Majority Rule: Each voter votes for one candidate. The candidate with the majority of the votes wins.

When there are *n* votes, the majority is $\frac{n}{2}+1$ [*n* even] or $\frac{n+1}{2}$ [*n* odd]

Plurality Method: Each voter votes for one candidate. The candidate with the most votes is the winner.

Borda Count: Each voter ranks the *n* candidates with *n* points assigned to the first choice, *n*–1 to the second choice and so on. The candidate with the most points wins.

Runoff Election: If there is no majority, another vote is taken after eliminating one or more of the candidates. The top two candidates could be in the runoff, or the Hare Method could be used.

Hare Method: If there is no majority winner, then the candidate with the fewest number of first place votes is eliminated. The election is held again and if no majority winner is found, the candidate with the fewest first place votes is eliminated and the election held again. Repeat until a majority winner is found.

Pairwise Comparison Method: Each voter ranks the candidates. Each candidate is compared to each of the other candidates and the candidate who is preferred gets one point. The candidate with the most points wins.

Tournament Method: Compare the entire slate of candidates two at a time, in a pre-determined order. The candidate with the fewest votes is eliminated and the winner goes on to compare with the third candidate. These pairwise comparisons continue until a winner is found.

Approval Method: Each voter votes for all the candidates they approve of. The candidate with the most votes wins.

FAIRNESS CRITERIA:

- **Majority**: If a candidate receives a majority of the first place votes, then that candidate should be declared the winner.
- **Condorcet**: If a candidate is favored when compared one-on-one with every other candidate, then that candidate should be declared the winner.
- **Monotonicity**: A candidate who wins a first election and then gains additional support without losing any of the original support should also win a second election.
- **Irrelevant Alternatives**: If a candidate is declared the winner of an election and in a second election one or more of the candidates is removed, then the previous winner should still be declared the winner.

Note that there may be a tie. With two candidates and an even number of votes, it is possible that each received $n/2$ votes. The method to break the tie should be in place before the election!

Ways to break a tie:
Flip a coin
Use the number of first place votes.
Introduce a new voter [the Senate uses the VP].

Example: Consider an election for Chief with 3 candidates, X, Y and Z. There were 12 voters and the voters were allowed to rank their choices for Chief. The results were

Choices	XYZ	XZY	YXZ	YZX	ZYX	ZXY
# votes						

Who won? A majority requires

Borda Count: 1st place votes = 3 points, 2nd place votes = 2 points, 3rd place votes = 1 point.

X has _____ (3 pts) + _____ (2 pts) + _____ (1 pt) = _____ points

Y has _____ (3 pts) + _____ (2 pts) + _____ (1 pt) = _____ points

Z has _____ (3 pts) + _____ (2 points) + _____ (1 pt) = _____ points

Heisman trophy uses Borda count. Baseball MVP uses modified Borda count with a first place vote worth 14, 2nd place is 9, 3rd place is 8 …. 10th place is 1.

Hare Method: Eliminate the candidate with the fewest first place votes. Move their 2nd place vote to first place. Here we would eliminate Y.

Choices	XYZ	XZY	YXZ	YZX	ZYX	ZXY
# votes						

X has _____ 1st place votes and Z has _____ 1st place votes so _____ wins with the Hare Method.

Pairwise Comparison Method:

X over Y: _____ Y over X: _____ _____ gets a point

X over Z: _____ Z over X: _____ _____ gets a point

Y over Z: _____ Z over Y: _____ _____ gets a point

Tournament Method: There are 3 ways to set up the tournament.

(X vs. Y) winner vs. Z gives (_____ vs. _____) so _____ advances.

(____ vs. Z) is (____ vs. ____) so _____ wins.

(X vs. Z) winner vs. Y gives (_____ vs. _____) so _____ advances.

(____ vs. Y) is (_____ vs. _____) so ____ wins

(Y vs. Z) winner vs. X gives (_____ vs. _____) so _____ advances.

(____ vs. X) is (_____ vs. _____) so ____ wins

Approval Method: Voters mark all the options they find acceptable. The option chosen most often wins.

Example: A family is deciding what to serve for dinner on Saturday and Sunday. Mom draws up a list and the votes are

	Mom	Dad	Boy	Girl 1	Girl 2	Total
Liver and Onions						
Lamb Stew						
Fish Sticks						
Fried Chicken						
Hamburgers						
Spaghetti						

VOTING DILEMMAS:

1. Majority Criterion: An election had the following results

XYZ	YZX	ZYX
_____	_____	_____

There were 3 rankings with no votes and a total of 31 voters.

X: _____ (3 points) + _____ (2 points) + _____ (1 point) = _____

Y: _____ (3 points) + _____ (2points) + _____ (1 point) = _____

Z: _____ (3 points) + _____ (2 points) + _____ (1 point) = _____

2. Condorcet Criterion: An election had the following results

DABC	ACBD	BCAD	CBDA	CBAD
_____	_____	_____	_____	_____

Pairwise Winner? Make a table,

	A	B	C	D
A				
B				
C				
D				

A vs. B is _____vs. _____so _____wins.

A vs. C is _____vs. _____so _____wins.

A vs. D is _____vs. _____so _____wins.

B vs. C is _____vs. _____so _____wins.

B vs. C is _____vs. _____so _____wins.

C vs. D is _____vs. _____so _____wins.

_____is the pairwise [Condorcet] winner.

Who is the majority winner? Who is the plurality winner?

DABC	ACBD	BCAD	CBDA	CBAD
_____	_____	_____	_____	_____

Borda count: 1st place is 4 points, 2nd place is 3 points, 3rd place is 2 points and 4th place is 1 point.

A = _____, B = _____, C = _____, and D = _____

_____ is the winner by Borda count.

Hare method: Eliminate candidate with the fewest 1st place votes. That is _____ in this election.

DABC	ACBD	BCAD	CBDA	CBAD
_____	_____	_____	_____	_____

___ has _____1st place votes,

___ has _____1st place votes and

___ has _____1st place votes.

___ is the winner

Pairwise method: Use the table and

A has ___ points, B has ___ points,

C has ___ points, and D has ___ points.

___ wins. This method will never violate the Condorcet criteria.

3. Monotonicity Criteria: Straw vote and then a binding vote. IOC voting for the Winter Olympics to be held in Quebec (Q), Salt Lake City (L), Ostersund (T) or Sion (S). A total of 87 votes [fictional].

Day 1

TLSQ	LQTS	QSTL	TQSL	TSLQ
___	___	___	___	___

Is there a majority winner? Need _____ votes. Use the Hare method.

T has ___ L has ___ Q has ___ S has ___

Eliminate ___ and still no majority. Eliminate ___.

Now ___ has _____and ___ has _____.

___ wins on the first day.

That night the Salt Lake City reps convince the ___ people who voted for ___ to move ___ to the top of their list.

Day 2

TLSQ	LQTS	QSTL	QTSL	QTSL
___	___	___	___	___

T has ___ 1st place votes L has ___ 1st place votes Q has ___ 1st place votes _____ eliminated

Next ___ is eliminated and we have

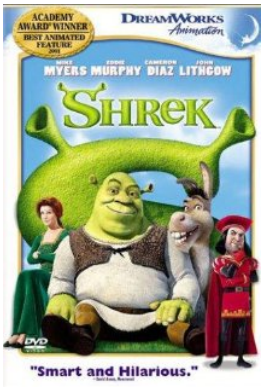
___ with _____1st place votes and ___ has _____ 1st place votes and _____ wins.

4. Irrelevant Alternatives Criteria: Have 5 candidates and votes

(BDCEA) (BDEAC) (EDABC) (ACEBD) (DECBA) (CBDEA) (CEDBA)

SINGLE TRANSFERABLE VOTE

A group of 20 people want to choose 3 movies to watch from a set of 5 movies. Each person could list their top 2 picks. The movies are



	1st choice	2nd choice
4 votes were	X-men	Shrek
2 votes were	Shrek	X-Men
8 votes were	Iron Man	Up
4 votes were	Iron Man	The Dark Knight
1 vote was	Up	
1 vote was	The Dark Knight	

What movies do they watch?