Quiz 1 (Notes, books, and calculators are not authorized)

Show all your work in the blank space you are given on the exam sheet. Always justify your answer. Answers with no justification will not be graded.

**Question 1:** Find a parametric representation of the line in $\mathbb{R}^4$ that passes through $P = (1, 2, 3, 4)$ in the direction of $u = (4, 3, 2, 1)$.

The line in question consists of the points $X = (x_1, x_2, x_3, x_4)$ in $\mathbb{R}^4$ that are such that there is $t \in \mathbb{R}$ so that $(X - P) = tu$. This means that

$$X - P = tu = (4t, 3t, 2t, t).$$

The parametric representation of the line is

$$x_1 = 1 + 4t, \quad x_2 = 2 + 3t, \quad x_3 = 3 + 2t, \quad x_4 = 4 + t.$$

or

$$L = \{(1 + 4t, 2 + 3t, 3 + 2t, 4 + t) \in \mathbb{R}^4, \ t \in \mathbb{R}\}.$$

**Question 2:** Let $u = (1 + i, 1 - i)$ and $v = (1, i)$ be two complex vectors in $\mathbb{C}^2$. Compute $\|u\|$, $\|v\|$, and $< u, v >$. What can you say about $u$ and $v$?

(a) By definition

$$\|u\|^2 = < u, u > = u \cdot \bar{u} = (1 + i)(1 - i) + (1 - i)(1 + i) = 1 - i^2 + 1 - i^2 = 4,$$

i.e., $\|u\| = 2.$

$$\|v\|^2 = < v, v > = v \cdot \bar{v} = 1 - i^2 = 2,$$

i.e., $\|v\| = \sqrt{2}.$

$$< u, v > = (1 + i) + (1 - i)(-i) = 1 + i - i - 1 = 0.$$

The two complex vectors $u$ and $v$ are orthogonal in $\mathbb{C}^2$. 

Question 3: Find an equation of the hyperplane in $\mathbb{R}^3$ that passes through $P = (1, 2, 3)$ and is orthogonal to the vector $n = (3, 2, 1)$.

By definition $H$ is the collection of points $X = (x_1, x_2, x_3)$ so that $(X - P) \cdot n = 0$. This means

$$0 = (X - P) \cdot (3, 2, 1) = (x_1 - 1, x_2 - 2, x_3 - 3) \cdot (3, 2, 1) = 3x_1 - 3 + 2x_2 - 4 + x_3 - 3 = 3x_1 + 2x_2 + x_3 - 10.$$

The equation of the hyperplane in question is

$$3x_1 + 2x_2 + x_3 = 10,$$

or

$$H\{(x_1, x_2, x_3) \in \mathbb{R}^3, \ 3x_1 + 2x_2 + x_3 = 10\}.$$

Question 4: Let $u = (\sqrt{2}, \sqrt{3}, 2)$. (a) Compute $\|u\|$. (b) How many vectors $v$ there are in $\mathbb{R}^3$ such that $\|u + v\| > 9 + \|v\|$? Justify your answer rigorously.

(a) It is clear that $\|u\| = \sqrt{2 + 3 + 4} = 3$. (b) Assume that there is a vector $v$ in $\mathbb{R}^3$ so that $\|u + v\| > 9 + \|v\|$, then we would have $\|u + v\| > \|u\| + \|v\|$ which contradicts the triangle inequality. This is absurd. As a result, the answer to the question is that there cannot be any such vector.