2.1 The Multiplication Principle and Permutations

Example
A fair coin is flipped three times. How many different outcomes are there for this experiment?

Multiplication Principle:
Suppose a task T_1 can be done N_1 ways and a task T_2 can be done N_2 ways and so on until task T_k can be done N_k ways. Then the number of ways of performing the tasks T_1, T_2, ... T_k is given by the product $N_1 \times N_2 \times ... \times N_k$

Factorials: $n! = 1 \times 2 \times 3 \times ... \times (n-1) \times n$. Note that $0! = 1$

Example
How many different license plates are possible where

a) The first characters is a letter, the next two characters are digits and the last three characters are letters?

b) The first two characters are letters, the next character is a digit, the next a letter and the last three are digits?

c) No characters are duplicated in part (b)

Example
How many ways can 10 students be seated in a row of 10 chairs?

Example
How many ways can 4 of 10 students be seated in a row of 4 chairs?

Permutations: If we have a finite set of n elements and we want to place r of them in an arrangement, we say the number of permutations of n things arranged r at a time is $P(n, r)$.

Example
How many ways can gold, silver and bronze medals be awarded in a race of 12 people?

Example
A sales person has 6 prospects. How many ways can she arrange her schedule to see all 6?

Example
You have a group of 15 different books. Five are math books, four are chemistry and six are history books. How many different arrangements are possible if books of the same type are kept together?

Example
How many different 4 digit numbers can be made from the digits
        1, 2, 3, 4, 5, 6, 7

a) If there are no restrictions?

b) If the number must be even?

c) If it is even and there are no repeats?

d) If four of the same digit is not allowed?

2.2 Combinations

Example
How many ways can a **group** of 4 students be chosen from 10 students?

Combinations: If we have a finite set of n elements and we want to take r of them in an group, we say the number of combinations of n things grouped r at a time is $C(n, r)$.

Example
How many ways can a hand of 6 clubs be chosen from a standard deck?

Example
From a group of 12 people, how many ways can a committee of 4 be formed if one person is the chair of the committee?

Example

A bag contains 6 blue, 1 green and 3 pink jelly beans. You choose 3 at random. How many samples are possible in which

a) the jelly beans are all blue?

b) the jelly beans are all green?

c) the jelly beans are all pink?

d) there are 2 blue and 1 pink?

e) How many ways to choose 3 jelly beans?

f) How many ways to choose no blue?

g) How many ways to choose at least one blue?

Example
A class of 12 students will divide into 3 teams of 4. How many ways can this be done?

Example
A school is putting together a committee. The committee will have a chair and an assistant chair chosen from a group of 10 teachers, two parents chosen from a group of 15 parents and two students chosen from a group of 20 students. How many different committees are possible?

Example
You are dealt a hand of four cards from a well-shuffled standard deck of 52 cards.
(a) How many ways can you be dealt at least 3 spades?

(b) How many ways can you be dealt exactly two diamonds or exactly two clubs?

Example
Seven children stop at a restaurant where they have a choice of a cheeseburger, a hot dog, pizza or a burrito. How many different purchases are possible?

2.3 Probability Applications of Counting Principles

Recall that $P(E) = \dfrac{n(E)}{n(S)}$

Example
Suppose we have a jar with 8 blue and 6 green marbles. What is the probability that in a sample of 2, both will be blue?

Find the probability distribution table for the number of blue marbles in the sample of 2 marbles:

What is the probability there is at least one blue marble?

Example

A lottery chooses 6 of 54 numbers be chosen and the order doesn't matter. What is the probability of choosing no winning numbers?

What is the probability of choosing at least 3 winning numbers?

Example

A stack of 100 copies has 3 defective papers. What is the probability that in a sample of 10 there will be no defective papers?

Example

A family has 5 children. What is the probability that exactly 3 of the children are girls?

Example
A box contains 7 red, 6 green, 5 black and 3 purple marbles. What is the
probability that in a sample of 6,

a) The balls are all the same color?

b) There are exactly three red and two purple balls?

c) There are exactly three red or two purple balls?

Example
An exam consists of 25 questions in which exactly 12 must be answered.
What is the probability that a student answered exactly 2 of the first 5
questions and at least 4 of the last 5 questions?

Example
We have 4 identical red marbles, 3 identical green marbles and one blue marble. How many distinguishable arrangements of the 8 marbles are there?

Distinguishable Permutations: If there are n_1 items of type 1 and n_2 items of type 2 and ... n_r items of type r , then the number of distinguishable permutations of the n=$n_1 + n_2 +...+ n_r$ items is

$$\frac{n!}{n_1! \times n_2! \times \ldots \times n_r!}$$

Example
How many "words" can be made from the letters *MANHATTAN*?

2.4   Bernoulli Trials

In a Bernoulli trial we have the following:

- The same experiment repeated several times.
- The only possible outcomes of these experiments are success or failure.
- The repeated trials are independent so the probability of success remains the same for each trial.

Example
A student takes a 3 question multiple choice test by guessing. Each question has 4 possible answers. Find the probability distribution table for the number of correct answers.

Example

During a fifteen day period in the summer, a town had a 20% chance of rain each day. If the rain chances are independent,

a) what is the probability of no rain?

b) what is the probability of at most two rain days?

c) what is the probability of between 2 and 6 rain days?

d) what is the probability of more than 4 rain days?

Example

A new drug being tested causes a serious side effect in 5% of the patients using the drug. What is the probability that in a group of 10 patients,

a) none will get the side effect?

b) all will get the side effect?

c) at least one person will get the side effect?

d) between 1 and 4 people get the side effect?