CHAPTER 7: PROBABILITY

7.1: Experiments, Sample Spaces and Events

An EXPERIMENT is an activity with an observable result.

Tossing coins, rolling dice and choosing cards are all probability experiments.

The result of the experiment is called the OUTCOME or SAMPLE POINT.

So the two possible outcomes from tossing a coin

The set of all outcomes or sample points is called the SAMPLE SPACE of the experiment.

An EVENT is a subset of a sample space. That is, an event can contain one or more outcomes that are in the sample space.

Consider tossing a coin. The sample space is S = {H,T}.

The events that are possible in this experiment are

When 2 outcomes are in the sample space, there are 4 different events [subsets].

If a 6-sided die is rolled, the sample space is

Consider tossing a coin 3 times.

These sample spaces are all UNIFORM.

A sample space in which each of the outcomes has the same chance of occurring is called a UNIFORM SAMPLE SPACE.

A uniform sample space has equally likely outcomes.

A non-uniform sample space for tossing a coin three times is

The uniform sample space is

Find the event E where E = {x|x has exactly one head}

Find the event E where E = {x|x has two or more heads}

Find the event E where E={x|x has more than 3 heads}

Rolling two fair six-sided dice.

1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

These sample spaces have all been finite. That is, we can list all the elements.

An infinite sample space has to be described; you can't list all the elements:

What is the sample space for the time spent working on a homework set?

Describe the event of spending between one and two hours on a homework set.

7.2 Definition of Probability

When we toss a fair coin the two outcomes in the sample space S = {H, T} are equally likely, so the probability of each outcome is 1/2.

This is a THEORETICAL PROBABILITY based on the sample space having equally likely outcomes.

In general, this is the way we will find probability, by using a sample space of EQUALLY LIKELY OUTCOMES.

The probability of an event, P(E) is a number between 0 and 1,

We can also calculate the EMPIRICAL PROBABILITY of an event by doing an experiment many times.

For example, you could toss a coin and note how many times it comes up heads (shown in book)

or you could roll a die and count how many times a 1 is rolled.

number of tosses (m)	number of 1's rolled (n)	relative frequency (n/m)

(SUBJECTIVE PROBABILITY)

UNIFORM SAMPLE SPACE $S=\{s_1,s_2, ...,s_n\}$,

$\{s_1\}, \{s_2\} . . . \{s_n\}$ are called SIMPLE events

Notice these simple events are MUTUALLY EXCLUSIVE as only one can occur.

PROBABILITY DISTRIBUTION TABLE:

Event	probability

Properties of probability distribution tables:

Toss a coin three times. There are 8 equally likely outcomes:

Event Probability

Find the probability distribution table for the number of heads when a coin is tossed 3 times.

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

What is the probability of 2 or more heads?

Suppose the instructor of a class polled the students about the number of hours spent per week studying math during the previous week. The results were 69 students studied two hours or less, 128 students studied more than two hours but 4 or less hours, 68 students studied more than 4 hours but less than or equal to 6 hours, 30 students studied more than 6 hours but less than or equal to 8 hours and 14 students studied more than 8 hours.

Arrange this information into a PDT and find the probability that a student studied more than 4 hours per week

What is the probability of rolling a sum 2 or a sum of 12 using two fair die?

1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

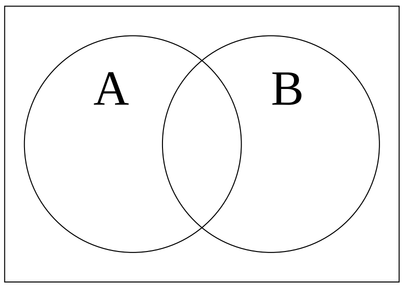
What is the probability of rolling a sum of 7?

1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

7.3 Rules of Probability

If event A and event B are mutually exclusive then

In general, A and B have some outcomes in common so we have the union rule for probability:

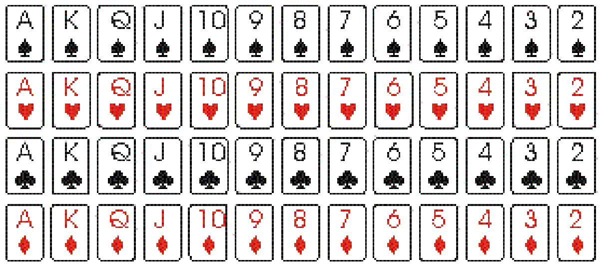


Example: Let
E={x|x is a sum of 7} =
and
F={x|x is a 6 on the green die} =

1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

What is the probability that you have a sum of 7 OR a 6 on the green die?

A standard deck of 52 cards has 4 suits, each with 13 cards. The suits are spades, ♠, hearts, ♥, clubs, ♣ ,and diamonds, ♦. The cards in each suit are numbered from Ace, King, Queen, Jack, ten down to 2.



Example - If a single card is drawn from a standard deck of cards, what are the probabilities of

a) a 9 or a 10? Let

b) a black card or a 3?

Example - a survey gave the following results: 45% of the people surveyed drank diet drinks (D) and 25% drank diet drinks and exercised ($D \cap E$) and 24% did not exercise and did not drink diet drinks ($D^c \cap E^c$). Find the probability that:

a) a person does not drink diet drinks (D^c).

b) does not exercise and drinks diet drinks ($E^c \cap D$).

c) exercises and does not drink diet drinks ($E \cap D^c$).

Draw the diagram.

Use of Counting Techniques in Probability

Let S be a uniform sample space and E be any event in S. Then

$$P(E) = \frac{\text{number of outcomes in event E}}{\text{number of outcomes in the Sample Space}} = \frac{n(E)}{n(S)}$$

Example - suppose we have a jar with 8 blue and 6 green marbles. What is the probability that in a sample of 2, both will be blue?

What is the probability there is at least one blue marble?

Find the probability distribution table for the number of blue marbles in the sample of 2 marbles:

Example - a stack of 100 copies has 3 defective papers. What is the probability that in a sample of 10 there will be no defective papers?

Example - A student takes a true/false test with 5 questions by guessing (choose answer at random). Write a probability distribution table for the number of correct answers.

7.5 Conditional Probability and Independent Events

A survey is done of people making purchases at a gas station. Most people buy gas (Event A) or a drink (Event B).

	buy drink (B)	no drink (B^c)	total
buy gas (A)			
no gas (A^c)			
total			

What is the probability that a person bought gas and a drink?

What the probability that *a person who buys a drink* also buys gas? In other words, **given** that a person *bought a drink* (B), what is the probability that they bought gas (A)?

NOTATION: P(A|B) = the probability of A given B

The CONDITIONAL PROBABILITY of event E given event F is

$$P(E \mid F) = \frac{n(E \cap F)}{n(F)} = \frac{n(E \cap F)/n(S)}{n(F)/n(S)} = \frac{P(E \cap F)}{P(F)}$$

What is the probability that *a person who buys gas* also buys a drink?

PRODUCT RULE: $P(E \cap F) = P(E)P(F \mid E)$

TREE DIAGRAMS

At a party, 1/3 of the guests are women. 75% of the women wore sandals and 25% of the men wore sandals. What is the probability that a person chosen at random at the party is a man wearing sandals, $P(M \cap S)$? What is the probability that a randomly chosen guest is wearing sandals?

A finite stochastic process is one in which the next stage of the process depends on the state you are in for the previous stage.

Consider drawing 3 cards from a standard deck of 52 cards without replacement.
(a) What is the probability that the three cards are hearts?
(b) What is the probability that the third card drawn is a heart given the first two cards are hearts?

INDEPENDENT EVENTS: P(E|F) = P(E)

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} = P(E) \quad \Rightarrow$$

$$P(E \cap F) = P(E) \cdot P(F) \; iff \; E \text{ and } F \text{ are independent}$$

Example - A certain part on airplane has a 1% chance of failure. If they carry two back-ups to this part, what is the probability that they will all fail?

Example - A medical experiment showed the probability that a new medicine was effective was 0.75, the probability of a certain side effect was 0.4 and the probability for both occurring is 0.3. Are these events independent?

7.6 Baye's Theorem

Given P(E|F), can we find P(F|E)? YES! Just use the definition,

Example - We are to choose a marble from a cup or a bowl. We need to flip a coin to decide to choose from the cup or the bowl. The bowl contains 1 red and 2 green marbles. The cup contains 3 red and 2 green marbles. What is the probability that a marble came from the cup given that it is red?

Example - Given the following table with income levels in "bucks", what is the probability that a family of two wage earners selected at random will have a family income of less than 10,000 "bucks"?

family income in "bucks"	proportion of families	proportion with 2 wage earners (E)
<10,000 (F_1)	0.08	0.10
10,000- 20,000 (F_2)	0.20	0.20
20,001 - 30,000 (F_3)	0.40	0.30
30,001 - 40,000 (F_4)	0.20	0.20
>40,001 (F_5)	0.12	0.20