

## LOGIC

A *proposition* or *statement* is a declarative sentence that can be classified as either true or false but not both.

### Examples

- San Francisco is the capital of California. *Statement  $\Rightarrow$  false*
- Be quiet! *Not a statement*
- Texas is east of California *or* it is west of California. *statement*
- When is the next exam? *Not a statement*
- The Exam 1 results were good. *define good then its a statement*
- $x + 1 = 4$  *define  $x$ , then its a statement*
- Miami Beach has  $10^{10}$  grains of sand. *statement*

A *prime* or *simple* proposition expresses one thought

Join *prime or simple* propositions with *logical connectives* to form compound propositions.

*conjunction*      $\wedge$      *and*

*negation*         $\sim$         *not*

*disjunction*     $\vee$         *or*

$p$ : San Francisco is the capitol of California. **F**

$q$ : Austin is the capitol of Texas. **T**

What is  $p \wedge q$  in words? Is this proposition true or false?

SF is the cap of CA and Austin is the cap of TX

In general,

The conjunction  $p \wedge q$  is true only if both  $p$  and  $q$  are true

Show this in a *truth table*,

$p$	$q$	$p \wedge q$
T	T	<b>T</b>
T	F	<b>F</b>
F	T	<b>F</b>
F	F	<b>F</b>

$p$ : San Francisco is the capitol of California.

$q$ : Austin is the capitol of Texas.

SF is the cap of CA  
or Austin is the  
cap of TX

What is  $p \vee q$  in words? Is this proposition true or false?

**TRUE**

In general,

The disjunction  $p \vee q$  is false only if both  $p$  and  $q$  are false

$p$	$q$	$p \vee q$
T	T	<b>T</b>
T	F	<b>T</b>
F	T	<b>T</b>
F	F	<b>F</b>

This is called the *inclusive disjunction*. This is also the mathematical *or*.

**Exclusive disjunction** is  $\underline{\vee}$ . This is true only if exactly one of the two statements is true.

$p$ : San Francisco is the capitol of California.

$q$ : Austin is the capitol of Texas.

What is  $p \underline{\vee} q$  in words? Is this proposition true or false?

*Either SF is the cap of CA or Austin is the cap of TX*

*TRUE*

$r$ : Sacramento is the capitol of California. *(True)*

What is  $r \underline{\vee} q$  in words? Is this proposition true or false?

*Either Sac is the cap of CA or Austin is the cap of TX*  
*False*

Show this in a **truth table**,

$p$	$q$	$r \underline{\vee} q$
T	T	<del>F</del>
T	F	T
F	T	T
F	F	F

Negation:  $\sim p$  means not  $p$ . Show this in a truth table:

$p$	$\sim p$
T	F
F	T

Write the following statements symbolically and find the truth table.

The car is blue or has a moon roof.

$p$ : The car is blue

$q$ : The car has a moon roof

$p$	$q$	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The book is not red and the subject is history.

$p$ : The book is red

$q$ : The subject is history

$p$	$q$	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

The sky is not blue or the grass is not purple.

$p$ :

$q$ :

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Define the following statements:

$p$ : The student is a girl.

$q$ : The student is a biology major.

$r$ : The student is enrolled in a math class.

Write the following statements symbolically and find the truth table.

The student is a boy and is not a biology major or enrolled in a math class.

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A statement is a *contradiction* if the truth value of the statement is always false.

Example: Find the truth table for  $p \wedge \sim p$

A statement is a *tautology* if the truth value of the statement is always true.

Example: Find the truth table for  $p \vee (\sim p \vee q)$