

CombinaTexas 2022

March 4-5, 2022

(Central Standard Time Zone)

Welcome to the Aggieland. All the lectures will be held in the Department of Mathematics, Blocker building, 3rd floor.

Friday evening, March 4, 2022

- 05:00–05:25pm: Registration
- 05:25–05:30pm: **Opening Remarks**
- 05:30–07:00pm: Contributed Session I

Saturday Morning, March 5, 2022

- 08:00–08:30am: **Registration open**
- 08:30–09:20am: **Plenary Talk 1: Jian Shen**
The Towers of Hanoi Problem and the Frame-Stewart Conjecture
- 09:30–10:30am: Contributed Session II
- 10:30–11:00am: **Break**
- 11:00–11:50am: **Plenary Talk 2: Nathan Williams**
Pop-Tsack Torsing

- 12:00–01:30pm: **Lunch Break**

- 01:30–02:20pm: **Plenary Talk 3: Kassie Archer**
Permutations from Dynamical Systems
- 02:30–03:30pm: Contributed Session III
- 03:30–03:50pm: **Break**
- 03:50–04:40pm: **Plenary Talk 4: Anton Dochtermann**
Homomorphism complexes, reconfiguration, and homotopy for digraphs
- 04:50–05:50pm: Contributed Session IV

Schedule for Contributed Talks

Friday Evening, Contributed Session I

- 05:30–05:50 Matthew Samuel. A Molev-Sagan-type formula for double Schubert polynomials.
- 05:50–06:10 Byeongsu Yu. When is the quotient of a semigroup ring by a monomial ideal Cohen-Macaulay
- 06:10–06:20 Break
- 06:20–06:40 Noureen Khan. Waste Number of Space-Efficient Knot Mosaics for Prime Knots with mosaic number 6
- 06:40–07:00 Ashley Tharp. Noncrossing arc diagrams beyond type A

Saturday Morning, Contributed Session II

- 09:30–09:50 Katie Anders. Characteristics of graphs admitting only constant splines over \mathbb{Z}_m
- 09:50–10:10 Leon Bankston. Combinatorial Structure in the Pauli Group
- 10:10–10:30 Padmapano Seveviratne. Quantum codes from metacirculant graphs

Saturday Afternoon, Contributed Session III

- 02:30–02:50 Jena Gregory. Iterated Generalized Rascal Triangles
- 02:50–03:10 Brandt Kronholm. The unimodality of Gaussian polynomials $\begin{bmatrix} N+m \\ m \end{bmatrix}$ for a few small values of m .
- 03:10–03:30 Ayo Adeniran. Pattern avoidance in parking functions

Sunday Afternoon, Contributed Session IV

- 04:50–05:10 Charles Burnette. Involution factorizations of Ewens random permutations
- 05:10–05:30 Jacob White. New inequalities for chromatic polynomials
- 05:30–05:50 Evangelos Nastas. On Probabilistic Distance and Diameter in Graphs

Abstracts of Plenary Talks

1. **Kassie Archer**, University of Texas–Tyler

Title: *Permutations from Dynamical Systems*

Abstract: We can obtain permutations, or patterns, of a dynamical system on a totally ordered set from the relative order of the initial segments of orbits. For a given map, we call a pattern "allowed" if it realized this way and "forbidden" if not. We will discuss interesting properties of allowed and forbidden patterns and some consequences for enumerative combinatorics.

2. **Anton Dochtermann**, Texas State University

Title: *Homomorphism complexes, reconfiguration, and homotopy for digraphs*

Abstract: The study of graph colorings (and more general homomorphisms) has been an active area of research for many years, with numerous applications and various tools brought to bear. Topological methods were introduced to the field via Lovász's neighborhood complex, and more general homomorphism (Hom) complexes of graphs were further studied by Babson and Kozlov. In joint work with Anurag Singh, we initiate the detailed study of Hom complexes for *directed graphs* (digraphs), which have applications in the study of graded posets and resolutions of monomial ideals. We relate the topological properties of Hom complexes to various digraph operations and also introduce the notion of a neighborhood complex for a directed graph. Our complexes provide a natural notion of reconfiguration in this setting, where one is interested in sampling from the space of digraph homomorphisms via walks in the 1-skeleton. We in particular study the connectivity of Hom complexes into tournaments T_n , obtaining a complete answer for the case of transitive T_n . Finally we use paths in the internal hom objects of directed graphs to define various notions of homotopy, and discuss connections to the topology of Hom complexes.

3. **Jian Shen**, Texas State University

Title: *The Towers of Hanoi Problem and the Frame-Stewart Conjecture*

Abstract: The multi-peg Towers of Hanoi problem consists of k pegs mounted on a board together with n disks of different sizes. Initially these disks are placed on one peg in order of size, with the largest on the bottom. The rules of the problem allow disks to be moved one at a time from one peg to another as long as no disk is placed on the top of a smaller disk. The goal of the problem is to transfer all the disks to another peg with the minimum number of moves, denoted $H(n,k)$. An easy recursive argument shows that $H(n, 3) = 2^n - 1$. However, the value of $H(n, k)$ is unknown for each $k > 3$. In this talk, we will discuss some upper and lower bounds on $H(n, k)$. Some variations of the Tower of Hanoi problems will also be considered.

4. **Nathan Williams**, University of Texas at Dallas

Title: *Pop-Tsack Torsing*

Abstract: Given a finite irreducible Coxeter group W , we use the W -noncrossing partition lattice to define a Bessis dual version of C . Defant's notion of a Coxeter

pop-stack sorting operator. We show that if W is coincidental or of type D , then the identity element of W is the unique periodic point of this operator and the maximum size of a forward orbit is the Coxeter number of W . In each of these types, we obtain a natural lift from W to the dual braid monoid of W . This is joint work with C. Defant.

Abstracts–Contributed Talks

Friday Evening, Contributed Session I

- Matthew Samuel, Prudential Financial

Title: A Molev-Sagan-type formula for double Schubert polynomials

Abstract: In 1997, Molev and Sagan published a positive, combinatorial formula for multiplying pairs of factorial Schur polynomials with different sets of coefficient variables. We present a formula that is a generalization of this to double Schubert polynomials that has the benefit of expressing the coefficients as polynomials in the negative roots with nonnegative coefficients in the specialization where the sets of coefficient variables are the same. This contributes to a solution to the longstanding unsolved problem of finding a positive combinatorial formula for the structure constants, in other words a Littlewood-Richardson rule for double Schubert polynomials, on which, objectively speaking, little progress has been made (though formulas for important classes of special cases have been found, such as the equivariant Pieri formula and puzzle rules for the Grassmannian and two-step flag variety). In addition to the case covered by Molev and Sagan’s formula, our formula generalizes to the Molev-Sagan case some known formulas for ordinary Schubert polynomials as well as providing a formula for previously unknown cases in both ordinary and double Schubert calculus.

- Byeongsu Yu, Texas A&M University

Title: When is the quotient of a semigroup ring by a monomial ideal Cohen-Macaulay?

Abstract: We give a new combinatorial criterion for quotients of affine semigroup rings by monomial ideals to be Cohen-Macaulay, by computing the homology of finitely many polyhedral complexes. This provides a common generalization of well-known criteria for affine semigroup rings and monomial ideals in polynomial rings. This is joint work with Laura Matusevich.

- Noureen Khan, University of North Texas at Dallas.

Title: Waste Number of Space-Efficient Knot Mosaics for Prime Knots with mosaic number 6

Abstract: Lomonaco and Kauffman introduced knot mosaics during their work on quantum knots in 2008. The theory of knot mosaics was later shown to be equivalent to the theory of tame knots. We calculate the waste number of space-efficient knot mosaics of prime knots and compile a list of the waste number of prime knots with mosaic number 6.

- Ashley Tharp, North Carolina State University

Title: Noncrossing arc diagrams beyond type A

Abstract: Noncrossing arc diagrams are a visual representation of permutations. They naturally encode canonical join representations and contain complete information on lattice congruences and lattice quotients on the weak order of type A. This talk will provide an overview of the existing results on noncrossing arc diagrams for permutations and present analogous results (joint with Barnard and Reading) on noncrossing arc diagrams for signed permutations (type B) and even signed permutations (type D). These results lead to work in progress exploring sublattice relationships between shard intersection orders on different finite Coxeter groups.

Saturday Morning, Contributed Session II

- Katie Anders, University of Texas at Tyler

Title: Characteristics of graphs admitting only constant splines over \mathbb{Z}_m

Abstract: We study the generalized graph splines introduced by Gilbert, Tymoczko, and Viel. We consider graphs that admit only constant splines and provide a characterization for splines on such graphs over the ring \mathbb{Z}_m .

- Leon Bankston, Tulane University

Title: Combinatorial Structure in the Pauli Group

Abstract: The Pauli group is a finite group that is well-studied in quantum information. We define two graphs from the commutation relations of the Pauli group and explore some of their properties.

- Padmapano Seveviratne, Texas A&M University-Commerce

Title: Quantum codes from metacirculant graphs

Abstract: In this research, we work on three closely connected objects, namely a metacirculant graph G , a symplectic self-dual additive code C over the finite field F_4 , and its corresponding quantum stabilizer code Q . We use this method to construct, for the first time, $[[l, 0, d]]$ qubit codes with parameters $(l, d) \in \{(78, 20), (90, 21), (91, 22), (93, 21), (96, 21)\}$.

Saturday Afternoon, Contributed Session III

- Jena Gregory, University of Texas Rio Grande Valley

Title: Iterated Generalized Rascal Triangles

Abstract: We introduce a sequence of number triangles, $\{R_i\}_{i=0}^{\infty}$, such that the entries of each share a common generalized recurrence relation. R_1 is the Rascal triangle and as i grows large, R_i becomes Pascal's triangle. For all i , we provide a combinatorial interpretation and find closed-term formulas for the entries of R_i . Additionally, we extend these results to a collection of Phil Hotchkiss' "Generalized Rascal Triangles." Our proofs rely on generating functions and other combinatorial arguments.

- Brandt Kronholm, University of Texas Rio Grande Valley

Title: The unimodality of Gaussian polynomials $\begin{bmatrix} N+m \\ m \end{bmatrix}$ for a few small values of m .

Abstract: In this presentation I will give new proofs of the unimodality of Gaussian polynomials $\begin{bmatrix} N+4 \\ 4 \end{bmatrix}$, $\begin{bmatrix} N+3 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} N+2 \\ 2 \end{bmatrix}$. Our proofs are completed by examining a novel collection of generating functions for these Gaussian polynomials. Surprisingly, the partitions of n into at most three parts plays an important role in the proof for the unimodality of $\begin{bmatrix} N+4 \\ 4 \end{bmatrix}$.

- Ayo Adeniran, Colby College

Title: Pattern avoidance in parking functions

Abstract: Parking functions have been well-studied in combinatorics. We extend the classical definition of patterns in permutations to parking functions. In particular, we study parking functions that avoid permutations of length 3. A number of well-known combinatorial sequences arise in our analysis, and this talk will highlight several enumeration results and conjectures. This project is joint work with Lara Pudwell.

Saturday Afternoon, Contributed Session IV

- Charles Burnette, Xavier University

Title: Involution factorizations of Ewens random permutations

Abstract: An involution is a bijection that is its own inverse. Given a permutation σ of $[n]$, let $\text{invol}(\sigma)$ denote the number of ways to express σ as a composition of two involutions. In this talk, we will prove that the statistic invol is asymptotically lognormal when the symmetric groups S_n are each equipped with Ewens Sampling Formula probability measures of some fixed positive parameter θ . A functional refinement and convergence rate for the Gaussian

- Jacob White, University of Texas Rio Grande Valley

Title: New inequalities for chromatic polynomials

Abstract: It is a long standing problem to find necessary and sufficient conditions for determining whether or not a given polynomial is a chromatic polynomial of some graph. We discuss some new conditions a chromatic polynomial satisfies. We also discuss conditions for determining whether a given symmetric function is the chromatic symmetric function of a graph. The underlying conditions come from studying generalized coloring complexes.

- Evangelos Nastas, SUNY

Title: On Probabilistic Distance and Diameter in Graphs

Abstract: This talk is devoted to exploring a probabilistic method for distance and diameter in Graphs. The objective here is to explore two different graph theory properties in the Erdős–Rényi model. First, the property isomorphic to the distance graph; second, the property isomorphic to the diameter of graphs.