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## 1.6 Conditional Probability

A survey is done of people making purchases at a gas station. Most people buy gas (Event $A$) or a drink (Event $B$).

| | buy drink ($B$) | no drink ($B^c$) | total |
|---|---|---|---|
| buy gas ($A$) | 45 | 25 | 70 |
| no gas ($A^c$) | 20 | 10 | 30 |
| total | 65 | 35 | 100 $n(S)$ |

What is the probability that a person bought gas and a drink?

$$P(A \cap B) = 45/100 = \frac{n(A\cap B)}{n(S)}$$

What the probability that a person who buys a drink also buys gas? In other words, given that a person bought a drink ($B$), what is the probability that they bought gas ($A$)?

$$P = \frac{45}{65} = \boxed{\frac{n(A\cap B)}{n(B)} = P(A|B)}$$ in general

***Notation***: $P(E\,|\,F)$ = the probability of $A$ given $B$

The ***conditional probability*** of event $E$ given event $F$ is

$$P(E|F) = \frac{n(E\cap F)}{n(F)} = \frac{n(E\cap F)/n(S)}{n(F)/n(S)} = \boxed{\frac{P(E\cap F)}{P(F)} = P(E|F)}$$ in general

What is the probability that a person who buys gas also buys a drink?

$$P(B\,|\,A) = \frac{n(A\cap B)}{n(A)} = \frac{45}{70}$$

$P(F) * P(E|F) = \frac{P(E \cap F)}{P(F)} * P(F)$



***The Product Rule:***

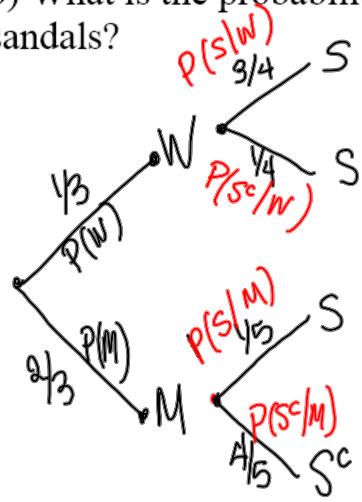
$$P(E \cap F) = P(F) * P(E|F)$$

*Example*

At a party, 1/3 of the guests are women. 75% of the women wore sandals and 20% of the men wore sandals.

a) What is the probability that a person chosen at random at the party is a man wearing sandals? $P(M \cap S) = P(M) \cdot P(S|M)$

b) What is the probability that a randomly chosen guest is wearing sandals?




$P(W \cap S) = \frac{1}{3} * \frac{3}{4} = \frac{3}{12}$

$P(W \cap S^c) = \frac{1}{3} * \frac{1}{4} = \frac{1}{12}$

$P(M \cap S) = \frac{2}{3} * \frac{1}{5} = \frac{2}{15}$

$P(M \cap S^c) = \frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$

$\left.\right\} 1$


$P(S) = P(W \cap S) + P(M \cap S) = \frac{1}{4} + \frac{2}{15} = \frac{8}{15}$

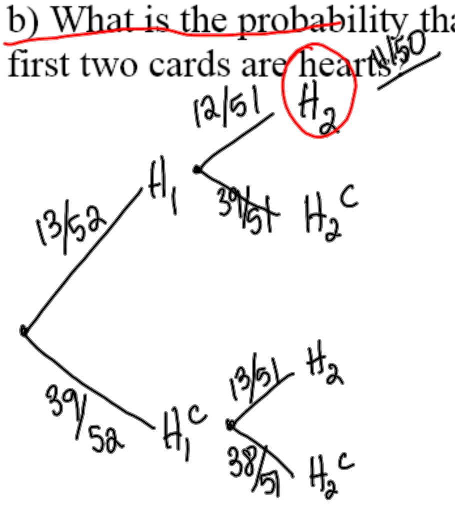
*Example*

Consider drawing 3 cards from a standard deck of 52 cards without replacement.

$P(H_1 \cap H_2 \cap H_3)$

* a) What is the probability that the three cards are hearts?

b) What is the probability that the third card drawn is a heart given the first two cards are hearts?

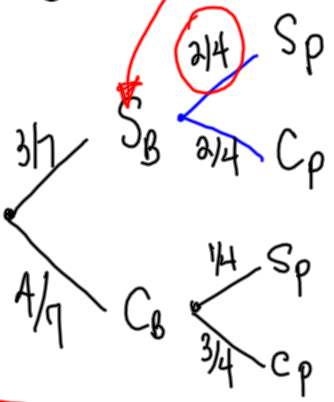


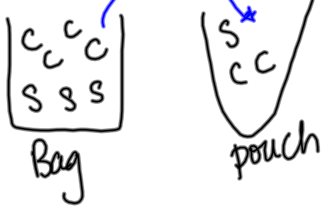

(a) $\frac{13}{52} * \frac{12}{51} * \frac{11}{50} = \frac{11}{850}$

(b) $\frac{11}{50} = P(H_3 | H_1 \cap H_2)$

*Example*

A bag has 3 silver and 4 copper coins. A pouch has 1 silver and 2 copper coins. A coin is drawn at random from the bag and placed in the pouch. A coin is then drawn from the pouch. What is the probability that a silver coin is drawn from the pouch given that a silver coin was chosen from the bag?



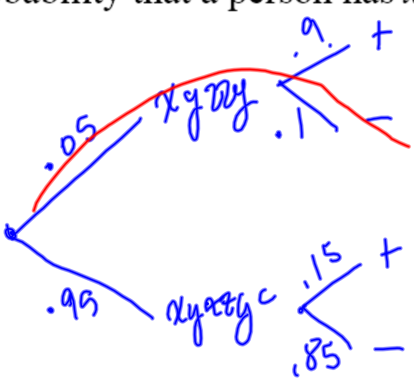




this happened

$$P(S_p \mid S_B) = \frac{2}{4}$$

*Example*

A medical test has been developed to detect *xyzzy* disease. It is estimated that 5% of the patients who come in for the test have the disease. When the test is given to a patient who has *xyzzy* disease, it is detected (positive) 90% of the time. When given to a patient who does not have *xyzzy* disease, a positive result is returned 15% of the time. What is the probability that a person has *xyzzy* disease and tests negative?




$$P(xyzzy \cap -) = .05 \times .1$$
$$= .005$$

***Independent Events:*** Events $E$ and $F$ are independent if $P(E \mid F) = P(E)$

$$\boxed{P(E|F) = \frac{P(E\cap F)}{P(F)} = P(E)} \Rightarrow P(E\cap F) = P(E)\cdot P(F)$$

iff E and F are indep

*Example*

A medical experiment showed the probability that a new medicine was effective was 0.75, the probability of a certain side effect was 0.4 and the probability for both occurring is 0.3. Are these events independent?

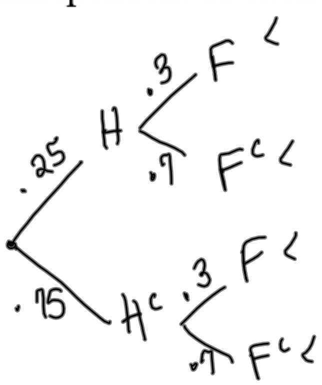
$P(E)$ = prob it is effective = .75

$P(F)$ = prob of side effect = .4

$P(E\cap F) = .3 \stackrel{?}{=} .75 \times .4 = .3 \Rightarrow$ E and F are indep

*Example*

The side effects of a certain medicine include a 25% chance of headaches and 30% chance of fatigue. What is the probability that a person taking this medicine will suffer exactly one of these side effects if they are independent of each other?




$P(H\cap F) = .25 \times .3$

$P(H\cap F^c) = .25 \times .7$

$P(H^c\cap F) = .75 \times .3$

$P(H^c\cap F^c) = .75 \times .7$

$P(H\cap F^c) + P(H^c\cap F)$

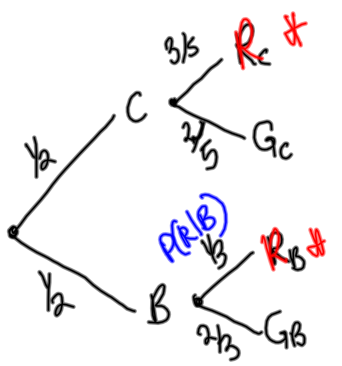
$= .25 \times .7 + (.75) \times .3$

$= .4$

## 1.7 Bayes' Theorem

Given $P(E|F)$, can we find $P(F|E)$? Yes

*Example*
We are to choose a marble from a cup or a bowl. ① We need to flip a coin to decide to choose from the cup or the bowl. The bowl contains 1 red and 2 green marbles. The cup contains 3 red and 2 green marbles. What is the probability that a marble came from the bowl given that it is red?





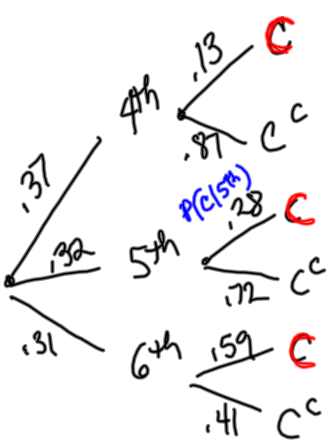
$$P(B|R) = \frac{P(B\cap R)}{P(R)}$$

$$= \frac{P(B\cap R)}{P(B\cap R)+P(C\cap R)} = \frac{(1/2)(1/3)}{(1/2)(1/3)+(1/2)(3/5)}$$

$$=$$

*Example*
A survey of the local middle school found the percent of students in each grade who own a calculator. The results are below. What is the probability that a student with a calculator is in the 5th grade?

| Grade | 4th | 5th | 6th |
|---|---|---|---|
| Percent of student body | 37 | 32 | 31 |
| Percent that own a calculator | 13 | 28 | 59 |



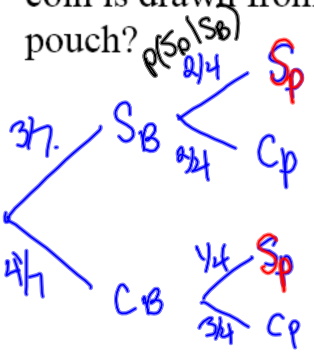

$$P(5^{th}|C) = \frac{P(5^{th}\cap C)}{P(C)}$$

$$= \frac{(.32)(.28)}{(.37)(.13)+(.32)(.28)+(.31)(.59)}$$

$$= \frac{64}{229} \quad (\approx .2795)$$

*Example*

A bag has 3 silver and 4 copper coins. A pouch has 1 silver and 2 copper coins. A coin is drawn at random from the bag and placed in the pouch. A coin is then drawn from the pouch. What is the probability that a silver coin is drawn from the bag given that a silver coin was chosen from the pouch?

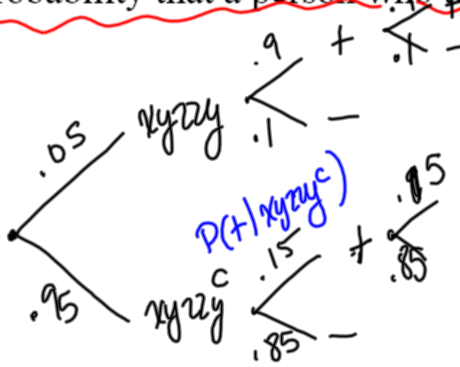



$$P(S_B|S_p) = \frac{P(S_B \cap S_p)}{P(S_p)} = \frac{(3/7)(2/4)}{(3/7)(2/4)+(4/7)(1/4)}$$

$$= \frac{3}{5} = .6$$

*Example*

A medical test has been developed to detect *xyzzy* disease. It is estimated that 5% of the patients who come in for the test have the disease. When the test is given to a patient who has *xyzzy* disease, it is detected (positive) 90% of the time. When given to a patient who does not have *xyzzy* disease, a positive result is returned 15% of the time. What is the probability that a person who tests positively does not have *xyzzy* disease?




$$P(xyzzy^c|+) = \frac{P(x^c \cap +)}{P(+)}$$

$$= \frac{P(x^c \cap +)}{P(x \cap +) + P(x^c \cap +)}$$

$$= \frac{.95 * .15}{.95 * .15 + .05 \times .9} = .76$$