Derivatives of Vector Functions (Section 3.7)

**EXAMPLE 1**
Sketch the following vector equations and include the direction of the curve. Find the equation of a unit tangent vector to the curve at the given value of $t$.

$r(t) = \langle t, t^2 \rangle, \quad t = 1$

$r(t) = \langle 2 \cos t, 3 \sin t \rangle, \quad t = \pi/2$

**EXAMPLE 2**
Find $r'(t)$ and the domain of $r(t)$ and $r'(t)$ for

$r(t) = \langle t^2 - 4, \sqrt{9-t} \rangle$

**Domain of $r(t)$:** $t \leq 9$

$r'(t) = \langle 2t, \frac{-1}{2\sqrt{9-t}} \rangle$

**Domain of $r'(t)$:** $t < 9$
EXAMPLE 3
The position (in feet) of an object at time \( t \) (in seconds) is given by
\[
\mathbf{r}(t) = (t, 25t - 5t^2) \quad \mathbf{v}(t) = (1, 25 - 10t) = \mathbf{v}(t)
\]
(a) Find the position, velocity, and speed at the time \( t = 1 \)
\[
\mathbf{r}(1) = (1, 25 - 5) = (1, 20)
\]
\[
\mathbf{v}(1) = (1, 25 - 10) = (1, 15)
\]
\[
|\mathbf{v}(1)| = \sqrt{1^2 + 15^2} = \sqrt{226}
\]
(b) When does the item strike the ground and with what speed?

Strikes ground when \( y = 0 \)
\[
y = 25t - 5t^2 = 0
\]
\[
\Rightarrow 5t(5 - t) = 0
\]
\[
\Rightarrow t = 5
\]
\[
\mathbf{v}(5) = (1, 25 - 10(5)) = (1, -25)
\]
\[
|\mathbf{v}(5)| = \sqrt{1^2 + (-25)^2} = \sqrt{626}
\]

EXAMPLE 4
Find the angle of intersection of the curves \( \mathbf{r}(t) = (1-t, 3+t^2) \) and \( \mathbf{s}(u) = (u-2, u^2) \).

1. Intersect when \( x = x \) and \( y = y \)
\[
\Rightarrow 1-t = u-2 \quad \text{and} \quad 3+t^2 = u^2
\]
\[
\Rightarrow u = 3-t \quad 3+t^2 = (3-t)^2
\]

2. Check \( t = 1, u = 2 \)
\[
3+t^2 = 3 - 6t + t^2
\]
\[
\Rightarrow 6t = 6 \Rightarrow t = 1
\]
\[
\mathbf{r}(1) = (0, 4) \quad \mathbf{s}(1) = (0, 4)
\]

3. \( \mathbf{r}'(t) = (-1, 2t) \), \( \mathbf{s}'(1) = (-1, 2) \)
\[
|\mathbf{r}'(1)| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}
\]
\[
|\mathbf{s}'(2)| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}
\]
\[
\cos \Theta = \frac{\mathbf{r}'(1) \cdot \mathbf{s}'(2)}{|\mathbf{r}'(1)| |\mathbf{s}'(2)|}
\]
\[
= \frac{(-1, 2) \cdot (1, 4)}{\sqrt{5} \cdot \sqrt{5} = \frac{-1 + 8}{5} = \frac{7}{5}}
\]
\[
\Theta = \cos^{-1} \left( \frac{7}{5} \right) \approx 41^\circ