right)^{-2} \cdot \frac{\partial \tau^1(w)}{\partial \lambda^1}. \end{aligned} \tag{122}$$

Thus, applying (120) and (121) to (122) reveals

$$\frac{\partial \tau^1(w)}{\partial \xi^1} > 0 \ \text{ and } \ \frac{\partial \tau^1(w)}{\partial \lambda^1} < 0, \ \forall w < w_b^1(w_m) \ \text{ or } \ \forall w > w_B^1(w_m). \tag{123}$$

Additionally, using (120) yields:

$$\frac{\partial^2 \frac{\tau^1(w)}{1-\tau^1(w)}}{\partial \xi^1 \partial \lambda^1} = \frac{1+\varepsilon(w)}{wf(w)} \cdot \frac{\xi^1-(1+\lambda^1)}{(1+\xi^1+\lambda^1)^3}, \ \forall w < w_b^1(w_m). \tag{124}$$

Similarly, using (121) yields:

$$\frac{\partial^2 \frac{\tau^1(w)}{1-\tau^1(w)}}{\partial \xi^1 \partial \lambda^1} = \frac{1+\varepsilon(w)}{wf(w)} \cdot \frac{1+\xi^1-\lambda^1}{(1+\xi^1+\lambda^1)^3}, \ \forall w > w_B^1(w_m). \tag{125}$$

Differentiating both sides of the first equality in (122) with respect to λ^1 yields:

$$\frac{\partial^2 \frac{\tau^1(w)}{1-\tau^1(w)}}{\partial \xi^1 \partial \lambda^1} = 2\left(1-\tau^1(w)\right)^{-3} \cdot \frac{\partial \tau^1(w)}{\partial \lambda^1} \frac{\partial \tau^1(w)}{\partial \xi^1} + \left(1-\tau^1(w)\right)^{-2} \cdot \frac{\partial^2 \tau^1(w)}{\partial \xi^1 \partial \lambda^1}.$$

Regrouping terms and using (123), we arrive at

$$\underbrace{\left(1-\tau^1(w)\right)^{-2}}_{+} \cdot \frac{\partial^2 \tau^1(w)}{\partial \xi^1 \partial \lambda^1} = \frac{\partial^2 \frac{\tau^1(w)}{1-\tau^1(w)}}{\partial \xi^1 \partial \lambda^1} - \underbrace{2\left(1-\tau^1(w)\right)^{-3} \cdot \frac{\partial \tau^1(w)}{\partial \lambda^1} \frac{\partial \tau^1(w)}{\partial \xi^1}}_{-}. \tag{126}$$

Therefore, applying (124) and (125) to (126) immediately leads to the desired assertion. ■

Appendix B: The Case with only the Participation Constraint for Top Talent

In order to identify the impact of imposing either one of these two participation constraints on equilibrium income redistribution, here we present the analysis for the case in which only the participation constraint for top talent is taken into consideration. Thus, type k's problem reads as

$$\max_{\{x(w),y(w)\}_{w\in[\underline{w},\overline{w}]}} U(k) \tag{127}$$

subject to (3), (4), (5), (6) and (8). The set of alternatives that are voted on consists of allocations that solve (127) for some $k \in [\underline{w},\overline{w}]$. As in the text, we begin our analysis of Problem (127) by temporarily ignoring the SOIC (5), and so the reduced problem becomes

$$\max_{\{x(w),y(w)\}_{w\in[\underline{w},\overline{w}]}} U(k) \tag{128}$$

subject to (3), (4), (6) and (8).

As previously, the selfishly optimal schedule of before-tax incomes, denoted by $\tilde{y}(\cdot)$, for Problem (128) is obtained by solving

$$\begin{aligned} &\max_{y(\cdot)} \int_{\underline{w}}^{k} G^M(w,y(w))dw + \int_{k}^{\overline{w}} G^R(w,y(w))dw \\ &\text{subject to } \int_{\underline{w}}^{\overline{w}} G^M(w,y(w))dw \geq \overline{u}, \end{aligned} \tag{129}$$

where $G^M(\cdot)$ and $G^R(\cdot)$ are given by (11). The Lagrange function of Problem (129) is

$$\begin{aligned} \widetilde{\mathcal{L}}^k &= \int_{\underline{w}}^{k} G^M(w,y(w))dw + \int_{k}^{\overline{w}} G^R(w,y(w))dw + \lambda\left[\int_{\underline{w}}^{\overline{w}} G^M(w,y(w))dw - \overline{u}\right] \\ &= \int_{\underline{w}}^{k} (1+\lambda)G^M(w,y(w))dw + \int_{k}^{\overline{w}} \left[G^R(w,y(w)) + \lambda G^M(w,y(w))\right] dw - \lambda\overline{u}, \end{aligned} \tag{130}$$

where λ (depends on k) is the nonnegative Lagrange multiplier associated to the participation constraint (8). Simple differentiation of (130) with respect to $y(w)$ yields the FOCs:

$$\begin{aligned} \theta^M(w,y(w)) &= 0,\ \forall w \in [\underline{w},k), \\ \theta^R(w,y(w)) + \lambda\theta^M(w,y(w)) &= 0,\ \forall w \in (k,\overline{w}], \end{aligned} \tag{131}$$

where $\theta^M(\cdot)$ and $\theta^R(\cdot)$ are given by (14).

To ensure that Lagrangian (130) is strictly concave in income, we impose:

Assumption 6.1 *$\theta^M_y(w,y(w)) < 0$, $\theta^R_y(w,y(w)) < 0$, and $\theta^R_y(w,y(w)) + \lambda\theta^M_y(w,y(w)) < 0$, for $\forall(w,y) \in [\underline{w},\overline{w}] \times \mathbb{R}_+$.*

Using (2), (131) and (14), we get the MTRs corresponding to the maxi-max income allocation, to the maxi-min income allocation with $\lambda > 0$, and to the maxi-min income allocation

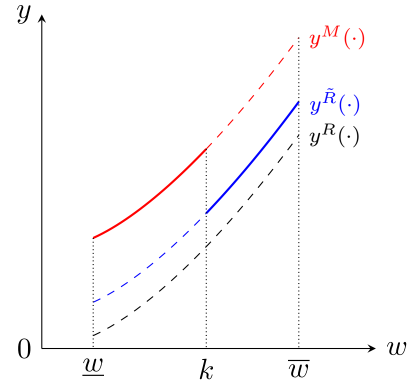

Figure 3: A downward jump discontinuity emerges at the proposer's skill type

with $\lambda = 0$, respectively, as follows:

$$
\begin{aligned}
\tau^M(w) &= T'\left(y^M(w)\right) = -\frac{F(w)}{f(w)}\left[\frac{y(w)}{w^3}h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2}h'\left(\frac{y(w)}{w}\right)\right] \le 0,\ \forall w \in [\underline{w}, \overline{w}],\\
\tau^{\tilde{R}}(w) &= T'\left(y^{\tilde{R}}(w)\right)\\
&= \left[\frac{1}{1+\lambda} - F(w)\right]\frac{1}{f(w)}\left[\frac{y(w)}{w^3}h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2}h'\left(\frac{y(w)}{w}\right)\right],\ \forall w \in [\underline{w}, \overline{w}],\\
\tau^R(w) &= T'\left(y^R(w)\right) = \frac{1-F(w)}{f(w)}\left[\frac{y(w)}{w^3}h''\left(\frac{y(w)}{w}\right) + \frac{1}{w^2}h'\left(\frac{y(w)}{w}\right)\right] \ge 0,\ \forall w \in [\underline{w}, \overline{w}].
\end{aligned}
\tag{132}
$$

Thus, we have $\tau^M(w) < \tau^{\tilde{R}}(w) < \tau^R(w)$ for all $w \in [\underline{w}, \overline{w}]$. Further, using (132) yields that all types but the bottom type receive positive transfers under the maxi-max tax schedule, and that all types but the top type pay positive taxes under the maxi-min tax schedule with $\lambda = 0$; under the maxi-min tax schedule with $\lambda > 0$, however, there exists a threshold of skill type such that types below this threshold pay positive taxes, while types above this threshold receive positive transfers. Consequently, the income allocation under the maxi-max tax scheme is generally distorted upward, the income allocation under the maxi-min tax scheme with $\lambda = 0$ is generally distorted downward, and the income allocation under the maxi-min tax scheme with $\lambda > 0$ is in between in terms of the size of tax distortion. Graphically speaking, income schedule $y^M(w)$ lies above income schedule $y^R(w)$, while income schedule $y^{\tilde{R}}(w)$ lies somewhere between these two schedules. Combining this observation with the FOCs (131) reveals that there must be a downward jump discontinuity at the proposer's skill type in the selfishly optimal income schedule, as shown in Figure 3. To guarantee incentive compatibility, we need to endogenously identify a bunching region such that the SOIC (5) is satisfied, as illustrated by Figure 4.

We now derive the incentive compatible income allocation to type k's problem as follows.

Proposition 6.1 *The optimal schedule of before-tax incomes, $y^*(\cdot)$, for type k's problem is continuous on $[\underline{w}, \overline{w}]$, and is given by*

$$
y^*(w) = \begin{cases} y^M(w) & \text{for } w \in [\underline{w}, w_b),\\ y^M(w_b) & \text{for } w \in [w_b, w_B] \text{ if } w_b > \underline{w},\\ y^{\tilde{R}}(w_B) & \text{for } w \in [w_b, w_B] \text{ if } w_B < \overline{w},\\ y^{\tilde{R}}(w) & \text{for } w \in (w_B, \overline{w}]. \end{cases} \tag{133}
$$

The optimal values of the Lagrange multiplier $\lambda > 0$ and the bridge endpoints w_b and w_B are

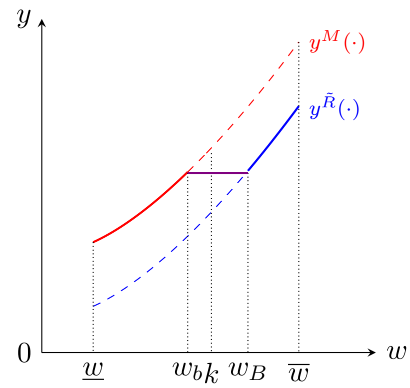

Figure 4: The incentive compatible income schedule with a bridge

determined by solving the binding participation constraint

$$\int_{\underline{w}}^{\overline{w}} G^M(w, y^*(w))dw - \overline{u} = 0,$$

together with either the first-order condition

$$(1+\lambda)\int_{w_b}^{k} \theta^M\left(w, y^M(w_b)\right)dw + \int_{k}^{w_B} \theta^R\left(w, y^M(w_b)\right)dw + \lambda\int_{k}^{w_B} \theta^M\left(w, y^M(w_b)\right)dw = 0$$

if $w_b > \underline{w}$*, or the first-order condition*

$$(1+\lambda)\int_{w_b}^{k} \theta^M\left(w, y^{\tilde{R}}(w_B)\right)dw + \int_{k}^{w_B} \theta^R\left(w, y^{\tilde{R}}(w_B)\right)dw + \lambda\int_{k}^{w_B} \theta^M\left(w, y^{\tilde{R}}(w_B)\right)dw = 0$$

if $w_B < \overline{w}$*. Moreover,* $\lambda > 0$ *holds true for all* $k \in [\underline{w}, \overline{w})$*.*

Proof. Following Brett and Weymark (2016), we will solve type k's problem (127) in two steps. In the first step, for fixed values of λ, w_b, and w_B, the optimal before-tax incomes outside of the bridge are determined by solving the FOCs given by (131), which are re-expressed as follows:

$$\begin{aligned} \theta^M(w, y(w)) &= 0, \ \forall w \in [\underline{w}, w_b), \\ \theta^R(w, y(w)) + \lambda\theta^M(w, y(w)) &= 0, \ \forall w \in (w_B, \overline{w}]. \end{aligned} \tag{134}$$

For a fixed value of $\lambda > 0$, we let $\tilde{y}(w, \lambda)$ denote the solution to (134). As is obvious, for $w < w_b$, we have $\tilde{y}(w, \lambda) = y^M(w)$, which does not depend on λ. Let $\bar{y}$ denote the before-tax income allocated to those types on the bridge. Then, the Lagrangian for the second step is

$$\begin{aligned} &\int_{\underline{w}}^{w_b} G^M\left(w, y^M(w)\right)dw + \int_{w_b}^{k} G^M\left(w, \bar{y}\right)dw + \int_{k}^{w_B} G^R\left(w, \bar{y}\right)dw \\ &+ \int_{w_B}^{\overline{w}} G^R\left(w, \tilde{y}(w, \lambda)\right)dw + \lambda\int_{\underline{w}}^{w_b} G^M\left(w, y^M(w)\right)dw \\ &+ \lambda\left(\int_{w_b}^{w_B} G^M\left(w, \bar{y}\right)dw + \int_{w_B}^{\overline{w}} G^M\left(w, \tilde{y}(w, \lambda)\right)dw - \overline{u}\right). \end{aligned} \tag{135}$$

There are three cases.

Case 1: $\underline{w} < w_b < w_B < \overline{w}$. The continuity of the income schedule yields

$$\bar{y} = y^M(w_b) = \tilde{y}(w_B, \lambda). \tag{136}$$

The choice of λ, w_b, and w_B is determined by simultaneously solving

$$y^M(w_b) - \tilde{y}(w_B, \lambda) = 0, \tag{137}$$

$$\begin{aligned}\int_{\underline{w}}^{w_b} G^M\left(w, y^M(w)\right) dw \ + \int_{w_b}^{w_B} G^M\left(w, y^M(w_b)\right) dw \\ + \int_{w_B}^{\overline{w}} G^M\left(w, \tilde{y}(w, \lambda)\right) dw - \overline{u} = 0,\end{aligned} \tag{138}$$

and

$$\begin{aligned}(1+\lambda)\int_{w_b}^{k} \theta^M\left(w, y^M(w_b)\right) dw + \int_{k}^{w_B} \theta^R\left(w, y^M(w_b)\right) dw \\ + \lambda \int_{k}^{w_B} \theta^M\left(w, y^M(w_b)\right) dw = 0.\end{aligned} \tag{139}$$

In particular, equation (139) is obtained by differentiating the Lagrangian above with respect to $\bar{y}$ and then, in light of (136), by evaluating the resulting derivative at $\bar{y} = y^M(w_b)$.

Case 2: $\underline{w} < w_b < w_B = \overline{w}$. Using continuity of the selfishly optimal income schedule, the types in $[w_b, \overline{w}]$ receive $y^M(w_b)$. Thus, the two variables w_b and λ are determined by solving

$$\int_{\underline{w}}^{w_b} G^M\left(w, y^M(w)\right) dw + \int_{w_b}^{\overline{w}} G^M\left(w, y^M(w_b)\right) dw - \overline{u} = 0, \tag{140}$$

and

$$\begin{aligned}(1+\lambda)\int_{w_b}^{k} \theta^M\left(w, y^M(w_b)\right) dw + \int_{k}^{\overline{w}} \theta^R\left(w, y^M(w_b)\right) dw \\ + \lambda \int_{k}^{\overline{w}} \theta^M\left(w, y^M(w_b)\right) dw = 0.\end{aligned} \tag{141}$$

Comparing equation (141) with equation (139) yields that the first-order condition over the bunching region is essentially the same whenever $w_b > \underline{w}$, regardless of whether $w_B < \overline{w}$ or $w_B = \overline{w}$.

Case 3: $\underline{w} = w_b < w_B < \overline{w}$. Now, the types in $[\underline{w}, w_B]$ receive $\tilde{y}(w_B, \lambda)$. Hence, w_B and λ are determined by solving

$$\int_{\underline{w}}^{w_B} G^M\left(w, \tilde{y}(w_B, \lambda)\right) dw + \int_{w_B}^{\overline{w}} G^M\left(w, \tilde{y}(w, \lambda)\right) dw - \overline{u} = 0, \tag{142}$$

and

$$\begin{aligned}(1+\lambda)\int_{\underline{w}}^{k} \theta^M\left(w, \tilde{y}(w_B, \lambda)\right) dw + \int_{k}^{w_B} \theta^R\left(w, \tilde{y}(w_B, \lambda)\right) dw \\ + \lambda \int_{k}^{w_B} \theta^M\left(w, \tilde{y}(w_B, \lambda)\right) dw = 0.\end{aligned} \tag{143}$$

Using (136), it is easy to see that equation (143) and equation (139) are essentially identical to each other given $w_B < \overline{w}$.

Following the same procedure used in proving Proposition 4.1, one can show that the Lagrange multiplier associated with the participation constraint for top talent satisfies $\lambda > 0$ for $\forall k \in [\underline{w}, \overline{w})$. We omit the technical details to economize on the space. ■

We have provided the full characterization of selfishly optimal nonlinear income tax schedules, and hence the next step is to apply the majority voting process to select a certain schedule that will be implemented in the voting equilibrium. In order to establish the existence of a voting equilibrium, we need to make use of the following result:

Lemma 6.1 *Suppose Assumption 6.1 holds. Then, for all* $k \in [\underline{w}, \overline{w})$, *a marginal increase in* k *results in*

(1) a decrease in $\lambda(k)$*;*

(2) an increase in $w_b(k)$ *and* $w_B(k)$ *if* $\underline{w} < w_b(k) < w_B(k) < \overline{w}$*;*

(3) no change in $w_b(k)$ *if* $\underline{w} < w_b(k) < w_B(k) = \overline{w}$*;*

(4) an increase in $w_B(k)$ *if* $\underline{w} = w_b(k) < w_B(k) < \overline{w}$*.*

Proof. Outside of the bridge on the income schedule proposed by type k, we have $y^*(w,k) = \tilde{y}(w, \lambda(k))$ that is a solution to equation (134). Applying the implicit function theorem and Assumption 6.1 to (134) yields:

$$\frac{\partial \tilde{y}(w, \lambda(k))}{\partial \lambda} = \frac{\partial y^M(w,k)}{\partial \lambda} = 0, \ \forall w < w_b(k) \tag{144}$$

and

$$\frac{\partial \tilde{y}(w, \lambda(k))}{\partial \lambda} = -\frac{\theta^M(w, y^*(w,k))}{\theta^R_y(w, y^*(w,k)) + \lambda \theta^M_y(w, y^*(w,k))} > 0, \ \forall w > w_B(k). \tag{145}$$

As previously, we consider three cases:

Case 1: $\underline{w} < w_b(k) < w_B(k) < \overline{w}$. To derive the comparative statics, we first express the left-hand sides of equations (137)-(139) as implicit functions of w_b, w_B, λ and k, and then rewrite these equations as

$$\begin{aligned}
\Upsilon(w_b, w_B, \lambda, k) &\equiv y^M(w_b) - \tilde{y}(w_B, \lambda) = 0, \\
\Psi(w_b, w_B, \lambda, k) &\equiv \int_{\underline{w}}^{w_b} G^M\left(w, y^M(w)\right) dw + \int_{w_b}^{w_B} G^M\left(w, y^M(w_b)\right) dw \\
&\quad + \int_{w_B}^{\overline{w}} G^M\left(w, \tilde{y}(w,\lambda)\right) dw - \overline{u} = 0, \\
\Phi(w_b, w_B, \lambda, k) &\equiv (1+\lambda) \int_{w_b}^{k} \theta^M\left(w, y^M(w_b)\right) dw + \int_{k}^{w_B} \theta^R\left(w, y^M(w_b)\right) dw \\
&\quad + \lambda \int_{k}^{w_B} \theta^M\left(w, y^M(w_b)\right) dw = 0.
\end{aligned} \tag{146}$$

Expressed in matrix form, using again the implicit function theorem yields:

$$\begin{bmatrix} \Upsilon_{w_b} & \Upsilon_{w_B} & \Upsilon_\lambda \\ \Psi_{w_b} & \Psi_{w_B} & \Psi_\lambda \\ \Phi_{w_b} & \Phi_{w_B} & \Phi_\lambda \end{bmatrix} \begin{bmatrix} \mathrm{d}w_b \\ \mathrm{d}w_B \\ \mathrm{d}\lambda \end{bmatrix} = \begin{bmatrix} -\Upsilon_k \\ -\Psi_k \\ -\Phi_k \end{bmatrix} \mathrm{d}k. \tag{147}$$

Using (146), we are able to compute the entries in (147). Firstly, by (5), (145), continuity of the selfishly optimal income schedule, as well as the strictly increasing monotonicity of the selfishly optimal income schedule outside of the bridge, we obtain

$$\begin{aligned}
\Upsilon_{w_b} &= \frac{\partial y^M(w_b)}{\partial w_b} > 0, \quad \Upsilon_{w_B} = -\frac{\partial \tilde{y}(w_B, \lambda)}{\partial w_B} < 0, \\
\Upsilon_\lambda &= -\frac{\partial \tilde{y}(w_B, \lambda)}{\partial \lambda} < 0, \text{ and } \Upsilon_k = 0.
\end{aligned} \tag{148}$$

Secondly, using Assumption 6.1, (137), and (145), the implicit differentiation of $\Psi(w_b, w_B, \lambda, k)$, and simplification of the resulting algebra yields:

$$\begin{aligned}
\Psi_{w_b} &= \int_{w_b}^{w_B} \theta^M\left(w, y^M(w_b)\right) dw \cdot \frac{\partial y^M(w_b)}{\partial w_b} > 0, \\
\Psi_{w_B} &= G^M\left(w_B, y^M(w_b)\right) - G^M\left(w_B, \tilde{y}(w_B, \lambda)\right) = 0, \\
\Psi_\lambda &= \int_{w_B}^{\overline{w}} \theta^M\left(w, \tilde{y}(w,\lambda)\right) \cdot \frac{\partial \tilde{y}(w,\lambda)}{\partial \lambda} dw > 0, \text{ and } \quad \Psi_k = 0.
\end{aligned} \tag{149}$$

And thirdly, exploiting Assumption 6.1, (14), (134), (137), (145), and the continuity and monotonicity of the selfishly optimal income schedule, we can analogously obtain:

$$\begin{aligned}
\Phi_{w_b} &= (1+\lambda) \int_{w_b}^{k} \theta_y^M\left(w, y^M(w_b)\right) dw \cdot \frac{\partial y^M(w_b)}{\partial w_b} \\
&\quad + \int_k^{w_B} \left[\theta_y^R\left(w, y^M(w_b)\right) + \lambda \theta_y^M\left(w, y^M(w_b)\right)\right] dw \cdot \frac{\partial y^M(w_b)}{\partial w_b} < 0, \\
\Phi_{w_B} &= \theta^R\left(w_B, y^M(w_b)\right) + \lambda \theta^M\left(w_B, y^M(w_b)\right) = 0, \\
\Phi_\lambda &= \int_{w_b}^{w_B} \theta^M\left(w, y^M(w_b)\right) dw > 0, \\
\Phi_k &= \theta^M\left(k, y^M(w_b)\right) - \theta^R\left(k, y^M(w_b)\right) > 0.
\end{aligned} \tag{150}$$

Using (148)-(150), we can simplify (147) as

$$\underbrace{\begin{bmatrix} \Upsilon_{w_b} & \Upsilon_{w_B} & \Upsilon_\lambda \\ \Psi_{w_b} & 0 & \Psi_\lambda \\ \Phi_{w_b} & 0 & \Phi_\lambda \end{bmatrix}}_{\mathcal{A}} \begin{bmatrix} \mathrm{d}w_b \\ \mathrm{d}w_B \\ \mathrm{d}\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\Phi_k \end{bmatrix} \mathrm{d}k. \tag{151}$$

Thus, using (148)-(151) yields the determinant of matrix $\mathcal{A}$ as

$$\det(\mathcal{A}) = \underbrace{-\Upsilon_{w_B}}_{+} \cdot \left(\underbrace{\Psi_{w_b}}_{+} \cdot \underbrace{\Phi_\lambda}_{+} - \underbrace{\Phi_{w_b}}_{-} \cdot \underbrace{\Psi_\lambda}_{+} \right) > 0. \tag{152}$$

By Cramer's rule and (152), we arrive at:

$$\frac{\mathrm{d}w_b(k)}{\mathrm{d}k} = \frac{1}{\det(\mathcal{A})} \begin{vmatrix} 0 & \Upsilon_{w_B} & \Upsilon_\lambda \\ 0 & 0 & \Psi_\lambda \\ -\Phi_k & 0 & \Phi_\lambda \end{vmatrix} = \frac{\overbrace{-\Phi_k}^{-} \cdot \overbrace{\Upsilon_{w_B}}^{-} \cdot \overbrace{\Psi_\lambda}^{+}}{\det(\mathcal{A})} > 0, \tag{153}$$

$$\frac{\mathrm{d}w_B(k)}{\mathrm{d}k} = \frac{1}{\det(\mathcal{A})} \begin{vmatrix} \Upsilon_{w_b} & 0 & \Upsilon_\lambda \\ \Psi_{w_b} & 0 & \Psi_\lambda \\ \Phi_{w_b} & -\Phi_k & \Phi_\lambda \end{vmatrix} = \frac{\overbrace{\Phi_k}^{+} \cdot \left(\overbrace{\Upsilon_{w_b}}^{+} \cdot \overbrace{\Psi_\lambda}^{+} - \overbrace{\Psi_{w_b}}^{+} \cdot \overbrace{\Upsilon_\lambda}^{-} \right)}{\det(\mathcal{A})} > 0, \tag{154}$$

and

$$\frac{\mathrm{d}\lambda(k)}{\mathrm{d}k} = \frac{1}{\det(\mathcal{A})} \begin{vmatrix} \Upsilon_{w_b} & \Upsilon_{w_B} & 0 \\ \Psi_{w_b} & 0 & 0 \\ \Phi_{w_b} & 0 & -\Phi_k \end{vmatrix} = \frac{\overbrace{-\Phi_k}^{-} \cdot \left(\overbrace{-\Psi_{w_b}}^{-} \cdot \overbrace{\Upsilon_{w_B}}^{-} \right)}{\det(\mathcal{A})} < 0. \tag{155}$$

Case 2: $\underline{w} < w_b < w_B = \overline{w}$. Using (140) and (141) gives rise to

$$\begin{aligned}
\hat{\Psi}(w_b, \lambda, k) \;&\equiv \int_{\underline{w}}^{w_b} G^M\left(w, y^M(w)\right) dw + \int_{w_b}^{\overline{w}} G^M\left(w, y^M(w_b)\right) dw - \overline{u} = 0, \\
\hat{\Phi}(w_b, \lambda, k) \;&\equiv (1+\lambda) \int_{w_b}^{k} \theta^M\left(w, y^M(w_b)\right) dw + \int_{k}^{\overline{w}} \theta^R\left(w, y^M(w_b)\right) dw \\
&\quad + \lambda \int_{k}^{\overline{w}} \theta^M\left(w, y^M(w_b)\right) dw = 0.
\end{aligned} \tag{156}$$

Differentiating $\hat{\Psi}(\cdot)$ and $\hat{\Phi}(\cdot)$ with respect to the arguments and simplifying the algebra yields:

$$\hat{\Psi}_{w_b} = \int_{w_b}^{\overline{w}} \theta^M\left(w, y^M(w_b)\right) dw \cdot \frac{\partial y^M(w_b)}{\partial w_b} > 0, \text{ and } \hat{\Psi}_\lambda = \hat{\Psi}_k = 0; \tag{157}$$

and also

$$\begin{aligned}
\hat{\Phi}_{w_b} \;&= (1+\lambda) \int_{w_b}^{k} \theta_y^M\left(w, y^M(w_b)\right) dw \cdot \frac{\partial y^M(w_b)}{\partial w_b} \\
&\quad + \int_{k}^{\overline{w}} \left[\theta_y^R\left(w, y^M(w_b)\right) + \lambda \theta_y^M\left(w, y^M(w_b)\right)\right] dw \cdot \frac{\partial y^M(w_b)}{\partial w_b} < 0, \\
\hat{\Phi}_\lambda \;&= \int_{w_b}^{\overline{w}} \theta^M\left(w, y^M(w_b)\right) dw > 0, \\
\hat{\Phi}_k \;&= \theta^M\left(k, y^M(w_b)\right) - \theta^R\left(k, y^M(w_b)\right) > 0.
\end{aligned} \tag{158}$$

Expressed in matrix form, total differentiation of (156) gives

$$\begin{bmatrix} \hat{\Psi}_{w_b} & \hat{\Psi}_\lambda \\ \hat{\Phi}_{w_b} & \hat{\Phi}_\lambda \end{bmatrix} \begin{bmatrix} \mathrm{d}w_b \\ \mathrm{d}\lambda \end{bmatrix} = \begin{bmatrix} -\hat{\Psi}_k \\ -\hat{\Phi}_k \end{bmatrix} \mathrm{d}k. \tag{159}$$

Using (157) and (158), (159) can be simplified as

$$\underbrace{\begin{bmatrix} \hat{\Psi}_{w_b} & 0 \\ \hat{\Phi}_{w_b} & \hat{\Phi}_\lambda \end{bmatrix}}_{\hat{\mathcal{A}}} \begin{bmatrix} \mathrm{d}w_b \\ \mathrm{d}\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ -\hat{\Phi}_k \end{bmatrix} \mathrm{d}k.$$

Noting that $\det(\hat{\mathcal{A}}) = \hat{\Psi}_{w_b} \cdot \hat{\Phi}_\lambda > 0$ under (157) and (158), we thus have:

$$\frac{\mathrm{d}w_b(k)}{\mathrm{d}k} = 0 \;\text{ and }\; \frac{\mathrm{d}\lambda(k)}{\mathrm{d}k} = -\frac{\hat{\Psi}_{w_b} \cdot \hat{\Phi}_k}{\det(\hat{\mathcal{A}})} < 0. \tag{160}$$

Case 3: $\underline{w} = w_b < w_B < \overline{w}$. Using (142) and (143) gives rise to

$$\begin{aligned}
\check{\Psi}(w_B, \lambda, k) \;&\equiv \int_{\underline{w}}^{w_B} G^M\left(w, \tilde{y}(w_B, \lambda)\right) dw + \int_{w_B}^{\overline{w}} G^M\left(w, \tilde{y}(w, \lambda)\right) dw - \overline{u} = 0, \\
\check{\Phi}(w_B, \lambda, k) \;&\equiv (1+\lambda) \int_{\underline{w}}^{k} \theta^M\left(w, \tilde{y}(w_B, \lambda)\right) dw + \int_{k}^{w_B} \theta^R\left(w, \tilde{y}(w_B, \lambda)\right) dw \\
&\quad + \lambda \int_{k}^{w_B} \theta^M\left(w, \tilde{y}(w_B, \lambda)\right) dw = 0.
\end{aligned} \tag{161}$$

The partial derivatives of the two implicit functions defined in (161) with respect to w_B, λ, and k can be derived as previously:

$$\begin{aligned} \check{\Psi}_{w_B} &= \int_{\underline{w}}^{w_B} \theta^M \left(w, \tilde{y}(w_B, \lambda)\right) dw \cdot \frac{\partial \tilde{y}(w_B, \lambda)}{\partial w_B} > 0, \\ \check{\Psi}_{\lambda} &= \int_{\underline{w}}^{w_B} \theta^M \left(w, \tilde{y}(w_B, \lambda)\right) dw \cdot \frac{\partial \tilde{y}(w_B, \lambda)}{\partial \lambda} \\ &\quad + \int_{w_B}^{\overline{w}} \theta^M \left(w, \tilde{y}(w, \lambda)\right) \cdot \frac{\partial \tilde{y}(w, \lambda)}{\partial \lambda} dw > 0, \text{ and } \check{\Psi}_k = 0; \end{aligned} \tag{162}$$

and

$$\begin{aligned} \check{\Phi}_{w_B} &= (1+\lambda) \int_{\underline{w}}^{k} \theta_y^M \left(w, \tilde{y}(w_B, \lambda)\right) dw \cdot \frac{\partial \tilde{y}(w_B, \lambda)}{\partial w_B} \\ &\quad + \int_{k}^{w_B} \left[\theta_y^R \left(w, \tilde{y}(w_B, \lambda)\right) + \lambda \theta_y^M \left(w, \tilde{y}(w_B, \lambda)\right)\right] dw \cdot \frac{\partial \tilde{y}(w_B, \lambda)}{\partial w_B} < 0, \\ \check{\Phi}_{\lambda} &= \int_{\underline{w}}^{w_B} \theta^M \left(w, \tilde{y}(w_B, \lambda)\right) dw > 0, \\ \check{\Phi}_k &= \theta_y^M \left(k, \tilde{y}(w_B, \lambda)\right) - \theta_y^R \left(k, \tilde{y}(w_B, \lambda)\right) > 0. \end{aligned} \tag{163}$$

Similar to the previous case, we have:

$$\underbrace{\begin{bmatrix} \check{\Psi}_{w_B} & \check{\Psi}_{\lambda} \\ \check{\Phi}_{w_B} & \check{\Phi}_{\lambda} \end{bmatrix}}_{\check{\mathcal{A}}} \begin{bmatrix} \mathrm{d}w_B \\ \mathrm{d}\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ -\check{\Phi}_k \end{bmatrix} \mathrm{d}k.$$

Hence, in light of (162) and (163) we arrive at

$$\det\left(\check{\mathcal{A}}\right) = \underbrace{\check{\Psi}_{w_B}}_{+} \cdot \underbrace{\check{\Phi}_{\lambda}}_{+} - \underbrace{\check{\Phi}_{w_B}}_{-} \cdot \underbrace{\check{\Psi}_{\lambda}}_{+} > 0,$$

which gives rise to

$$\frac{\mathrm{d}w_B(k)}{\mathrm{d}k} = \frac{\check{\Phi}_k \cdot \check{\Psi}_{\lambda}}{\det\left(\check{\mathcal{A}}\right)} > 0 \quad \text{and} \quad \frac{\mathrm{d}\lambda(k)}{\mathrm{d}k} = -\frac{\check{\Phi}_k \cdot \check{\Psi}_{w_B}}{\det\left(\check{\mathcal{A}}\right)} < 0. \tag{164}$$

Therefore, the desired assertion in Lemma 6.1 follows immediately from (164) and (160), together with (153)-(155). ■

Further, analogous to Lemma 4.4, we obtain the following:

Lemma 6.2 *Suppose Assumption 6.1 holds. Then, individual preferences represented by (17) are single-peaked on the set of skill types.*

Proof. It follows from (4) and (8) that

$$U(w,k) = \overline{u} - \int_{w}^{\overline{w}} h'\left(\frac{y^*(t,k)}{t}\right) \frac{y^*(t,k)}{t^2} dt. \tag{165}$$

Using (165) and (14), partial derivative of $U(w,k)$ with respect to k gives

$$\begin{aligned} \frac{\partial U(w,k)}{\partial k} &= -\int_{w}^{\overline{w}} \left[h''\left(\frac{y^*(t,k)}{t}\right) \frac{y^*(t,k)}{t^3} + h'\left(\frac{y^*(t,k)}{t}\right) \frac{1}{t^2}\right] \frac{\partial y^*(t,k)}{\partial k} dt \\ &= -\int_{w}^{\overline{w}} \left[\theta^M(t, y^*(t,k)) - \theta^R(t, y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt. \end{aligned} \tag{166}$$

As previously, we consider three cases:

Case 1: $\underline{w} < w_b(k) < w_B(k) < \overline{w}$. Exploiting Lemma 6.1, Proposition 6.1, (144), and (145) yields:

$$\begin{aligned}\frac{\partial y^*(w,k)}{\partial k} &= \frac{\partial \tilde{y}(w,\lambda(k))}{\partial \lambda}\cdot\frac{\mathrm{d}\lambda(k)}{\mathrm{d}k} < 0 \;\; \forall w > w_B(k),\\ \frac{\partial y^*(w,k)}{\partial k} &= \frac{\partial y^M(w_b(k))}{\partial w_b}\cdot\frac{\mathrm{d}w_b(k)}{\mathrm{d}k} > 0 \;\; \forall w \in (w_b(k), w_B(k)),\\ \frac{\partial y^*(w,k)}{\partial k} &= \frac{\partial y^M(w)}{\partial k} = 0 \;\; \forall w < w_b(k).\end{aligned} \tag{167}$$

(a) Suppose $w < k$, then single-peakedness requires that $\partial U(w,k)/\partial k < 0$. If $w > w_b(k)$, then (166) can be rewritten as

$$\begin{aligned}\frac{\partial U(w,k)}{\partial k} &= -\int_w^k \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt\\ &\quad -\int_k^{w_B(k)} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt\\ &\quad -\int_{w_B(k)}^{\overline{w}} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt\\ &= -\int_w^k \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt\\ &\quad -\int_k^{w_B(k)} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt + \int_k^{w_B(k)} \theta^R(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt\\ &\quad -(1+\lambda(k))\int_{w_B(k)}^{\overline{w}} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt\end{aligned} \tag{168}$$

where we have applied (134). It follows from (139) that

$$\begin{aligned}&\int_k^{w_B} \theta^R\left(w,y^*(w,k)\right)dw\\ &= -\lambda\int_k^{w_B} \theta^M\left(w,y^*(w,k)\right)dw - (1+\lambda)\int_{w_b}^{k} \theta^M\left(w,y^*(w,k)\right)dw.\end{aligned} \tag{169}$$

Noting that $\partial y^*(t,k)/\partial k$ is constant for all $t \in (w_b, w_B)$. Thus, substituting (169) into (168) and collecting terms, we arrive at

$$\begin{aligned}\frac{\partial U(w,k)}{\partial k} &= -\int_w^k \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]\frac{\partial y^*(t,k)}{\partial k}dt\\ &\quad -(1+\lambda(k))\int_{w_b(k)}^{w_B(k)} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt\\ &\quad -(1+\lambda(k))\int_{w_B(k)}^{\overline{w}} \theta^M(t,y^*(t,k))\frac{\partial y^*(t,k)}{\partial k}dt.\end{aligned} \tag{170}$$

In light of (167), differentiating (138) with respect to k gives

$$-\int_{w_B(k)}^{\overline{w}} \theta^M\left(w,y^*(w,k)\right)\frac{\partial y^*(w,k)}{\partial k}dw = \int_{w_b(k)}^{w_B(k)} \theta^M\left(w,y^*(w,k)\right)\frac{\partial y^*(w,k)}{\partial k}dw. \tag{171}$$

Substituting (171) into (170) and simplifying, in view of (167) we thus obtain

$$\frac{\partial U(w,k)}{\partial k} = -\int_w^k \underbrace{\left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]}_{+}\cdot\underbrace{\frac{\partial y^*(t,k)}{\partial k}}_{+}dt < 0$$

as desired.

If $w \leq w_b(k)$, then (166) can be rewritten as

$$
\begin{aligned}
\frac{\partial U(w,k)}{\partial k} = & -\int_{w}^{w_b(k)} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt \\
& -\int_{w_b(k)}^{k} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt \\
& -\int_{k}^{w_B(k)} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt \\
& -\int_{w_B(k)}^{\overline{w}} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt.
\end{aligned} \tag{172}
$$

Hence, applying (134), (167), (169), and (171) to (172), and simplifying the algebra as previously, we have

$$
\frac{\partial U(w,k)}{\partial k} = -\int_{w_b(k)}^{k} \underbrace{\left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]}_{+} \cdot \underbrace{\frac{\partial y^*(t,k)}{\partial k}}_{+} dt < 0
$$

as desired.

(b) Suppose $w > k$, then single-peakedness requires that $\partial U(w,k)/\partial k > 0$. If $w \geq w_B(k)$, then combining (167) with (166) immediately shows:

$$
\frac{\partial U(w,k)}{\partial k} = -\int_{w}^{\overline{w}} \underbrace{\left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]}_{+} \cdot \underbrace{\frac{\partial y^*(t,k)}{\partial k}}_{-} dt > 0
$$

as desired. If, however, $w < w_B(k)$, then (166) can be rewritten as follows:

$$
\begin{aligned}
\frac{\partial U(w,k)}{\partial k} = & -\int_{w}^{w_B(k)} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt \\
& -\int_{w_B(k)}^{\overline{w}} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt \\
= & -\int_{w}^{w_B(k)} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt \\
& -\int_{k}^{w} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt \\
& +\int_{k}^{w} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt \\
& -\int_{w_B(k)}^{\overline{w}} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt.
\end{aligned} \tag{173}
$$

Thus, (173) can be further expressed as

$$
\begin{aligned}
\frac{\partial U(w,k)}{\partial k} = & -\int_{k}^{w_B(k)} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt \\
& +\int_{k}^{w} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt \\
& -\int_{w_B(k)}^{\overline{w}} \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt.
\end{aligned} \tag{174}
$$

As before, applying (134), (167), (169), and (171) to (174), and simplifying the algebra, we arrive at

$$\frac{\partial U(w,k)}{\partial k} = \int_k^w \underbrace{\left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right]}_{+} \cdot \underbrace{\frac{\partial y^*(t,k)}{\partial k}}_{+} dt > 0$$

as anticipated.

Case 2: $\underline{w} < w_b(k) < w_B(k) = \overline{w}$. Only minor modifications are needed to the proof of Case 1. The terms in which $w > w_B(k)$ drop out as they are no longer relevant. For instance, (171) now becomes

$$0 = -\int_{w_B(k)}^{\overline{w}} \theta^M\left(w, y^*(w,k)\right) \frac{\partial y^*(w,k)}{\partial k} dw = \int_{w_b(k)}^{w_B(k)} \theta^M\left(w, y^*(w,k)\right) \frac{\partial y^*(w,k)}{\partial k} dw.$$

Case 3: $\underline{w} = w_b(k) < w_B(k) < \overline{w}$. Now, using Proposition 6.1 and (164) reveals that the derivative of income on the bridge with respect to k amounts to

$$\begin{aligned}\frac{\partial y^*(w,k)}{\partial k} &= \frac{\partial \tilde{y}\left(w_B(k),\lambda(k)\right)}{\partial k} \\ &= \frac{\partial \tilde{y}\left(w_B(k),\lambda(k)\right)}{\partial w_B} \cdot \frac{\mathrm{d}w_B(k)}{\mathrm{d}k} + \frac{\partial \tilde{y}\left(w_B(k),\lambda(k)\right)}{\partial \lambda} \cdot \frac{\mathrm{d}\lambda(k)}{\mathrm{d}k} \\ &= \frac{\overbrace{\check{\Phi}_k}^{+}}{\det\left(\check{\mathcal{A}}\right)} \cdot \left(\underbrace{\check{\Psi}_\lambda \cdot \frac{\partial \tilde{y}\left(w_B(k),\lambda(k)\right)}{\partial w_B}}_{+} - \underbrace{\check{\Psi}_{w_B} \cdot \frac{\partial \tilde{y}\left(w_B(k),\lambda(k)\right)}{\partial \lambda}}_{+} \right).\end{aligned} \tag{175}$$

Further, using (162) reveals that

$$\begin{aligned}&\check{\Psi}_\lambda \cdot \frac{\partial \tilde{y}\left(w_B(k),\lambda(k)\right)}{\partial w_B} - \check{\Psi}_{w_B} \cdot \frac{\partial \tilde{y}\left(w_B(k),\lambda(k)\right)}{\partial \lambda} \\ &= \left(\int_{w_B}^{\overline{w}} \theta^M\left(w, \tilde{y}(w,\lambda)\right) \cdot \frac{\partial \tilde{y}(w,\lambda)}{\partial \lambda} dw \right) \cdot \frac{\partial \tilde{y}\left(w_B(k),\lambda(k)\right)}{\partial w_B} > 0.\end{aligned}$$

Consequently, we have

$$\begin{aligned}&\frac{\partial y^*(w,k)}{\partial k} > 0 \quad \forall w \in [\underline{w}, w_B(k)), \text{ and} \\ &\frac{\partial y^*(w,k)}{\partial k} = \frac{\partial \tilde{y}(w,\lambda(k))}{\partial \lambda} \cdot \frac{\mathrm{d}\lambda(k)}{\mathrm{d}k} < 0 \quad \forall w > w_B(k).\end{aligned} \tag{176}$$

(a) Suppose $w < k$, then single-peakedness of individual preferences calls for $\partial U(w,k)/\partial k < 0$. Using (134), (166) can be rewritten as

$$\begin{aligned}\frac{\partial U(w,k)}{\partial k} &= -\int_w^k \left[\theta^M(t,y^*(t,k)) - \theta^R(t,y^*(t,k))\right] \frac{\partial y^*(t,k)}{\partial k} dt \\ &\quad - \int_k^{w_B(k)} \theta^M(t,y^*(t,k)) \frac{\partial y^*(t,k)}{\partial k} dt \\ &\quad + \int_k^{w_B(k)} \theta^R(t,y^*(t,k)) \frac{\partial y^*(t,k)}{\partial k} dt \\ &\quad - (1+\lambda(k)) \int_{w_B(k)}^{\overline{w}} \theta^M(t,y^*(t,k)) \frac{\partial y^*(t,k)}{\partial k} dt.\end{aligned} \tag{177}$$

Performing differentiation of (142) with respect to k yields

$$-\int_{w_B(k)}^{\overline{w}} \theta^M\left(w, y^*(w,k)\right) \cdot \frac{\partial y^*(w,k)}{\partial k} dw = \int_{\underline{w}}^{w_B(k)} \theta^M\left(w, y^*(w,k)\right) \cdot \frac{\partial y^*(w,k)}{\partial k} dw. \tag{178}$$

And, it follows from (143) that

$$\begin{aligned} &\int_{k}^{w_B(k)} \theta^R\left(w, y^*(w,k)\right) dw \\ =\ & -\left(1+\lambda(k)\right) \int_{\underline{w}}^{k} \theta^M\left(w, y^*(w,k)\right) dw - \lambda(k) \int_{k}^{w_B(k)} \theta^M\left(w, y^*(w,k)\right) dw. \end{aligned} \tag{179}$$

Plugging the terms of (178) and (179) in (177) gives, after some re-grouping of terms,

$$\frac{\partial U(w,k)}{\partial k} = -\int_{w}^{k} \underbrace{\left[\theta^M(t, y^*(t,k)) - \theta^R(t, y^*(t,k))\right]}_{+} \cdot \underbrace{\frac{\partial y^*(t,k)}{\partial k}}_{+} dt < 0,$$

as anticipated. Making use of (176), (178) and (179), the remaining proof can be analogously completed, and therefore is omitted to economize on the space. ■

Because of Lemma 6.2, we apply again the median voter theorem (Black 1948) and obtain the following:

Proposition 6.2 *Suppose Assumption 6.1 holds. Then, the selfishly optimal income tax schedule for the median skill type is a Condorcet winner when majority voting is restricted to the income tax schedules represented by (133) that are selfishly optimal for some skill type.*

Consequently, the analysis for the case with only the participation constraint for top talent has been completed.