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Numerical Study of Cooling by Ferrofluids in an Electrical Transformer Using an Axisymmetric Model

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The benefit of transformer oil seeded with magnetic nanoparticles (forming a transformer oil-based ferrofluid) is numerically studied in an axisymmetric model of 40 kVA transformer. The maximum temperature in the system is decreased by 2.2 °C when using ferrofluid instead of transformer oil. This effect is mainly due to the magnetic body force, which modifies the convective flow in the system and enhances heat transfer. The leakage magnetic field is highly localized and limits the effect of thermomagnetic convection. Varying the geometry of the primary coil assembly allows to increase this effect and to decrease the maximum temperature.

Index Terms—**Electrical transformer, ferrofluid, finite-element method, thermomagnetic convection.**

I. Introduction

FERROFLUIDS are suspensions of magnetic nanoparticles in a liquid carrier, which exhibit a super-paramagnetic behavior. When magnetic field and temperature gradients are applied to a ferrofluid, thermomagnetic convection may arise and enhance heat transfer. A recent work [1] for instance numerically studied the cooling by ferrofluid convection in the presence of alternating magnetic field. Experimental studies have shown the benefit of ferrofluids for the cooling of electrical transformers [2]–[5], but numerical studies are limited (only [3] that we know of). In this work, we numerically assess the benefit of ferrofluid for the cooling of a 40 kVA transformer. An axisymmetric model is used to compute the leakage magnetic field, the convective flow, and the temperature in the system. The ferrofluid model considers a temperature dependence contrary to that of [3], which is essential to capture the development of thermomagnetic convection.

II. Considered Transformer and Modeling

A. Considered Transformer

We consider a 40 kVA (20 kV/400 V) transformer, meaning a transformer that could be used for distribution purpose. The tank is cylindrical with flat top and bottom surfaces (with heat transfer fins). The primary and secondary coils are winded around the same branch of the ferromagnetic core. In the primary coil, there are 10 000 turns, the root mean square (rms) voltage is 20 kV, and the rms current in each turn is 2 A. In the secondary coil, there are 200 turns, the rms voltage is 400 V, and the rms current in each turn is 100 A. An axisymmetric model (Fig. 1 and Table I) is used to compute the leakage magnetic field, the Joule effect generated by the coils, the induced convection in the oil, and the temperature rise in the whole system. A primary coil with a rectangular cross section is shown in Fig. 1, but other geometries of the primary coil assembly are also tested in the article.

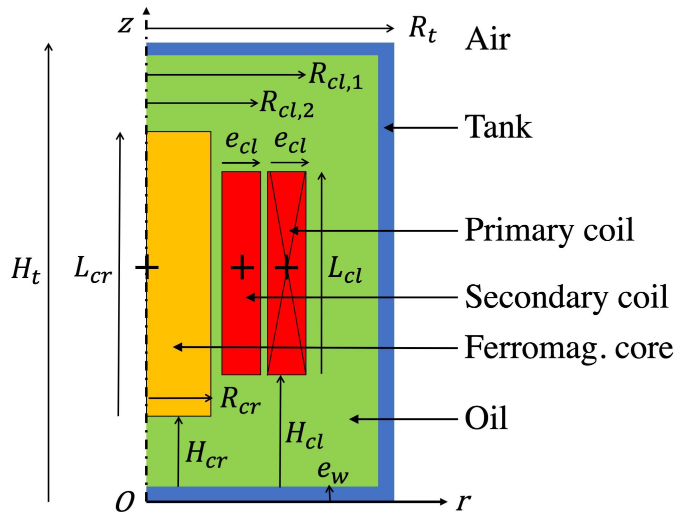

Fig. 1. Meridian section of the transformer model (geo. 1). Other shapes of the primary coil section are tested (geo. 2–4), see the associated temperature fields in Fig. 3(d)–(f).

TABLE I
Dimensions of the Transformer Model in cm

Parameter	Dim.	Parameter	Dim.
Tank height (H_t)	46	Tank radius (R_t)	21.1
Core height (H_{cr})	7	Coils height (H_{cl})	11
Wall thickness (e_w)	1	Coils thickness (e_{cl})	4
Core length (L_{cr})	32	Coils length (L_{cl})	24
Pr. coil ext. radius ($R_{cl,1}$)	14.1	Core radius (R_{cr})	4.5
Sec. coil ext. radius ($R_{cl,2}$)	9.6		

B. Governing Equations

The ferrofluid is considered as a continuum medium with homogeneous properties. A strong assumption of this work is

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that the time associated with the fluctuation of the magnetic field is much shorter than the time associated with the heating/cooling of the system. Thus, the temperature is not affected by the time variations in the magnetic field and the magnetic force generated in the ferrofluid. Therefore, the current density in the coils is assumed constant, with a value that corresponds to the rms value of the actual current density.

The displacement currents are neglected. The magnetic field is governed by the law of Maxwell–Ampère

$$\nabla \times \mathbf{H} = \mathbf{j} \tag{1}$$

and the divergence-free condition

$$\nabla \cdot \mathbf{B} = 0 \tag{2}$$

where $\mathbf{H}$ is the magnetic field, $\mathbf{j}$ is the current density, and $\mathbf{B}$ is the induction field. The current density $\mathbf{j}$ is equal to $j_0\mathbf{e}_\theta$ in the primary coil, $-j_0\mathbf{e}_\theta$ in the secondary coil (j_0 constant), and zero elsewhere. The constitutive equation is

$$\mathbf{B} = \mu_0\mu_r\mathbf{H} \tag{3}$$

where μ_0 is the magnetic permeability of vacuum and μ_r is the relative magnetic permeability (values given next). On the boundaries of the computational domain (a vacuum domain is considered around the tank), we enforce

$$\mathbf{H} \times \mathbf{n} = \mathbf{0} \tag{4}$$

where $\mathbf{n}$ is the outer normal unit vector.

Assuming that the ferrofluid has a Newtonian behavior, we use the Navier–Stokes equations. We consider only small temperature differences in the system so that it is possible to work under the Boussinesq approximation. The momentum equation and the continuity equation are

$$\begin{aligned}\rho\partial_t\mathbf{u} + \rho(\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p - \nabla\cdot(\eta(T)(\nabla\mathbf{u} + (\nabla\mathbf{u})^T))&\\ = \rho\alpha(T - T_0)g\mathbf{e}_z + \mathbf{f}_m& \quad (5)\\ \nabla\cdot\mathbf{u} = 0& \quad (6)\end{aligned}$$

where ρ is the density, $\mathbf{u}$ is the velocity, p is the pressure, η is the dynamic viscosity, T is the temperature, α is the thermal expansion coefficient, T_0 is the initial temperature, g is the gravity, and $\mathbf{f}_m$ is the magnetic body force. The magnetic body force is defined following the Kelvin force model:

$$\mathbf{f}_m = \mu_0(\mathbf{M}\cdot\nabla)\mathbf{H} \tag{7}$$

where $\mathbf{M}$ is the ferrofluid magnetization. On the boundaries of the fluid domain, we enforce the non-slip boundary condition

$$\mathbf{u} = \mathbf{0} \tag{8}$$

that accounts for the fluid viscosity.

We assume that the ferrofluid magnetization is collinear to the magnetic field and we use the assumption of linear magnetic material. The magnetization intensity is

$$M = \chi(T)H \tag{9}$$

where the magnetic susceptibility χ is given by an approximation of Langevin's law

$$\chi(T) = \frac{\phi\mu_0\pi d^3 M_{s,p}(T)^2}{18k_BT} \tag{10}$$

with ϕ the volume fraction of magnetic nanoparticles, d the particle diameter, $M_{s,p}$ the particle magnetization, and k_B Boltzmann's constant. The particle magnetization is a function of the temperature following Bloch's law:

$$M_{s,p}(T) = \begin{cases} M_0\left(1 - \left(\frac{T}{T_C}\right)^{1.5}\right)\left(1 - \left(\frac{T_0}{T_C}\right)^{1.5}\right)^{-1}, \\ \quad\text{if } T \le T_C \\ 0, \quad \text{if } T \ge T_C \end{cases} \tag{11}$$

where M_0 is the particle magnetization at T_0. Note that in (10) and (11), the temperatures are in kelvin. Based on the collinearity between $\mathbf{H}$ and $\mathbf{M}$ and (9), the magnetic body force [see (7)] can be simplified into

$$\mathbf{f}_m = \mu_0\chi(T)\nabla\left(\frac{H^2}{2}\right). \tag{12}$$

The viscous dissipation is neglected in the temperature equation, which is

$$\rho c\partial_t T + \rho c\mathbf{u}\cdot\nabla T - \nabla\cdot(\lambda\nabla T) = f_T \tag{13}$$

where c is the specific heat, λ is the thermal conductivity, and f_T is the heat source, equal to the Joule effect $\rho_e j_0^2$ in the coils (ρ_e the electrical resistivity of copper) and zero elsewhere. Eddy currents and associated Joule heating in other parts of the system are neglected. On the top and lateral walls of the electromagnetic system, we enforce Robin conditions

$$-\lambda\nabla T\cdot\mathbf{n} = h(T - T_0) \tag{14}$$

where h is the air convection coefficient. On the bottom wall, we enforce a homogeneous Neumann condition

$$\partial_{\mathbf{n}}T = \nabla T\cdot\mathbf{n} = 0. \tag{15}$$

C. Physical Properties

The ferromagnetic core is made of iron, the coils of copper, and the tank of steel. The fluid is either a (vegetable) transformer oil or the same transformer oil seeded with magnetite nanoparticles. The thermophysical properties used in the simulations are presented in Table II. The dynamic viscosity of the transformer oil is approximated by using the expression

$$\eta(T) = A\exp\left(\frac{B}{T}\right) \tag{16}$$

where $A \simeq 1.3\times10^{-6}$ Pa · s, $B \simeq 3.1\times10^3$ K, and T in K. The thermophysical properties of the transformer oil-based ferrofluid are functions of that of the transformer oil and that of magnetite, following the laws presented in [6]. The volume fraction of nanoparticles is $\phi = 1\%$ to maintain reasonable dielectric properties. The electrical resistivity of the coils is $\rho_e = 1.68\times10^{-8}\ \Omega\cdot$m.

The diameter of the magnetic nanoparticles is $d = 10$ nm, their saturation magnetization at 20 °C is $M_0 = 4.46\times10^5$ A/m, and their Curie temperature is $T_C = 580$ °C.

We consider that the tank is equipped with heat transfer fins, which form a larger exchange surface, and consequently use a large value of air convection coefficient $h = 150$ W/m$^2\cdot$K.

The relative magnetic permeability is $\mu_r = 100$ in the (steel) tank, $\mu_r = 5000$ in the (iron) ferromagnetic core,

TABLE II

MATERIAL PROPERTIES. OIL: TRANSFORMER OIL. FF: TRANSFORMER OIL-BASED FERROFLUID

Property	Copper	Oil	FF	Steel	Iron
Density (kg/m^3)	8933	922	965	7850	7870
Therm. expansion (1/K)	-	7.4e-4	7.3e-3	-	-
Heat capacity (J/K · kg)	385	1970	1898	475	447
Therm. cond. (W/m · K)	401	0.166	0.171	44.5	80.2

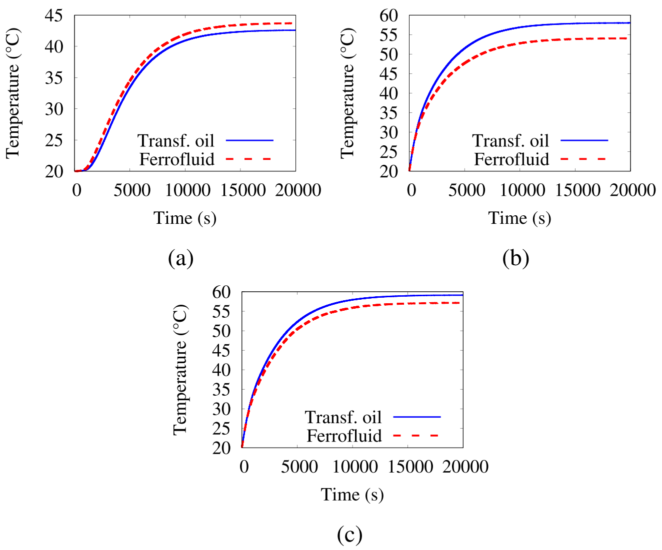


Fig. 2. Geometry 1—temperature evolution at the center of each component (cross marks in Fig. 1). The temperature is rather homogeneous in these metallic parts. (a) Ferromagnetic core. (b) Secondary coil. (c) Primary coil.

$\mu_r = 1+\chi(T_0)$ in the ferrofluid, and $\mu_r = 1$ in the transformer oil and in the vacuum around the tank.

III. SFEMANS CODE

The equations are solved by spectral/finite element for Maxwell and Navier–Stokes (SFEMaNS) code. Dedicated to axisymmetric geometries and based on cylindrical coordinates, SFEMaNS uses a spectral decomposition in the θ-direction and finite elements in the meridian plane (see [6]). We show here simulations on one spectral mode: the solution is axisymmetric. Three-dimensional computations on several modes do not have a major effect on the temperature field in this problem.

IV. RESULTS

A. Transformer Oil Versus Ferrofluid Cooling in Case of a Rectangular Primary Coil Section

The numerical simulations show that a permanent regime establishes itself around 10 000–20 000 s, see the time evolutions of the temperature at various points shown in Fig. 2. In the permanent regime, the temperature of the coils (at the monitored points) is lower with ferrofluid than with transformer oil (≈2 °C for the primary and 5 °C for the secondary). It is the opposite in the core (≈1 °C difference). Thus, the temperature is decreased in the hot parts and made more homogeneous in the overall system when using ferrofluid.

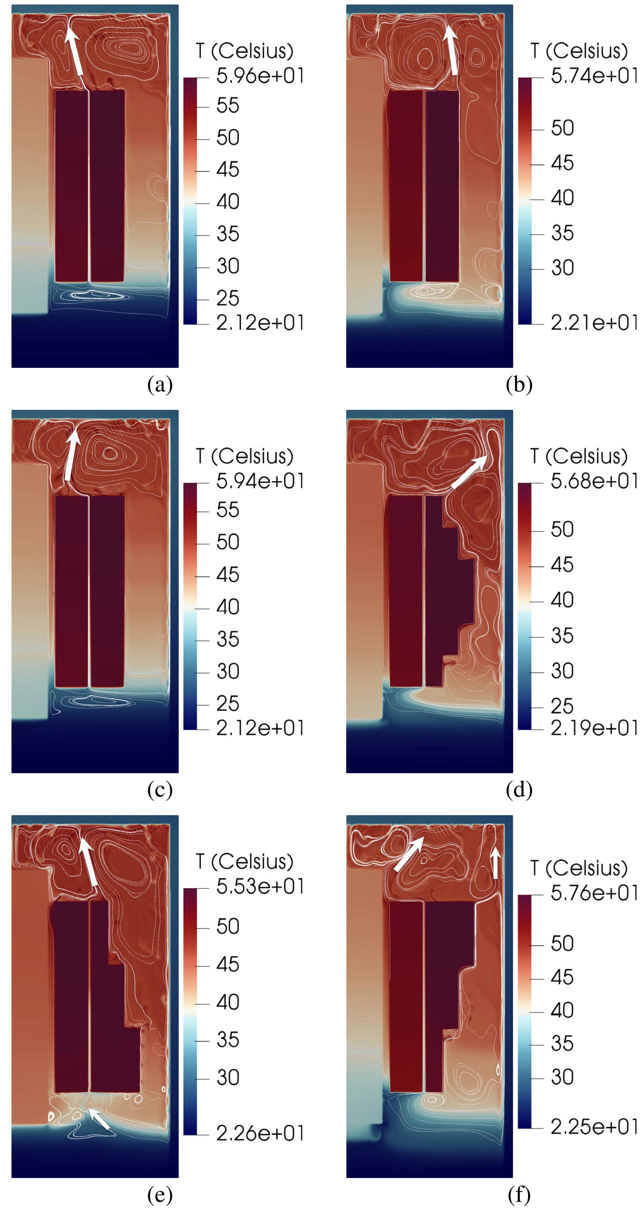
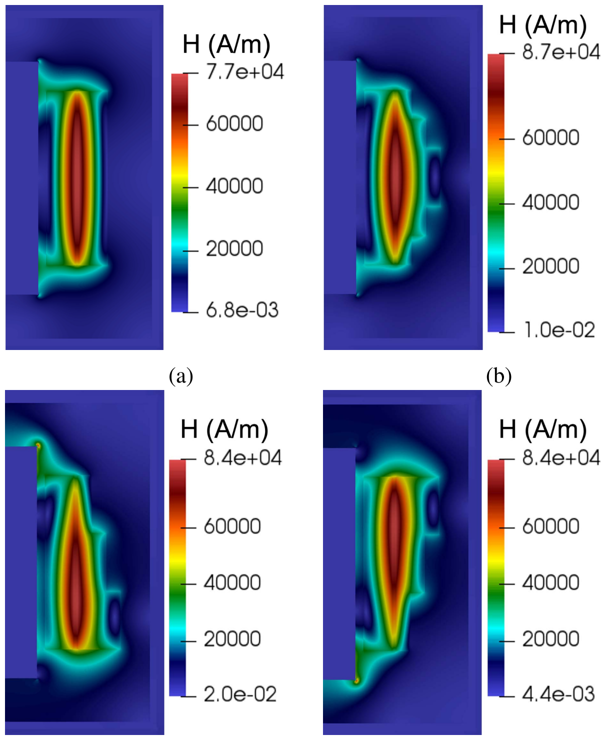


Fig. 3. Temperature and streamlines in a meridian section (symmetry axis on the left) in permanent regime ($t \approx 20\,000$ s). The arrows show the flow direction. (a) Geo. 1—Transformer oil. (b) Geo. 1—Ferrofluid. (c) Geo. 1—Ferrofluid – $H = 0$. (d) Geo. 2—Ferrofluid. (e) Geo. 3—Ferrofluid. (f) Geo. 4—Ferrofluid.

These temperature differences are confirmed by the views of the temperature fields in the permanent regime in Fig. 3(a) and (b). Recall that the solution is axisymmetric, so a view of the meridian section is enough. The maximum temperature in the system is 2.2 °C lower with ferrofluid, which confirms that it is more efficient than transformer oil. A relatively hot area at the bottom of the coils in the case of ferrofluid cooling also indicates that the temperature distribution is more homogeneous. The streamlines superimposed on the temperature fields show that the convection pattern is different with ferrofluid: the convection cells above the coils are radially shifted and the convection cell under the coils is more intense.

The temperature difference between the two kinds of cooling is explained by: 1) the difference of thermophysical



(c) (d)

Fig. 4. Ferrofluid cooling. Magnetic field intensity in a meridian section (symmetry axis on the left) at any time. (a) Geometry 1. (b) Geometry 2. (c) Geometry 3. (d) Geometry 4.

properties of the fluids (see Table II) and 2) the presence of the magnetic body force operating on the ferrofluid [see (7]. To assess the impact of each factor separately, we show in Fig. 3(c) the temperature field in a virtual case of ferrofluid cooling where the magnetic field is artificially zero (the magnetic body force is inactive). The maximum temperature is 0.2 °C lower compared to transformer oil cooling (factor 1 effect) and 2 °C greater compared to ferrofluid cooling (factor 2 effect). Hence, the improvement of the cooling with ferrofluid is mainly due to the magnetic body force. The change of the thermophysical properties has a marginal effect.

The magnetic nanoparticles have a positive influence on heat transfer (lower maximum temperature), but their influence is not substantial (small temperature reduction with ferrofluid). This observation can be explained by analyzing the magnetic field distribution [see Fig. 4(a)], which produces the magnetic body force. The nonlinear effect is neglected in the magnetic problem, and the magnetic field is the sum of the magnetic fields generated by each coil separately. Since the coils carry opposite currents (see Fig. 1), the magnetic field is highly localized in the gap between the coils and rather weak and uniform in the rest of the domain. This configuration, therefore, limits the intensity of the magnetic body force, and a strong thermomagnetic convection cannot develop.

B. Ferrofluid Cooling in Case of Different Geometries of the Primary Coil Section

We test various geometries of the primary coil assembly that may enhance thermomagnetic convection. The tested geometries have the same section area so that the current in the primary coil is preserved. They nevertheless lead to a different distribution of (total) magnetic field, which presents a stronger maximum of the intensity (see Fig. 4).

The time evolution of the temperature in the system is similar to that of Fig. 2 for every geometry. The temperature fields and streamlines in permanent regime obtained with the new geometries are shown in Fig. 3(d)–(f). The best geometry appears to be #3 [Fig. 3(e)] since the maximum temperature is decreased by 2.1 °C compared to the rectangular geometry [Fig. 3(b)]. This leads to a difference of 4.3 °C compared to the transformer oil cooling [Fig. 3(a)]. Geometries 2 and 4 [Fig. 3(d) and (f)] do not bring significant improvement. The new geometries nevertheless present a higher volume of primary coil and therefore generate more heat by the Joule effect. With transformer oil, the maximum temperature is decreased by 0.1 °C when geometry 3 is used instead of geometry 1. The positive influence of the geometry on transformer oil cooling is not as important as on ferrofluid cooling. Let us remark that these geometries, while interesting, would necessitate to adapt the ferromagnetic core in an actual transformer.

V. Conclusion

The benefit of a transformer oil-based ferrofluid ($\phi = 1\%$) on the cooling of a 40 kVA transformer has been studied with an axisymmetric model. The maximum temperature is decreased by 2.2 °C when ferrofluid instead of transformer oil is used, mainly due to the magnetic body force which modifies the convective flow. The slight temperature decrease is consistent with recent work [5]. The windings lead to an intense but localized magnetic field, which generates a localized magnetic force. Primary coils with various assembly geometries have been tested to optimize ferrofluid cooling, one leading to an additional temperature reduction of 2.1 °C.

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