

- 1 a) Regular - a transition matrix & no zeros
- b) Not a TM - col 2 > 1
- c) Not regular - an absorbing TM

2. a)

$$\begin{array}{c} \text{IN} \\ \text{OUT} \end{array} \begin{bmatrix} \text{IN} & \text{OUT} \\ .5 & .65 \\ .5 & .35 \end{bmatrix} \text{ (OR)} \begin{array}{c} \text{OUT} \\ \text{IN} \end{array} \begin{bmatrix} \text{OUT} & \text{IN} \\ .35 & .5 \\ .65 & .5 \end{bmatrix}$$

$$\text{b) } T^2 X_0 = \begin{bmatrix} .5 & .65 \\ .5 & .35 \end{bmatrix}^2 \begin{bmatrix} .25 \\ .75 \end{bmatrix} = \begin{bmatrix} .558125 \\ .441875 \end{bmatrix} \begin{array}{c} \text{IN} \\ \text{OUT} \end{array}$$

44.1875% chance of out of state vacation

$$\text{c) } TX_L = X_L$$

$$\begin{bmatrix} .5 & .65 \\ .5 & .35 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{array}{l} .5x + .65y = x \\ .5x + .35y = y \end{array}$$

$$\Rightarrow \begin{array}{l} -.5x + .65y = 0 \\ .5x - .65y = 0 \\ x + y = 1 \end{array} \xrightarrow{\text{REF}} \begin{array}{l} x = 13/23 \approx .5652 \\ y = 10/23 \approx .4348 \end{array}$$

Long term is 13/23 probability of in-state ($\approx 56.52\%$) and 10/23 probability of out of state ($\approx 44.48\%$) vacation

$$3. \begin{bmatrix} I & S \\ 0 & R \end{bmatrix} \Rightarrow \begin{array}{cc|cc} & \text{Paid} & \text{Bad} & & \\ \text{Paid} & 1 & 0 & \frac{1}{3} & \frac{1}{6} \\ \text{Bad} & 0 & 1 & 0 & \frac{1}{6} \\ \hline <30 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ <60 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{array}$$

$$\begin{array}{c} [A] \\ [B] \end{array} S(I-R)^{-1} = \begin{array}{cc} & \begin{array}{c} <30 & <60 \end{array} \\ \text{Paid} & \begin{bmatrix} \frac{5}{6} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{3} \end{bmatrix} \\ \text{Bad} & \end{array}$$

If <30 , $\frac{5}{6}$ th chance it is paid off and $\frac{1}{6}$ it is bad

If <60 , $\frac{2}{3}$ chance it is paid off and $\frac{1}{3}$ it is bad

$$F = (I-R)^{-1} = \begin{array}{cc} & \begin{array}{c} <30 & <60 \end{array} \\ \begin{array}{c} <30 \\ <60 \end{array} & \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \end{array}$$

Starting in <30 , spend ≈ 2 turns <30 and 1 turn in <60 . So about 3 turns before absorbed

Starting in <60 , spend ≈ 1 turn in <30 , 2 turns in <60 . About 3 turns before absorbed