Quiz 7 (Notes, books, and calculators are not authorized) Show all your work in the blank space you are given on the exam sheet. Always justify your answer. Answers with no justification will not be graded.

**Question 1:** Consider the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. Is it possible to find a set of two independent eigenvectors of $A$? Be very accurate and explicit when answering.

Since the matrix is symmetric, it is diagonalizable, which implies that, yes indeed, one can find a set of two independent eigenvectors of $A$.

**Question 2:** Consider the dynamical system $\frac{du}{dt} = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} u$. Is this dynamical system stable? Be very accurate and explicit when answering.

The two eigenvalues of the matrix are the roots of

$$0 = (4 - \lambda)(-1 - \lambda) - 6 = \lambda^2 - 3\lambda - 10.$$ 

The two roots are $\lambda_1 = 5$ and $\lambda_2 = -2$. The dynamical system is unstable since $\lambda_1 > 0$. 
Question 3: Let \( \frac{du}{dt} = F(u) \) be three-dimensional nonlinear dynamical system. Assume that \( u_0 \in \mathbb{R}^3 \) solves \( F(u_0) = 0 \) and assume that the dynamics in the neighborhood of \( u_0 \) is described by \( \frac{dz}{dt} = Az \) where \( A \) is \( 3 \times 3 \) real-valued matrix with eigenvalues \( -2 + 3i, -2 - 3i, \) and \( -3 \). Is \( u_0 \) a stable point? Be very accurate and explicit when answering.

The real part of the three eigenvalues are negative. As a result \( u_0 \) is a stable point.

Question 4: Let \( A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \). Find the (possibly complex-valued) eigenvalues and say if the matrix is diagonalizable in \( \mathbb{R}^2 \).

The characteristic polynomial is

\[
(1 - \lambda)^2 + 1 = 0.
\]

The two eigenvalues are complex conjugate \( \lambda_1 = 1 + i, \lambda_2 = 1 - i \). The matrix is not diagonalizable in \( \mathbb{R}^2 \). It is diagonalizable in \( \mathbb{C} \) though.