

## 4.3-4.4 Systems of Equations

A linear equation in 2 variables is an equation of the form  $ax + by = c$ .

A linear equation in 3 variables is an equation of the form  $ax + by + cz = d$ .

To solve a system of equations means to find the set of points that satisfy EVERY equation in the system.

A solution to a system of linear equations with 2 variables will be an ordered pair  $(e, f)$ .

A solution to a system of linear equations with 3 variables will be an ordered triple  $(e, f, g)$ .

The same ideas are true for 4 variables, etc.

We can use algebraic methods to solve systems of equations, such as substitution and elimination.

Solve the following system of equations algebraically.

$$x - 2y = -8$$

$$3x + y = -3$$

We would say this system of equations has 1 unique solution.

When solving any system of linear equations, there are ONLY 3 possibilities for the number of solutions:

1. Exactly One Solution (called a unique solution).
2. No Solution (This is called an *inconsistent* system.)
3. Infinitely Many Solutions (This is called a *dependent* system.)

Solving a system of linear equations means finding the points of intersection. Think about two lines: They will either intersect at one point (unique solution), at no points (parallel lines), or at infinitely many points (same line).

Instead of using algebra, we will learn a way to solve systems of equations using matrices and the calculator.

A **matrix** is just a rectangular array of numbers.

We can represent a system of equations with an **augmented matrix**. An augmented matrix is a short-hand way of representing a system without having to write the variables.

When writing an augmented matrix from a system of equations, line up all the variables on one side of the equal sign with the constants on the other side. Then, form a matrix with the coefficients and constants in the equations.

Examples: Represent the following systems of equations with an augmented matrix.

$$\begin{aligned} -3x + 2y - 6z &= 14 \\ 4x - 5y + 2z &= -3 \\ 7x + 9y + 3z &= 14 \end{aligned}$$

$$\begin{aligned} 2x - 4y &= 9 \\ -2y - z &= 0 \\ x + y + 4 &= 2z \\ 3y + z &= 5x \end{aligned}$$

If a matrix has  $m$  rows and  $n$  columns, we say the size of the matrix is  $m \times n$ . What are the sizes of the above matrices?

In order to solve a system, we need to “reduce” the matrix to a form where we can identify the solution. This form is called “reduced row echelon form.” It is equivalent to the original system, just simplified.

The calculator command *rref* will reduce the matrix for you without having to do algebra.

To enter a matrix into your calculator and find the row-reduced form:

- Press 2nd  $x^{-1}$ . (The TI-83 has a separate MATRX button.)
- Cursor over to EDIT and select where you want to store your matrix. Enter the size of the matrix and then enter in the matrix values.
- Go back to your home screen. Press MATRX and then cursor right to MATH. Scroll down and select *B:rref*.
- Select the matrix you want to reduce by pressing MATRX and then choosing your matrix under the NAMES column. Then close your parentheses and press enter. Note: The *rref* command only works when the number of rows is less than or equal to the number of columns.

Once you have the reduced form of the matrix, you can determine the solution by rewriting each row back as an equation.

Solve the first system from above:

$$\left[ \begin{array}{ccc|c} -3 & 2 & -6 & 14 \\ 4 & -5 & 2 & -3 \\ 7 & 9 & 3 & 14 \end{array} \right]$$

Solve the second system from above:

$$\left[ \begin{array}{ccc|c} 2 & -4 & 0 & 9 \\ 0 & -2 & -1 & 0 \\ 1 & 1 & -2 & -4 \\ -5 & 3 & 1 & 0 \end{array} \right]$$

Solve the following systems:

$$x + 2y = 2$$

$$-3x + 3y = -1$$

$$7x - 13y = -1$$

$$-x + 2y + 3z = 14$$

$$2x - y + 2z = 2$$

$$-x + 5y + 11z = 44$$

When a system has infinitely many solutions, we **parameterize** the solution (write the solution in parametric form):

A particular solution is a specific solution to the system. What are some particular solutions for this system?

Examples: The following reduced matrices represent systems of equations with variables  $x, y, z$ , and, if necessary,  $u$ . Determine the solutions to these systems.

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & -9 \\ 0 & 0 & 1 & 0 & 8 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 2 & 0 & -5 \\ 0 & 1 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 2 & -1 & 6 \\ 0 & 1 & -3 & 4 & 1 \end{array} \right]$$

Systems of equations are used to model problems. For the following examples, set up a system of equations to solve the problem and solve. ALWAYS DEFINE YOUR VARIABLES when setting up a system of equations.

Ben has a total of \$20,000 invested in two municipal bonds that have yields of 7% and 11% simple interest per year, respectively. If the total interest he receives from the bonds in 1 year is \$1880, how much does he have invested in each bond?

A theater has a seating capacity of 900 and charges \$2.50 for children, \$4 for students, and \$5.50 for adults. At a certain screening with full attendance, there were half as many adults as children and students combined. The total money brought in was \$3825. How many children, students, and adults attended the show?

A sporting goods store sells footballs, basketballs, and volleyballs. A football costs \$35, a basketball costs \$25, and a volleyball costs \$15. On a given day, the store sold 5 times as many footballs as volleyballs. They brought in a total of \$3750 that day, and the money made from basketballs alone was 4 times the money made from volleyballs alone. How many footballs, basketballs, and volleyballs were sold? Just set up the problem.

(Problem #40, Section 4.3) A furniture company makes loungers, chairs, and footstools out of wood, fabric, and stuffing. The number of units of each of these materials needed for each of the products is given in the table below. How many of each product can be made if there are 1110 units of wood, 880 units of fabric, and 660 units of stuffing available? Just set up the problem.

	Wood	Fabric	Stuffing
Lounger	40	40	20
Chair	30	20	20
Footstool	20	10	10

If a system of equations arising from a word problem has a parametric solution, sometimes there really aren't infinitely many solutions, since only some of them will make sense in the problem.

For example, if  $x$  represents the number of children that were at a movie, we would know that  $x \geq 0$  and that  $x$  must be a whole number [an integer].

(Problem #49, Section 4.4) A person has 36 coins made up of nickels, dimes, and quarters. If the total value of the coins is \$4. how many of each type of coin does this person have?

An instructor wants to write a quiz with 9 questions where each question is worth 3, 4, or 5 points based on difficulty. He wants the number of 3-point questions to be 1 more than the number of 5-point questions, and he wants the quiz to be worth a total of 35 points. How many 3, 4, and 5 point questions could there be?