MARKOV CHAINS

1. Classify the following matrices as
(A) regular transition matrix   (B) not regular transition matrix   (C) not a transition matrix

_____ $\begin{bmatrix} 0.5 & 0.4 \\ 0.5 & 0.6 \end{bmatrix}$ _____ $\begin{bmatrix} 0.5 & 0.8 \\ 0.5 & 0.6 \end{bmatrix}$ _____ $\begin{bmatrix} 0.75 & 0 \\ 0.25 & 1 \end{bmatrix}$

2. A study has shown that a family living in the state of Denial typically takes a vacation once per year. A family that takes an out-of-state vacation has a 35% chance of taking an out-of-state vacation the following year and a 65% chance of taking an in-state vacation. A family that has taken an in-state vacation has a 50% chance of taking an out-of-state vacation the next year and 50% chance of taking an in-state vacation.
(a) What is the transition matrix for the vacation decision?
(b) If the initial distribution of vacations is 25% in-state and 75% out-of-state and people, what is the probability that a family will take an out-of-state vacation in two years?
(c) What is the long term distribution of vacation locations?

3. A charge account is classified at the end of each month as "paid," "overdue less than 30 days," "overdue less than 60 days," or "overdue 60 days or more." In the latter case, the debt is judged "bad" and the charge account is canceled. Each month the debt can age by at most one month, keep the same or lesser age by means of a payment, or be paid off. If paid off, the account remains paid off. Over a period of time, the company has obtained figures that give the following transition matrix

	Paid	< 30	< 60	Bad
Paid	1	1/3	1/6	0
< 30	0	1/3	1/3	0
< 60	0	1/3	1/3	0
Bad	0	0	1/6	1

By looking at the long-term behavior, determine the probability that the debt will be paid off and the probability that the debt will default in each of the cases: < 30 and < 60.

GAME THEORY

1. A student has a quiz in his math class. He can study hard for the quiz, study a bit for the quiz or not study at all. The quiz can be challenging, moderate or easy. The student assigns a "satisfaction value" to each of the outcomes as shown in the matrix below:

a) What should the student do if he is an optimist? Why?

b) What should the student do if he is a pessimist? Why?

c) What is the expected value of studying a little if there is a 25% chance of a challenging quiz, 50% chance of a moderate quiz and 25% chance of an easy quiz?

$$
\begin{array}{cccc}
 & \text{challenging} & \text{moderate} & \text{easy} \\
\text{study hard} & 5 & 3 & 2 \\
\text{study a little} & 2 & 4 & 1 \\
\text{don't study} & -1 & 1 & 0
\end{array}
$$

2. Bill and Sue play a game with coins. Both flip a coin at the same time. If both coins show heads or both coins show tails, Bill wins $2 from Sue. If one coin shows heads and one shows tails, Sue wins $2 from Bill. Construct the payoff matrix for this game.

3. Using the payoff matrix below for a two-person zero-sum game, find the saddle point, the optimal strategy for the row player and the value of the game. Is the game fair? If not, why not?

$$
\begin{pmatrix}
1 & -4 & 5 \\
-2 & 0 & -1 \\
4 & 1 & 1 \\
-1 & -2 & -4
\end{pmatrix}
$$

4. Remove any dominated strategies from the matrix
$$
\begin{pmatrix}
2 & 3 & -1 & -5 \\
6 & 5 & 4 & 1 \\
1 & -4 & 0 & 3
\end{pmatrix}
$$

5. Given the payoff matrix $M = \begin{pmatrix} 2 & -3 \\ -5 & 6 \end{pmatrix}$

(a) What is the value of this game if the row player plays row 1 20% of the time and the column player plays column 1 70% of the time?

(b) What is the row player's optimum strategy?

(c) What is the value of the game if each player plays their optimum strategy?