<u>Linear Programming: A Geometric Approach</u>

3.1: Graphing Systems of Linear Inequalities in Two Variables

The general form for a line is ax+by+c = 0.

The general forms for linear inequalities are

$$ax \quad by \quad c \quad 0$$
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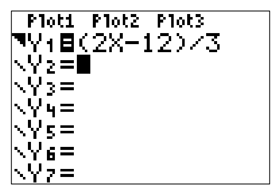
How do these inequalities look graphically?

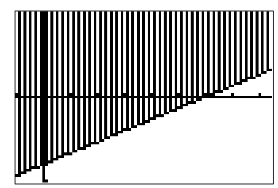
Example – Graph the inequality 2x-3y ≥ 12.
Answer - start with the "=" part.
Graph the line 2x - 3y = 12.

This line divides the plane into an upper half plane and a lower half plane. One side of the line is the solution.

NOTE - if your line passes through the origin, you must take a different point for a test point.

On the calculator,





If our inequality had ≥ or ≤ we draw the bounding line as a solid.

If our inequality had > or < we draw the bounding line as DASHED .

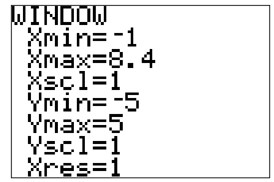
The region that satisfies our inequality is called the *feasible region*. This is the region that is white (unshaded). Please label it with an *S*.

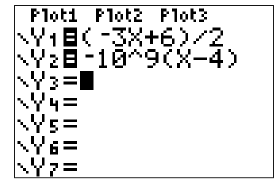
What if we have two inequalities (a system)? The feasible region will be where they are both true at the same time.

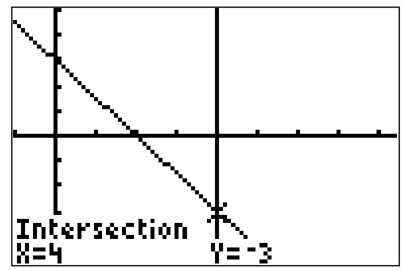
Example: Find the feasible region for the system

$$\begin{array}{llll} 3x & +2y & > & 6 \\ x & & \leq & 4 \end{array}$$

We can't really draw vertical lines on the calculator, so we want to draw one with a VERY large slope instead.







one corner at (4,-3).

It is *UNBOUNDED* because the feasible region cannot be enclosed in a circle.

If S can be enclosed by a circle, it is called *BOUNDED*.

Example: Find the feasible region and label the corner points for the following system of linear inequalities:

$$\begin{aligned} 4x - 3y &\le 12 \\ x + 2y &\le 10 \\ x &\ge 0 \\ y &\ge 2 \end{aligned}$$

This is a bounded region. Where are the corners?

3.2 – 3.3 Linear Programming Problems

Linear inequalities are important because we often want to minimize or maximize a quantity (called the *objective function*) subject to certain constraints (linear inequalities).

For example, we may want to maximize our profits but we are constrained by how much material and labor are available.

Or we may want to minimize the calories in our diet subject to getting a least our daily requirements of vitamins.

In this section the text only SETS-UP the problems. Solving them isn't until the next section.

We will do both at once.

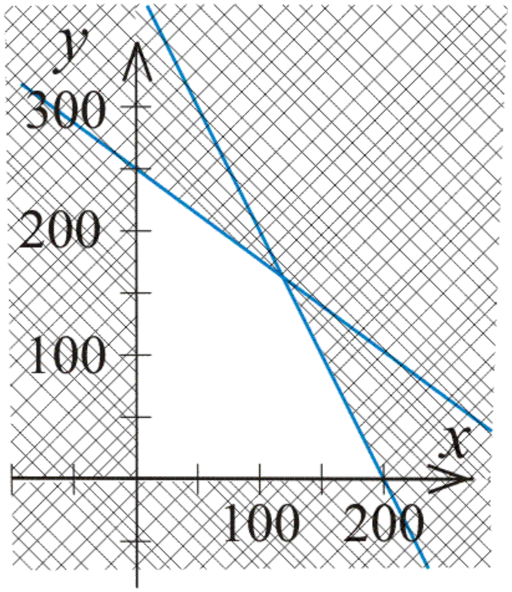
Production Scheduling: A company produces two models of hibachis, model A and model B.

- To produce each model A requires 3 lbs of cast iron and 6 minutes of labor.
- Each model B requires 4 lbs of cast iron and 3 minutes of labor.
- The profit for each model A is $2 and the profit for each model B is $1.50.
- If 1000 lbs of cast iron and 20 hours of labor are available each day, how many hibachis of each model should be produced to maximize profits?

Any point in *S* will satisfy the constraints, but which one will maximize our profit?

$3x+4y \leq 1000$ (pounds of iron)

$6x + 3y \leq 1200$ (minutes of labor)



Lowest possible profit is P = 0

How about a profit of $300?

How high can we go?

Theorem 1: If a linear programming problem has a solution, then it must occur at a vertex (or corner point) of the feasible region *S* associated with the problem.

Furthermore, if the objective function is optimized at two adjacent vertices of *S*, then it is optimized at every point on the line segment joining these vertices. In this case there are infinitely many solutions.

Theorem 2: Existence of a Solution

Suppose we are given a linear programming problem with a feasible set *S* and objective function P = *ax* + *by*.

If *S* is bounded then P has both a maximum and a minimum value on S.

If *S* is unbounded and both *a* and *b* are non-negative, then P has a minimum value on *S* provided that the constraints defining *S* include $x \geq 0$ and $y \geq 0$.

If *S* is the empty set, then the linear programming problem has no solution.

METHOD OF CORNERS:

1. Graph the feasible region (set), *S*.

2. Find the EXACT coordinates of all the vertices (corner points) of *S*.

3. Evaluate the objective function at each vertex.

4. Find the vertex that renders the objective function a maximum (or minimum).

If there is one such vertex, that is the unique solution to the linear programming problem.

If the objective function is maximized (or minimized) at two adjacent vertices of *S*, there are infinitely many optimal solutions give by the points on the line segment connecting the two vertices.

Now we can solve the Hibachi problem. Make a table.

A frozen lemonade stand has 5 cups of lemon juice, 8 cups of sugar and 16 cups of water available to make sweet and tangy frozen lemonades for tomorrow. A sweet frozen lemonade uses ¼ cup of lemon juice, ½ cup of sugar and ½ cup of water. The tangy lemonade uses ¼ cup of lemon juice, ¼ cup of sugar and 1 cup of water. If the price of a sweet lemonade is $1.25 and the tangy lemonade is $1.00, how many of each type of frozen lemonade should be made to maximize revenue? Is anything leftover?

Example: A dietitian is to prepare two foods in order to meet certain requirements. Each pound of food I contains 100 units of vitamin C, 40 units of vitamin D and 20 units of vitamin E and costs 20 cents. Each pound of food II contains 10 units of vitamin C, 80 units of vitamin D and 15 units of vitamin E and costs 15 cents. The mixture of the two foods is to contain at least 260 units of vitamin C, 320 units of vitamin D and 120 units of vitamin E. How many pounds of each type of food should be used in order to minimize the cost? Is there an excess of any vitamin?