1.3 Sample Spaces and Events

An ***experiment*** is an activity with an observable result.
Tossing coins, rolling dice and choosing cards are all probability experiments.

The result of the experiment is called the ***outcome*** or ***sample point***.

The set of all outcomes or sample points is called the ***sample space*** of the experiment.

Example

a) What is the sample space for flipping a fair coin?

$$S = \{H, T\}$$

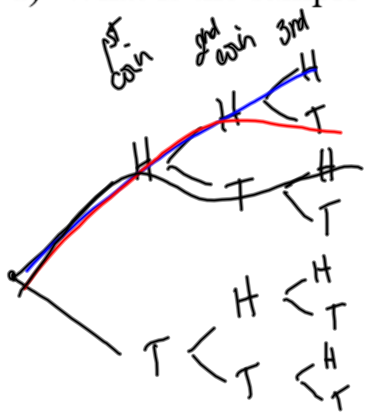
b) What is the sample space for rolling a fair six-sided die?

$$S = \{1, 2, 3, 4, 5, 6\}$$

c) What is the sample space for rolling two fair four-sided dice?

$$S = \left\{\begin{array}{l} 1\text{-}1,\ 1\text{-}2,\ 1\text{-}3,\ 1\text{-}4, \\ 2\text{-}1,\ 2\text{-}2,\ 2\text{-}3,\ 2\text{-}4, \\ 3\text{-}1,\ 3\text{-}2,\ 3\text{-}3,\ 3\text{-}4, \\ 4\text{-}1,\ 4\text{-}2,\ 4\text{-}3,\ 4\text{-}4 \end{array}\right\}$$

d) What is the sample space for flipping 3 fair coins?



$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

A sample space in which each of the outcomes has the same chance of occurring is called a UNIFORM SAMPLE SPACE.

Example a non-uniform is $S=\{M, I, S, P\}$

A letter is chosen from the word MISSISSIPPI. What is the uniform sample space for this experiment?

$S=\{M, I, S, S, I, S, S, I, P, P, I\}$, $n(S)=11$

An ***event*** is a subset of a sample space. That is, an event can contain zero, one, or more outcomes that are in the sample space.

Example $S=\{H, T\}$

What are the possible events when a single fair coin is flipped?

$\{H\}, \{T\}, \{H, T\}, \emptyset$

In general, if there are n outcomes in a sample space, there are 2^n events.

Example

A single marble is drawn from a cup with one red, one blue and one green marble. List all possible events in this experiment.

$S=\{r, b, g\}$ $n(S)=3$ # of events $= 2^3 = 8$

$\emptyset, \{r\}, \{b\}, \{g\}, \{r, b\}, \{r, g\}, \{b, g\}, \{r, b, g\}$

Two events are ***mutually exclusive*** if the sets are disjoint.

<u>*Example*</u>

Two fair six—sided dice are rolled. One is red and one is green.

1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

Let *E* be the event that the sum of the numbers shown uppermost is 7
F be the event that the red die shows a 1
G be the event that the green die shows a 6
H be the event that the sum of the numbers shown uppermost is 10

Which of the following pairs of events are mutually exclusive? If the events are not mutually exclusive, what outcomes do they have in common?

E and *F* Not ME, {1-6} in common

E and *G* " "

E and *H* ME

F and *G* Not ME, {1-6} in common

F and *H* ME

G and *H* Not ME, {4-6} in common

A simple event is an event that contains exactly one outcome

A ***continuous sample space*** has outcomes that are not restricted to certain values. In these cases, the sample space must be described.

Example

a) What is the sample space for the time spent working on a homework set?

$S = \{t \mid t \geq 0 \text{, } t \text{ in minutes}\}$

exclusive

b) Describe the event of spending between one and two hours on a homework set.

$E = \{t \mid 60 < t < 120\}$

Spending between one and two hours inclusive

$E = \{t \mid 60 \leq t \leq 120\}$

1.4 Basics of Probability

If S is a finite uniform sample space and E is any event, then the probability of E given by

$$P(E) = \frac{n(E)}{n(S)}$$

Example
Two fair six—sided dice are rolled. One is red and one is green.

1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

$n(S) = 36$

a) What is the probability that the sum of the numbers is 5?

$P = \frac{4}{36} \left(= \frac{1}{9} \right)$

b) What is the probability that the sum of the numbers shown is 8 and there is a 4 on the red die?

$\frac{1}{36}$

c) What is the probability the sum of the numbers shown is 1?

$P = \frac{0}{36} = 0$

d) What is the probability that the sum of the numbers shown uppermost is 2 or greater?

$P = \frac{36}{36} = 1$

We can also calculate the ***empirical probability*** of an event by doing an experiment many times.

Example
Roll a fair six-sided die and count how many times a 1 is rolled.

Rel. freq

# of tosses (m)	# of 1's rolled (n)	probability (n/m)
15	3	3/15 (=.2)
100	15	15/100 (= .15)
1000	170	170/1000 (=.17)

⇒ 1/6 (in theory)

Consider a uniform sample space $S = \{s_1, s_2, \ldots, s_n\}$.

Events that contain a single outcome such as $\{s_i\}$ are called ***simple*** events

✱ ***Probability distribution table:*** ✱

Event	probability
$\{s_1\}$	$1/n$
$\{s_2\}$	$1/n$
$\vdots$	
$\{s_n\}$	$1/n$

adds to 1

① The events list MUST BE Mut. excl.

② PROB MUST ADD TO 1

Properties of probability distribution tables:

Example S={HHH, HHT, HTH, THH, THT .... TTT}

Find the probability distribution table for the number of heads when a coin is tossed 3 times.

EVENT	PROB
3H	1/8
2H	3/8
1H	3/8
0H	1/8

$\frac{4}{8}$

What is the probability of 2 or more heads?

Example

Suppose the instructor of a class polled the students about the number of hours spent per week studying math during the previous week. The results were 69 students studied two hours or less, 128 students studied more than two hours but 4 or less hours, 68 students studied more than 4 hours but less than or equal to 6 hours, 30 students studied more than 6 hours but less than or equal to 8 hours and 14 students studied more than 8 hours.

Arrange this information into a PDT and find the probability that a student studied more than 4 hours per week

Let t be the time spent studying in hours

EVENT	# of students	PROB
$0 \le t \le 2$	69	69/309
$2 < t \le 4$	128	128/309
$4 < t \le 6$	68	68/309
$6 < t \le 8$	30	30/309
$t > 8$	14	14/309
	309	

$$P(t>4) = \frac{68}{309} + \frac{30}{309} + \frac{14}{309}$$

$$= \frac{112}{309}$$

What is the probability of rolling a sum 2 or a sum of 12 using two fair die?

1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

$P = \frac{1}{36} + \frac{1}{36} = \frac{2}{36}$

sum of 2 and a sum of 12

$P = 0$

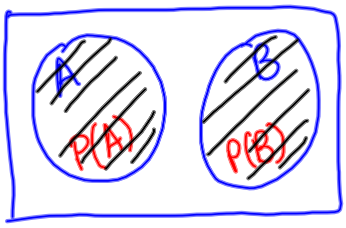
What is the probability of rolling a sum of 7?

1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

$\frac{6}{36}$

1.5 Rules for Probability

If events A and event B are mutually exclusive then



$$P(A \cup B) = P(A) + P(B)$$

In general, A and B have some outcomes in common so we have the union rule for probability:

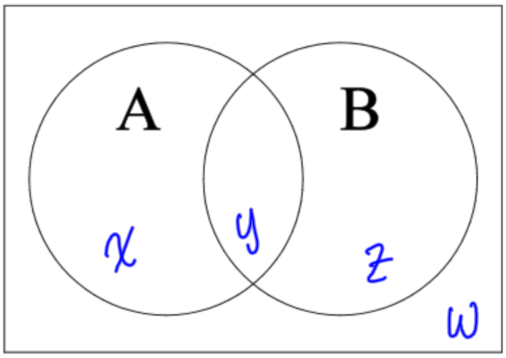
$x = P(A \cap B^c)$, $y = P(A \cap B)$, $z = P(A^c \cap B)$

$w = P((A \cup B)^c)$

$\boxed{x + y + z + w = 1}$

$P(A) = x + y$

$P(B) = y + z$



$$P(A \cup B) = x + y + z = (x+y) + (y+z) - y = P(A) + P(B) - P(A \cap B)$$

Example

Two fair six-sided dice are rolled. One die is red and one die is green.

1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

Let E be the event that the sum of the numbers shown uppermost is 7
F be the event that the red die shows a 1
G be the event that the green die shows a 6
H be the event that the sum of the numbers shown uppermost is 10

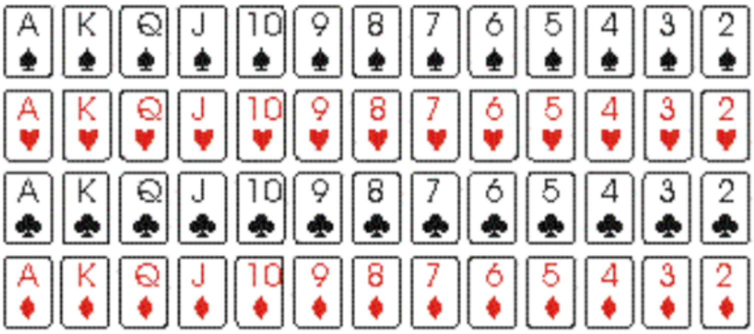
Find the following probabilities

a) $P(E \cup F) = \frac{11}{36} = P(E) + P(F) - P(E \cap F) = \frac{6}{36} + \frac{6}{36} - \frac{1}{36}$

b) $P(G \cup H) = \frac{8}{36} = \frac{6}{36} + \frac{3}{36} - \frac{1}{36}$

c) $P(H \cup F) = \frac{9}{36} = \frac{3}{36} + \frac{6}{36} - \frac{0}{36}$

A standard deck of 52 cards has 4 suits, each with 13 cards. The suits are spades, ♠, hearts, ♥, clubs, ♣ ,and diamonds, ♦. The cards in each suit are numbered from Ace, King, Queen, Jack, ten down to 2.



Example

A single card is drawn from a standard deck of cards. What are the probabilities of

a) a 9 or a 10?

$$P = \frac{4}{52} + \frac{4}{52} - \frac{0}{52} = \frac{8}{52}$$

b) a black card or a 3?

$$P = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52}$$

Example

$$\boxed{P(A)+P(A^c)=1}$$

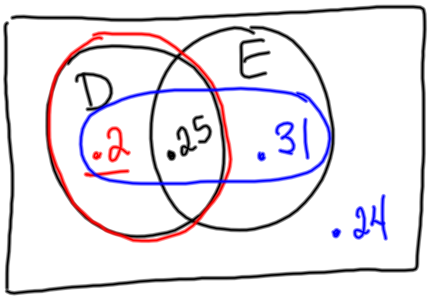
A survey gave the following results: 45% of the people surveyed drank diet drinks and 25% drank diet drinks and exercised and 24% did not exercise and did not drink diet drinks. Find the probability that:

a) A person does not drink diet drinks. $P(D^c)=1-P(D)=1-.45=.55$ or 55%

.31 b) A person exercises and does not drink diet drinks.

.96 c) A person exercises or drinks diet drinks.

$1-.2-.25-.24=.31$



The ***odds in favor*** of an event E are defined to be the ratio of $P(E)$ to $P(E^c)$

The ratio $P(E)/P(E^c)$ is often reduced to lowest terms, $\frac{a}{b}$ and then we say that the ***odds in favor*** of E are ***a:b*** or ***a to b***.

Example

Given the probability of rain is 30%, what are the odds in favor of rain?

$$\frac{P(E)}{P(E^c)}=\frac{.3}{1-.3}=\frac{.3}{.7}=\frac{3}{7}\Rightarrow 3:7 \text{ or } 3 \text{ to } 7$$

If the odds in favor of an event E are a:b, then $P(E)=\dfrac{a}{a+b}$

Example

If a horse has 1:9 odds that it will win, what is the probability the horse will win the race?

$$P=\frac{1}{1+9}=\frac{1}{10}=.1 \text{ or } 10\%$$