

LOGIC

A *proposition* or *statement* is a declarative sentence that can be classified as either true or false but not both.

Examples

- San Francisco is the capital of California.
- Be quiet!
- Texas is east of California or it is west of California.
- When is the next exam?
- The Exam 1 results were good.
- $x + 1 = 4$
- Miami Beach has 10^{10} grains of sand.

A *prime* or *simple* proposition expresses one thought

Join propositions with *logical connectives* to form compound propositions.

conjunction

negation

disjunction

p : San Francisco is the capitol of California.

q : Austin is the capitol of Texas.

What is $p \wedge q$ in words? Is this proposition true or false?

In general,

Show this in a *truth table*,

p	q	$p \wedge q$
T	T	
T	F	
F	T	
F	F	

p : San Francisco is the capitol of California.

q : Austin is the capitol of Texas.

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In general,

p	q	$p \vee q$
T	T	
T	F	
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This is called the *inclusive disjunction*. This is also the mathematical *or*.

Exclusive disjunction is $\underline{\vee}$. This is true only if exactly one of the two statements is true.

p : San Francisco is the capitol of California.

q : Austin is the capitol of Texas.

What is $p \underline{\vee} q$ in words? Is this proposition true or false?

r : Sacramento is the capitol of California.

What is $r \underline{\vee} q$ in words? Is this proposition true or false?

Show this in a *truth table*,

p	q	$r \underline{\vee} q$
T	T	
T	F	
F	T	
F	F	

Negation: $\sim p$ means not p . Show this in a truth table:

p	$\sim p$
T	F
F	T

Write the following statements symbolically and find the truth table.

The car is blue or has a moon roof.

p :

q :

The book is not red and the subject is history.

p :

q :

The sky is not blue or the grass is not purple.

p :

q :

Define the following statements:

p : The student is a girl.

q : The student is a biology major.

r : The student is enrolled in a math class.

Write the following statements symbolically and find the truth table.

The student is a boy and is not a biology major or enrolled in a math class.

A statement is a *contradiction* if the truth value of the statement is always false.

Example: Find the truth table for $p \wedge \sim p$

A statement is a *tautology* if the truth value of the statement is always true.

Example: Find the truth table for $p \vee (\sim p \vee q)$