

# White Paper on Combinatorics

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During the conference CombinaTexas'03, the organizing committee had met and discussed on the state of art, current directions, and emerging opportunities in the field of combinatorics. The following is a summary of the discussion.

The recent progress in graph drawing, random graphs, and computer-aided microchip design has been dramatic. Continuing research in graph theory and its applications will contribute advances in internet modeling, algorithm design, and bioinformatics, with each area being of independent and intense interest.

The following topics about graph theory were covered in the conference, which shed light on the future development of the subject.

1. *Graph Drawing.* Many real world problems are intractable, which are either NP-hard, or are undecided at all. A lot of problems in graph drawing are in this category. Graph drawing is to use graph as a model to solve real-world problems, (e.g., VLSI design), and to understand such problems by studying the structures of the graph. Current research contains network visualizations, in which people want to construct a network model and to analyze it; crossing minimization, in which people search for the extreme graph with the minimal number of crossings. It is also related to the Ulam's lattice point conjecture and homomorphisms of graph.

In the conference a new approach was suggested, that is, the use of computational geometry. It was proposed to use linear programming and integer programming techniques and algebraic characterizations to solve the geometric optimization problems. Future research involves drawing of infinite graphs, drawing in specified metric spaces, and drawing graphs with lattice points.

2. *Random Graphs.* The classical Erdos-Renyi model of random graphs ( $G(n, m)$  or  $G(n, p)$ ) uses probabilistic method to study the evolution properties of graphs, where the probability space consists of all graphs with  $n$  vertices. The current research focuses on some real-life graphs with a special degree sequence, for example, the random regular graphs and power-law graphs. It is much more difficult to study the class of random regular graphs, since there is a high dependence between pairs of edges present

in the graph. Almost all probabilistic techniques used in the classical random graph fail here.

Most results about random regular graphs  $G_{n,d}$ , even for constant  $d$ , are settled recently, due to a new approach to the problem: the graph process models. Several new models for random regular graphs were established, including the matching model, and the algorithm model. Graphs are generated by certain stochastic processes, which are compared with the uniform random regular graphs. Advanced techniques are introduced, including stochastic analysis, Sandwich theorem, and new sharp concentration inequalities. The future research would be on the comparison of various graph models, and on applications to random graphs with a given degree sequence  $(d_1, d_2, \dots, d_n)$ . This work has far-reaching applications in “massive networks”, including internet graphs, neural networks, and telecommunication graphs.

3. *Chromatic Sum of Graphs and Optimization.* In the minimum sum coloring problem one has to assign a color (positive integer) to each vertex of the graph such that the sum of these integers is minimal. The smallest sum that can be achieved is the chromatic sum of the graph. The concept was introduced by E. Kubicka in 1989. Schwenk and Kubicka have demonstrated the NP-completeness of this problem and have constructed a polynomial algorithm for determining the chromatic sum of an arbitrary tree  $T$ . However, as far as we know no other chromatic sum algorithm exists, for any class of graphs. In the minimum sum multicoloring problem each vertex has a demand which tells us how many colors have to be assigned to the vertex (minimum sum coloring is the special case where every demand is 1). As usual, the colors have to be assigned in such a way that a color cannot appear on two neighboring vertices. The optimization problem is the following: the response time of a vertex is defined to be the largest color assigned to it, and the objective is to minimize the sum of these finish times. This problem is motivated by applications in combinatorial optimization. In the context of distributed resource allocation the following optimization problem arises: given a conflict graph that represents the competition of processors over resources, find an allocation under which no two jobs with conflicting requirements are executed simultaneously. The objective is to minimize the average response time of the system. During the conference, different variations of this problems and their complexities over different families of graphs were discussed.
4. *Graph Dynamics.* In the last 30 years much research has been done in the so-called *graph operators* or *graph valued functions*. The most widely studied among them is the *line graph*, which is used in many applications in different fields. In order to study those applications and to make the line graph technique more versatile different

generalizations have been introduced. Some examples are total graphs, middle graphs, clique graphs or triangle graphs. Lately, other new generalizations of line graphs have appeared, such as path graphs or super line graphs, which were presented as alternatives to the line graph in interconnection networks design and analysis. The main problem is then to determine which of the properties of line graphs are inherited by these generalizations and which advantages they present in concrete applications. In the conference we discussed characterizations and isomorphisms of super line graphs, as well as some variations when the operator is applied to hypergraphs, directed graphs or multigraphs. Some topological properties were presented in relation to cycles, independence, pancyclicity, distances and connectivity. Distance properties of path graphs were also discussed, in particular, those regarding to the diameter. Finally, we talked about the iterated application of super line graphs and path graphs.