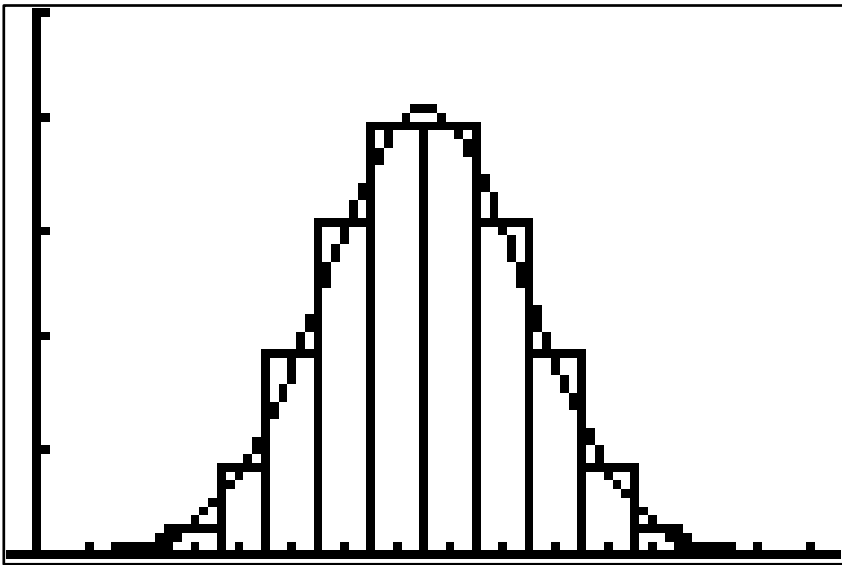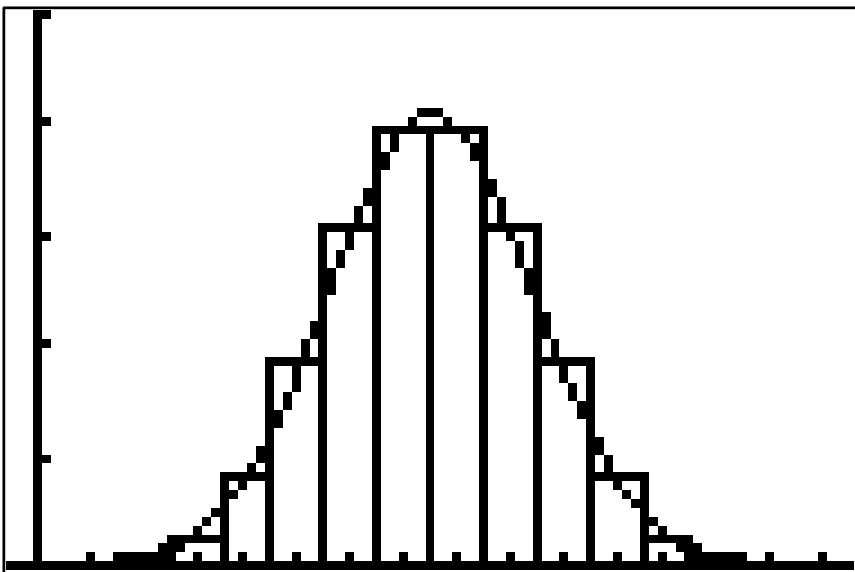## *Example*

Consider tossing a coin 15 times and let X=number of heads.

(a) What is the probability that you toss exactly 5 heads?

(b) What is the probability of more than 9 heads?

THE NORMAL CURVE APPROXIMATION TO THE BINOMIAL DISTRIBUTION

At a school 1000 children are exposed to the flu. There is a 35% chance of getting the flu if you are exposed. Use the normal curve approximation to the binomial distribution to estimate the probability that

(a) more than 360 children get the flu.

(b) fewer than 320 children get the flu.

(c) between 325 and 375 children get the flu.

The Poisson Distribution

The Poisson distribution is an infinite discrete probability distribution that tells you the probability of a certain number of events occurring in a fixed interval of time or space if these events occur with a known average rate and independent of each other.

You should think about the Poisson distribution for any situation that involves counting events such as
- How many cars arrive at a toll booth in an hour?
- How many items are used from an inventory in a week?
- How many red blood cells are in a cc of blood?

These are all events that we can try to find the probability of occurring, but not the probability that they don't occur. To study these we will use the Poisson distribution.

When the Poisson Probability Applies

The Poisson distribution is based on four assumptions. The term "interval" refers to either a time interval or an area.
1. The probability of observing a single event over a small interval is approximately proportional to the size of that interval.
2. The probability of two events occurring in the same narrow interval is negligible.
3. The probability of an event within a certain interval does not change over different intervals.
4. The probability of an event in one interval is independent of the probability of an event in any other non-overlapping interval.

When the Poison probability law will apply, there is a number λ which is the average rate that the events occur.

If the Poisson distribution applies, then the probability of x occurrences per unit measure is approximately

$$P_\lambda(x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

with a mean of $\mu = \lambda$ and $\sigma = \sqrt{\lambda}$

Example: Suppose on average there are 16 emergency patients on the 8A to 4P shift of a certain hospital. What is the probability that during any one hour of the shift that
a) zero patients arrive?
b) at most one patient?
c) more than one patient?
d) draw the histogram for x = 0 to 6

Example: On average 90 hamburgers are sold during the lunch hour at a fast food restaurant. What is the probability that during a certain minute of the lunch hour that
a) zero hamburgers will be sold?
b) at most two hamburgers will be sold?
c) more than 3 hamburgers will be sold?

The binomial distribution can be approximated by the Poisson distribution if p, the probability of success in a single trial, is small [generally less than 0.1]. In that case, using $\lambda = np$ will be a good approximation.

Example: The probability of theft on a subway is given as 0.001. What is the probability that 10 of the next 5000 passengers will be robbed? Use both the binomial and Poisson distributions.
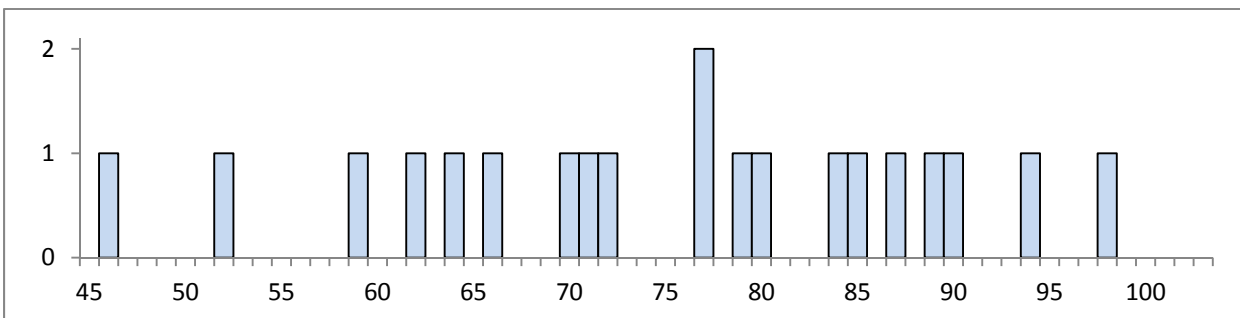
Example
An exam has a mean of $\mu=75$ and a population standard deviation of $\sigma=14$.
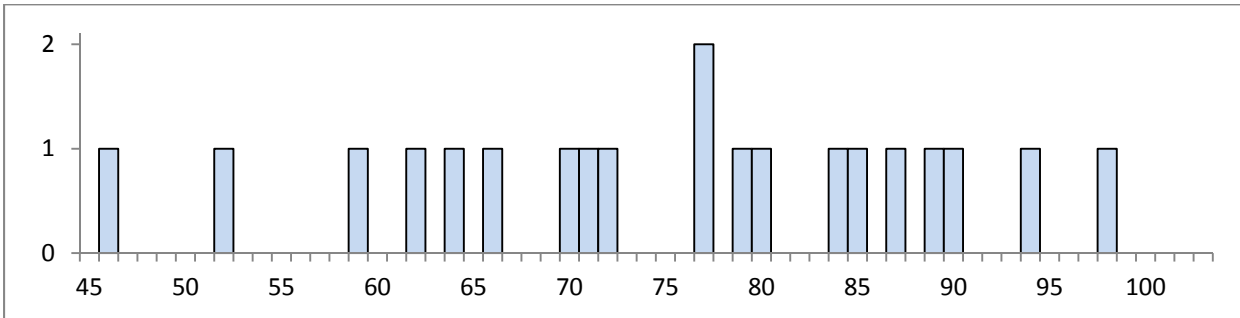
What is the probability that a randomly chosen data point is within 1 standard deviation of the mean?

What is the probability that a randomly chosen data point is within 2 standard deviations of the mean?

Chebychev's Theorem: For **any** data distribution with mean μ and standard deviation σ, the probability that a randomly chosen data point is within $\mu - k \cdot \sigma$ to $\mu + k \cdot \sigma$ is at least $1 - 1/k^2$. Or,

$$P(\mu - k \cdot \sigma \leq x \leq \mu + k \cdot \sigma) \geq 1 - \frac{1}{k^2}$$

Example

A probability distribution has a mean of 50 and a standard deviation of 5.

a) What is the probability that an outcome of the experiment lies between 35 and 65?

b) Find the value of k so that at least 93.75% of the data lies in the range 50-5k to 50+5k.