SECOND-ORDER INVARIANT DOMAIN PRESERVING APPROXIMATION OF THE EULER EQUATIONS USING CONVEX LIMITING (SUPPLEMENTARY MATERIAL)*

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5.8. Mach 3 step. Let us now illustrate the method on the classical Mach 3 flow in a wind tunnel with a forward facing step. The computational domain is $D = (0,1)\times(0,3) \setminus (0.6,3)\times(0,0.2)$; the geometry of the domain is shown in Figure 1. The initial data is $\rho = 1.4$, $p = 1$, $\boldsymbol{v} = (3,0)^\mathsf{T}$. The inflow boundary conditions are $\rho_{|\{x=0\}} = 1.4$, $p_{|\{x=0\}} = 1$, $\boldsymbol{v}_{|\{x=0\}} = (3,0)^\mathsf{T}$. The outflow boundary conditions are free, i.e., we do nothing at $\{x = 3\}$. On the top and bottom boundaries of the channel we enforce $\boldsymbol{v}\cdot\boldsymbol{n} = 0$.

The computation is done from $t = 0$ to $t = 4$. We show in Figure 1 a Schlieren-type snapshot of the density at $t = 4$ obtained with the three codes. The meshes used with Code 1 and Code 2 are nonuniform Delaunay triangulations composed of 207340 and 209741 $\mathbb{P}_1$ nodes, respectively. The mesh used with Code 3 is uniform and composed of quadrangles with a total of 310101 $\mathbb{Q}_1$ nodes. The Kelvin-Helmholtz instability of the contact discontinuity is clearly visible. The top left corner of the step is rounded for Code 1 and Code 2 (the corner is a quarter circle of radius 0.01) and it is sharp for Code 3; no regularization or smoothing is applied at the corner. We show a Schlieren-type snapshot of the density at $t = 4$ in Figure 1 for the three codes; that is, after defining the norm of the gradient as follows $r_i^n := m_i^{-1}\|\sum_{j\in\mathcal{I}(D_i)} \boldsymbol{c}_{ij}\rho_j^n\|_{\ell^2}$, for all $i \in \mathcal{I}$, we show the scalar field with point values $\exp(-\beta(r_i^n - \min_{j\in\mathcal{I}} r_j^n)/(\max_{j\in\mathcal{I}} r_j^n - \min_{j\in\mathcal{I}} r_j^n))$ where $\beta = 10$; see Banks et al. [1, Eq. (35)].

5.9. Two-dimensional double Mach reflection. In this section we solve the well-known double Mach reflection problem at Mach 10 with a gamma-law equation of state, $\gamma = \frac{7}{5}$. The shock impinges a wall with a 60 degree angle. The computational domain for this problem is the rectangle $\Omega = (0,3.2)\times(0,1)$. The post-shock values are $\rho = 8$, $p = 116.5$, $\boldsymbol{v} = (8.25\cos(30^o), -8.25\sin(30^o))^\mathsf{T}$, and the values ahead of the shock are $\rho = 1.4$, $p = 1$, $\boldsymbol{v} = (0,0)^\mathsf{T}$. The slip boundary condition is applied on the wall $\{x_1 \geq \frac{1}{6}; x_2 = 0\}$. No boundary condition is applied at the outflow boundary $\{x_1 = 3.2; x_2 > 0\}$. On the rest of the boundary, the post-shock values are applied if $x_1 < \frac{1}{6} + \frac{x_2+20t}{\sqrt{3}}$, and the values ahead of the shock are applied if $x_1 > \frac{1}{6} + \frac{x_2+20t}{\sqrt{3}}$.

The problem is solved until $t = 0.2$. The mesh for Code 1 is a nonuniform Delaunay triangulation with 453969 $\mathbb{P}_1$ nodes. The mesh for Code 2 is a uniform triangulation with $h_K = 1/400$, which in total gives 513861 $\mathbb{P}_1$ nodes. The mesh for Code 3 is composed of uniform quadrangles with 513600 $\mathbb{Q}_1$ nodes. We show a Schlieren-like representation of the density in Figure 2 for the three codes.

*Supplementary material for SISC MS#M114996.

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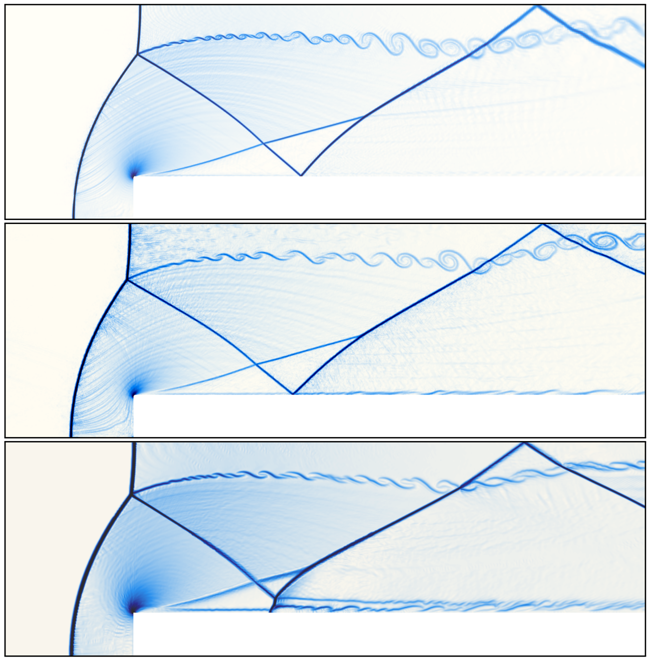

FIG. 1. *Mach 3 step, $t = 4$, density. Top: Code 1, unstructured grid, 207340 $\mathbb{P}_1$ nodes. Middle: Code 2, unstructured grid 209741 $\mathbb{P}_1$ nodes. Bottom: Code 3, structured grid, 310101 $\mathbb{Q}_1$ nodes.*

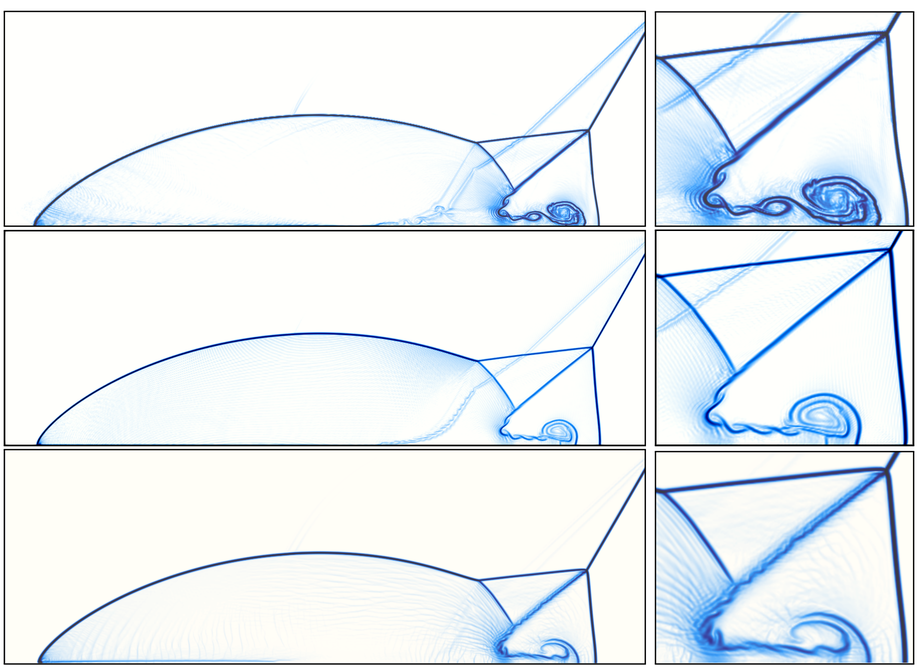

FIG. 2. *Mach 10 double reflection. Schlieren-like representation of the density. Top Code 1, unstructured mesh, 453969 $\mathbb{P}_1$ nodes. Center Code 2, structured mesh, 513861 $\mathbb{P}_1$ nodes. Bottom Code 3, structured mesh, 513600 $\mathbb{Q}_1$ nodes.*

References.

[1] J. W. Banks, W. D. Henshaw, D. W. Schwendeman, and A. K. Kapila. A study of detonation propagation and diffraction with compliant confinement. *Combust. Theory Model.*, 12(4):769–808, 2008.