①

Country	POP	q	⌊q⌋	N	q	⌊q⌋	
M	40650	40.2435	40	40	40.65	40	41
Q	38650	38.2635	38	38	38.65	38	38 **+1 = 39**
W	10400	10.296	10	11	10.4	10	10
G	10300	10.197	10	10	10.3	10	10
total	100,000			98	99		98

$S = 100000/99 = 1010.1010$ $S = 100000/100 = 1000$

Alabama paradox — W lost a seat even though the
number of seats increased

②

Language	POP	q	⌊q⌋	N
E	945	9.416	9	10
K	9055	90.22	90	90	← lost a section — New
V	1040	10.36	10	10	States Paradox
TOTAL	11040		109	

$S = 11040/110 = 100.4$

③

Person	Σ	q	⌊q⌋	d⁺
A	1320	26.4	26	1320/27 = 48.889
B	1515	30.3	30	1515/31 = 48.871
C	4935	(98.7)	98	4935/99 = 49.848 ← gets one. use as d
L	1118	22.36	22	1118/23 = 48.609
Y	1112	22.24	22	1112/23 = 48.348
	10,000		198	

quota
98 or ←
99

$S = 10,000/200 = 50$ 2 to give

	q		⌊q⌋	d⁺	
A	1320/ 48.848 = 26.48		26	1320/27 = 48.889	
B	1515/ " = 30.39		30	1515/31 = 48.871	violates
C	4935/ " = 99		99	4935/100 = 49.35 (+1)	quota rule
L	1118/ " = 22.43		22	1118/23 = 48.61	
Y	1112/ " = 22.31		22	1112/23 = 48.35	

④

SUBJECT	#	$\bar{g}$	web [g]	g^*	N	d^-	
1ˢᵗ	84	3.9375	4	$\sqrt{3x4} = 3.46$	4	$84/\sqrt{3x4} = 24.25$	
2ⁿᵈ	34	1.5938	2	$\sqrt{1x2} = 1.41$	2	$34/\sqrt{1x2} = 24.04$	(-1)
3rd	10	.46875	0	$\sqrt{1x0} = 0$	1	$10/\sqrt{1x0} = \infty$	
	128		6		7		
			OK				

$S = 128/6 = 21.3333$

⑤ a) total of 9 votes → 5 for a majority = A wins

b) 4 locations so 4pts 1ˢᵗ place, 3 pts 2nd place, etc

A has $5 \times 4 + 0 \times 3 + 2 \times 2 + 2 \times 1 = 26$ points

L has $3 \times 4 + 4 \times 3 + 2 \times 2 + 0 \times 1 = 28$ points ⟹ WINS

N has $0 \times 4 + 3 \times 3 + 3 \times 2 + 3 \times 1 = 18$ points

P has $1 \times 4 + 2 \times 3 + 2 \times 2 + 4 \times 1 = 18$ points

⑥ a)

H vs J   12 vs 14+7+6 ~~+7~~ = 27   J wins

H vs K   12 vs 27   K wins

H vs M   12 vs 27   +7   M wins

J vs K   12+6 = 18 vs 14 ~~+7~~ = 21   K wins

J vs M   12+7+6 = 25 vs 14 ~~+7+6~~   J wins

K vs M   12+7 = 19 vs 14+6 ~~+7~~ = 20   M wins

} no pairwise winner

b) Tournament

H̲ ̲J̲,K M → J̲ ̲K̲,M → M wins
   J   K

K̲ ̲M̲,J H → M̲ ̲J̲ H → J wins
   M   J

H̲ ̲M̲,J K → M̲ ̲J̲,K → K wins
   M   J

c) Hare method. Total of 39 votes. Need 20 to win

H has 12 1ˢᵗ, M has 14 1ˢᵗ place, K has 7, J has 6 → elim J

(HKM)	(MKH)	(KMH)	(MKH)	(MKH)	
12	10	7	6	4	⟹ M wins

① Athena 51 #3

Hera 50

Poseidon 55 #2

Zeus 63 #1

Medusa 1

⑧

OBJECT	S	L
Bike	25	(40)
Barbells	(25)	10
Row	(15)	10
Xbox	15	(18)
Books	20	(22)
sum	40	80
	B	A

ratio A/B
40/25 = 1.6

18/15 = 1.2
22/20 = 1.1

→ lowest → Keep x, give back $1-x$

Lucy's   Sandy's
$40 + 18 + 22x = 25 + 15 + 20(1-x)$
$58 + 22x = 40 + 20 - 20x$
$42x = 2 \Rightarrow x = 2/42 \approx .0476$

⇒ Sandy gets the barbells, rowing machine and 95% of
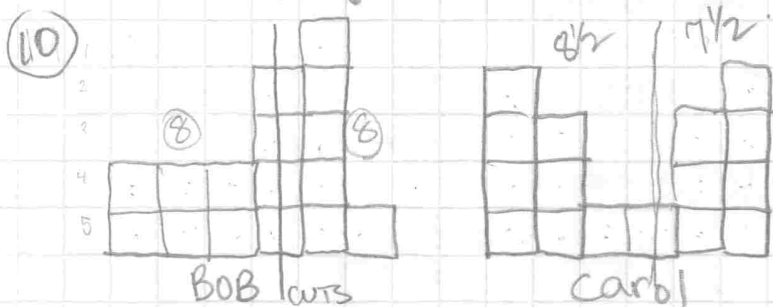the book. Lucy gets the bike, the xbox and 5% of the books

⑨ John gets the painting. His value is $25,200/3 = 8400$
so he puts $25200 - 8400 = 16800$ in a kitty.

Ken values the painting $21600, So his share is
$21,600/3 = 7200$ which he takes from the kitty

Linda values the painting $18000. So her share is
$18000/3 = 6000$ which she takes from the kitty.

$16800 - 7200 - 6000 = 3600$ left. Split 3 ways

⇒ John gets the painting and it costs him $16800 - 1200$
$= \$15,600$. He pays Ken $7200 + 1200 = \$8400$ and
he pays Linda $6000 + 1200 = \$7200$.

⑩

BOB cuts   Carol

Carol picks left half with
value of 8½. Bob keeps
right half with value 8

11. Cake has 12 units, do player 2 will divide 2 1/3 from the left and 2 1/3 from the right. This produces, in player 2's opinion, 3 pieces with value of 4. Player 1 takes the left piece and gets 5 2/3 units of cake, Player 3 takes the right piece and gets 6 2/3 units of cake. Player 2 gets the middle piece with value of 4.
12. Kelly will pick Spiderman if she picks first.
    Jan will pick Spiderman if she picks first.