Higher Derivatives (Section 3.8)

The second derivative of the function \( y = f(x) \) is

\[
y'' = f''(x) = \frac{d}{dx} \left( f'(x) \right) = \frac{d}{dx} \left( \frac{d}{dx} f(x) \right) = \frac{d^2 y}{dx^2} = D^2 f(x) = D_x^2 f(x)
\]

The third derivative of the function \( y = f(x) \) is

\[
y''' = f'''(x) = \frac{d}{dx} \left( f''(x) \right) = \frac{d^3 y}{dx^3} = D^3 f(x) = D_x^3 f(x)
\]

The \( n \)th derivative of the function \( y = f(x) \) is

\[
y^{(n)} = f^{(n)}(x) = \frac{d}{dx} \left( f^{(n-1)}(x) \right) = \frac{d^n y}{dx^n} = D^n f(x) = D_x^n f(x)
\]

**EXAMPLE 1**
Find the second derivative of the following functions

\[
g(u) = \frac{1}{\sqrt{1-u}}
\]

\[
g(x) = x^2 \cos x
\]

**EXAMPLE 2**
Find a formula for \( f^{(n)}(x) \) when \( f(x) = \frac{1}{(1-x)^2} \)

\[
D^{99} \sin 3x =
\]

The instantaneous rate of change of the velocity is the acceleration. So if \( s(t) \) is the position of an object at time \( t \), the acceleration is \( a(t) = v'(t) = s''(t) \)

**EXAMPLE 3**
Given the position in meters at time \( t \) in seconds of an object is given by \( s = 2t^3 - 9t^2 \), find the times when the acceleration is zero. At the times when the acceleration is zero, where is the object and what is the object’s velocity?
EXAMPLE 5
(a) Sketch the curve traced by the given vector equation
(b) Find $r'(t)$ and $r''(t)$
(c) Sketch the position vector $r(t)$, the tangent vector $r'(t)$ and $r''(t)$
for the given value of $t$.

$$r(t) = \langle 2\cos t, 3\sin t \rangle, \quad t = \pi/3$$

$$r(t) = t^3 i + t^2 j, \quad t = 1$$
**EXAMPLE 6**
Find \( f''(x) \) if \( f(x) = g(x^3) + (g(x))^3 \).

**EXAMPLE 7**
Find \( y'' \) by implicit differentiation for \( \sqrt{x} + \sqrt{y} = 1 \).

**EXAMPLE 8**
Find \( f' \) and \( f'' \). Sketch \( f, f' \) and \( f'' \) and determine their domains.
\[ f(x) = |x^2 - x| \]