
**Question 1**: Let $\Omega$ be a convex polygon in $\mathbb{R}^2$. Let $\{T_h\}_{h>0}$ be a conforming, shape-regular mesh family composed of triangles. Consider the $P_1$-Crouzeix-Raviart finite element approximation of 

$$-\nabla^2 u = f, \quad u|_{\partial \Omega} = 0.$$ 

(a) Write the definition of the Crouzeix-Raviart approximation space based on $T_h$, say $D_{0,h}$.

(b) Let $u_h$ be the Galerkin approximation of $u$ in $D_{0,h}$. Recall the definition of $u_h$.

(c) Let $C_{0,h}$ be the space composed of continuous piecewise $P_1$ functions based on $T_h$ and with zero trace on $\partial \Omega$.

(d) Let $w_h \in C_{0,h}$. Evaluate $\sum_{K \in T_h} \int_K \nabla w_h \cdot \nabla (u - u_h) dK$.

(e) Using the fact that there is $c > 0$, uniform with respect to $h$, so that

$$\|v - v\|_{L^2(F)} \leq c \frac{1}{h} \|\nabla v\|_{H^1(K)}, \quad \forall v \in H^1(K), \forall F \in F_h,$$

prove that $\|u - u_h\|_{L^2(\Omega)} \leq c' h \|u - u_h\|_{1,h}$ (i.e., finish the Nitsche-Aubin proof ...)

Let $K$ and $\hat{K}$ be a nondegenerate simplex and a reference simplex in $\mathbb{R}^d$, respectively. Let $a_1, \ldots, a_{d+1}$ be the vertices of $K$. Let $\hat{a}_1, \ldots, \hat{a}_{d+1}$ be the vertices of $\hat{K}$. Let $\lambda_1, \ldots, \lambda_{d+1}$ be the barycentric coordinate functions in $\hat{K}$. Give a simple expression of the affine transformation that maps $\hat{a}_1, \ldots, \hat{a}_{d+1}$ to $a_1, \ldots, a_{d+1}$, respectively.

**Question 2**: Let $K$ be a non-degenerate polyhedron in $\mathbb{R}^d$. Let $\hat{K}$ be the non-degenerate polyhedron obtained from $K$ by applying the homothety of ratio $1/h_K$, where $h_K := \text{diam}(K)$. Prove that there exists a uniform constant $c(\hat{K})$ so that

$$\|v\|_{L^2(\partial K)} \leq c(\hat{K}) \left( h_K^{-\frac{1}{2}} \|v\|_{L^2(\Omega)} + h_K^{\frac{1}{2}} \|
abla v\|_{L^2(\Omega)} \right), \quad \forall v \in \mathbb{P}_1(K).$$

**Question 3**: Let $K$ be a non-degenerate polyhedron in $\mathbb{R}^d$. Let $\hat{K}$ be the non-degenerate polyhedron obtained from $K$ by applying the homothety of ratio $1/h_K$, where $h_K := \text{diam}(K)$. Let $k$ be an integer, $k \geq 0$. Prove that there exists a uniform constant $c(k, \hat{K})$ so that

$$\|v\|_{L^2(\partial K)} \leq c(\hat{K}) h_K^{-\frac{1}{2}} \|v\|_{L^2(\Omega)}, \quad \forall v \in \mathbb{P}_k(K).$$