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Cyclic nonlinear dynamo action in a finite Taylor Couette flow

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Abstract.
We present calculations of dynamo action in a finite Taylor-Couette configuration. The first hydrodynamic bifurcation is an imperfect pitchfork bifurcation giving rise to Taylor vortices. This flow is used in kinematic dynamo computations showing a Hopf bifurcation towards a localized magnetic structure of typical length twice as long as the velocity typical length. Using Taylor vortices and the magnetic eigenvector obtained from the kinematic regime, the nonlinear dynamo shows a striking cyclic behaviour where the symmetries with respect to the median plane play a major role.

1. Introduction

The Taylor-Couette driving leads to one of the most intensively studied flows using either analytical, experimental or numerical tools (see [6] and references therein). In its MHD version, due to the difficulty to deal with magnetic boundary conditions, kinematic and nonlinear dynamo properties remain unexplored in a finite configuration in demand by the experimentalists. We consider in this paper a finite set-up that leads to a new cyclic type of nonlinear dynamo.

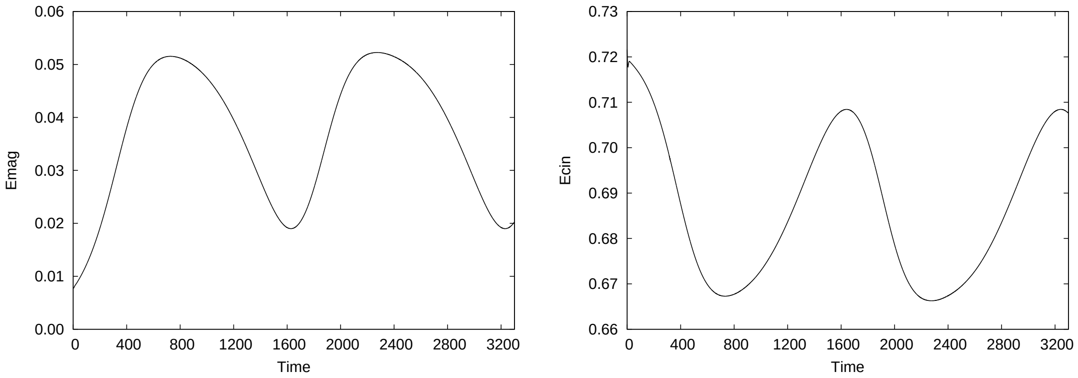
2. Model

We consider an incompressible fluid of kinematic viscosity ν contained between two coaxial cylinders of inner radius R_i (which rotates at imposed angular speed Ω_i) and static outer radius R_o of height L_z. We use $\delta = R_o - R_i$ as unit of length, U as unit of velocity and $U\sqrt{\rho/\mu}$ as unit of magnetic field (expressed in term of Alfvén speed) where the density ρ and the magnetic permeability μ are constant. The governing nondimensionalized parameters of the system are the kinetic Reynolds number Re, the magnetic Reynolds number Rm, the radius ratio η and the aspect ratio Γ,

$$Re = \frac{R_i \Omega_i \delta}{\nu},\ Rm = \mu\sigma R_i \Omega_i \delta,\ \eta = \frac{R_i}{R_o},\ \Gamma = \frac{L_z}{\delta}.$$

The nondimensionalized equations are

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U}\cdot\nabla)\mathbf{U} = -\nabla p + (\nabla\times\mathbf{H})\times\mathbf{H} + Re^{-1}\Delta\mathbf{U}, \nabla\cdot\mathbf{U} = 0, \quad (1)$$



(a) (b)

Figure 1. Nonlinear dynamo at $Re = 120$ and $Rm = 240$. Time evolution of (a) magnetic and (b) kinetic energies in the conducting region $R_i \leq r \leq R_o$ and $-\Gamma/2 \leq z \leq \Gamma/2$.

$$\frac{\partial \mathbf{H}}{\partial t} = \nabla\times(\mathbf{U}\times\mathbf{H}) + Rm^{-1}\Delta\mathbf{H}, \nabla\cdot\mathbf{H} = 0. \tag{2}$$

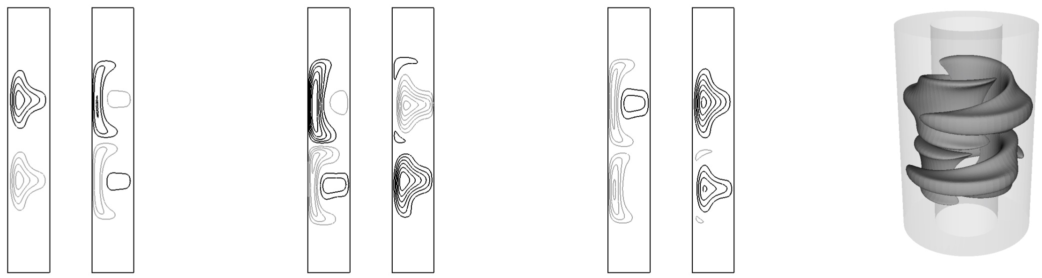
These equations are solved using a hybrid spectral-finite element method. They are written in cylindrical coordinates (r, θ, z). Each variable of the problem is decomposed in Fourier series in θ and polynomials in (r, z). The magnetic field in the vacuum is expressed as the gradient of a scalar potential and the continuities required on the interface between conducting and isulating domain are ensured by a penalization method (Interior Penalty Galerkin). This method was shown to be stable and convergent in [2] and adapted to the study of dynamo problem in [4].

In the following, the inner and outer cylinder radii are chosen as $R_i = 1$ and $R_o = 2$ such that $\delta = 1$, $\eta = 0.5$ and $\Gamma = L_z$. The inner cylinder rotates at unit angular speed $\Omega_i = 1$. We choose $\Gamma = 2\pi$ and the two cylinders are surrounded by vacuum bounded by a sphere of radius $R_s = 10$. Therefore the conducting domain Ω_c containing the fluid has dimensions: $r \in [1, 2]$, $\theta \in [0, 2\pi]$ and $z \in [-\Gamma/2, \Gamma/2]$.

3. Dynamo action

We first integrate the Navier-Stokes equations without the magnetic field. Above $Re^c = 65$, the azimuthal shear flow is destabilized by centrifugal instability and gives rise to the axisymmetric Taylor vortex flow consisting of three pairs of meridional rolls through an imperfect pitchfork bifurcation. This flow at $Re = 120$ is used to perform kinematic dynamo computations. Above the threshold of a Hopf bifurcation at $Rm^c = 187$, the magnetic eigenvector has a $m = 1$ structure and corresponds to a rotating wave which rotates in the same sense than the inner cylinder. Note that the magnetic wavelength is about twice that of the velocity as in the periodic case [5]; there is no scale separation in this kinematic dynamo. However, due to the value of the finite height, only one magnetic pair can fit in the domain (see figure 2 in the nonlinear regime). The Taylor vortex flow and the magnetic eigenvector are then used as initial condition for the nonlinear computation at $Re = 120$ and $Rm = 240$. We have chosen 12 azimuthal modes $(m = 0, ..., 11)$.

Time evolution of the kinetic and magnetic energies is reported on figure 1. From $t = 0$ to $t = 300$ (first phase), the magnetic energy grows exponentially with a growth rate similar to the kinematic one while the kinetic energy slightly decreases. Then, in the nonlinear regime, both magnetic and kinetic energies seem to saturate but, at $t = 800$, the magnetic energy decreases while the kinetic one increases (second phase for $800 \leq t \leq 1600$). The magnetic energy reaches



(a) $t = 200$ (b) $t = 750$ (c) $t = 1400$ (d) $t = 750$

Figure 2. Azimuthal magnetic field at $\theta = 0$ and $\theta = \pi/2$ (a) $t = 200$, (b) $t = 750$, (c) $t = 1400$. The magnetic field has odd azimuthal modes. The fields are normalized by the square root of the kinetic energy at $t = 1600$ and $-2 \leq H_\theta \leq 2$ (16 levels). (d) Isosurface of $|\mathbf{H}|$ (25% of maximum value) at $t = 750$

a minimum level at $t \approx 1600$ and increases again until $t = 2300$. We have reproduced this behavior for two cycles of about 1600 nondimensionalized time, where magnetic and kinetic energies oscillate in opposition between a minimum and a maximum level. It is instructive to follow in time the symmetry with respect to the $z = 0$ plane of the azimuthal component of the magnetic field. At initial condition, $H_\theta(r, \theta, -z, 0) = -H_\theta(r, \theta, z, 0)$. In the first phase, the magnetic field grows while keeping the eigenvector symmetry (see figure 2 a) until nonlinearities break this symmetry (see figure 2 b-d). The Lorentz force changes the symmetry of the velocity field and, therefore, the symmetry of the magnetic field. In the second phase, H_θ is nearly even in z (see figure 2 c) and the dynamo is killed. When the magnetic field is so weak that the Lorentz force fades out and perturbs less the Taylor vortex flow, the magnetic eigenvector with the same symmetry than the initial condition can grow again. This phenomenon repeats itself. It is a laminar cyclic dynamo regime. The nonlinear solution (see figure 3) corresponds to a modulated rotating wave with the wave period $T_{wave} = 170$ and the modulated period $T_{modulated} = 1600$.

4. Conclusion

The cyclic behaviour of the TC dynamo is related to the breaking of the equatorial symmetry of the flow induced by the growing magnetic field. This symmetry breaking leads to the fading of dynamo action. When the magnetic energy is sufficiently low, the flow is dominated by the $m = 0$ contribution with the adequate forcing equatorial symmetry. Dynamo action may begin again. This process differs from the "self-killing dynamos" which have been considered in [1, 3] where forcing leads to distinct hydrodynamic steady states with different dynamo properties.

We have studied the Taylor-Couette dynamo problem in a finite geometry in the nonlinear regime. In the finite configuration we have considered ($\eta = 0.5$, $\Gamma = 2\pi$, $Re = 120$), the axial scale of the most unstable magnetic mode (with azimuthal wavenumber $m = 1$) is about twice the one of the flow: this is in agreement with the periodic case, which was first examined in [5] in a kinematic case and extended to the nonlinear dynamo regime in [7]. However, the saturated state obtained in [7] is different: the magnetic energy saturates to a constant value corresponding to a rigidly rotating wave. In the finite case, the dynamo action shows a striking behaviour where the spatial symmetry about the equatorial plane of the velocity and magnetic fields play a major role. The dynamo is cyclic in time and the fields rotate rigidly with modulated amplitude. This result shows that, even in laminar regime ($Re = 120$, $Rm = 240$), realistic boundary conditions change the theoretical predictions and should stimulate experimental work.

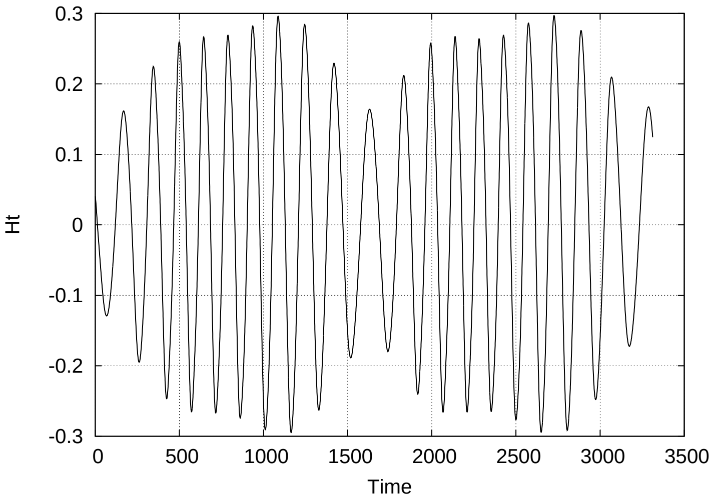


Figure 3. Time series of the azimuthal component of the magnetic field at the point $(r = 1.2, z = -1.5)$ corresponding to a modulated wave.

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