1.6 Conditional Probability

A survey is done of people making purchases at a gas station. Most people buy gas (Event A) or a drink (Event B).

	buy drink (B)	no drink (B^c)	total
buy gas (A)			
no gas (A^c)			
total			

What is the probability that a person bought gas and a drink?

What the probability that a person who buys a drink also buys gas? In other words, given that a person bought a drink (B), what is the probability that they bought gas (A)?

Notation: $P(E \mid F) =$ the probability of A given B

The ***conditional probability*** of event E given event F is

What is the probability that a person who buys gas also buys a drink?

The Product Rule:

Example

At a party, 1/3 of the guests are women. 75% of the women wore sandals and 20% of the men wore sandals.

a) What is the probability that a person chosen at random at the party is a man wearing sandals?

b) What is the probability that a randomly chosen guest is wearing sandals?

Example

Consider drawing 3 cards from a standard deck of 52 cards without replacement.
a) What is the probability that the three cards are hearts?
b) What is the probability that the third card drawn is a heart given the first two cards are hearts?

Example
A bag has 3 silver and 4 copper coins. A pouch has 1 silver and 2 copper coins. A coin is drawn at random from the bag and placed in the pouch. A coin is then drawn from the pouch. What is the probability that a silver coin is drawn from the pouch given that a silver coin was chosen from the bag?

Example
A medical test has been developed to detect *xyzzy* disease. It is estimated that 5% of the patients who come in for the test have the disease. When the test is given to a patient who has *xyzzy* disease, it is detected (positive) 90% of the time. When given to a patient who does not have *xyzzy* disease, a positive result is returned 15% of the time. What is the probability that a person has *xyzzy* disease and tests negative?

Independent Events: Events E and F are independent if $P(E\,|\,F) = P(E)$

Example
A medical experiment showed the probability that a new medicine was effective was 0.75, the probability of a certain side effect was 0.4 and the probability for both occurring is 0.3.   Are these events independent?

Example
The side effects of a certain medicine include a 25% chance of headaches and 30% chance of fatigue. What is the probability that a person taking this medicine will suffer exactly one of these side effects if they are independent of each other?

1.7 Bayes' Theorem

Given $P(E \mid F)$, can we find $P(F \mid E)$?

Example
We are to choose a marble from a cup or a bowl. We need to flip a coin to decide to choose from the cup or the bowl. The bowl contains 1 red and 2 green marbles. The cup contains 3 red and 2 green marbles. What is the probability that a marble came from the bowl given that it is red?

Example
A survey of the local middle school found the percent of students in each grade who own a calculator. The results are below. What is the probability that a student with a calculator is in the 5th grade?

Grade	4th	5th	6th
Percent of student body	37	32	31
Percent that own a calculator	13	28	59

Example

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Example

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